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PREFACE: SPECIAL ISSUE ON QUATERNION OPTIMIZATION AND DUAL QUATERNION OPTIMIZATION

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Quaternions, proposed by Hamilton in 1843, have been used in computer graphics, computer vision, robotics, control theory, signal processing, attitude control, physics, bioinformatics, molecular dynamics, computer simulation, orbital mechanics and many other fields; while dual quaternions, introduced by Clifford in 1873, have also been applied in many fields in recent years, including computer graphics, robotics, optimal control system, etc. The advantages of the quaternion or dual quaternion representation/parametrization among the involved application problems have been well exploited. This triggers the study on quaternion optimization and dual quaternion optimization. Existing works along this line are mostly tailored for specific motivating application problems, lacking of the systematic research on the basic theory and algorithms of quaternion optimization and dual quaternion optimization. Challenges come from the more difficult and complicated algebraic structures of the underlying quaternion division ring and the dual quaternion group, comparing to the traditional optimization in the real or complex field. Efforts are in great need for these types of optimization problems, in both theory and algorithms, to promote the quaternion- and dual quaternion-based computation and applications in the real world.

This special issue aims to attract original research results in theory and algorithms of quaternion optimization and dual quaternion optimization, and their applications. A total of 9 papers accounted below, which were reviewed according to the usual high standards of the journal, cover a wide range of theoretical, practical, and applied topics in quaternion optimization and dual quaternion optimization.

R. Xu and Y. Wei [1] introduce a two-sided random orthogonalization decomposition named quaternion matrix UTV (QUTV) decomposition to replace the QSVD in some applications of quaternion matrix optimization. The compressed randomized QUTV algorithm is simple and accurate in color image reconstruction. The blockrandomized QUTV algorithm outperforms QSVD in terms of efficiency and accuracy in the problem of eigenvalues. The QUTV decompositions are theoretically reliable and the numerical examples in quaternion matrix optimization verify their claims.

C. Li, Y. Chen and D. Li [2] consider dual quaternion optimization with applications in hand-eye calibration. To solve optimization of real function in dual quaternion variables, they utilize eight-dimensional vectors to represent dual quaternions. Then the unit dual quaternion constraint is turned into two quadratic equations. They derive an algorithm for computing the projection of dual quaternions onto a set of unit dual quaternions, and further propose a proximal linearized algorithm for optimization of real function in dual quaternion variables. The global convergence of the proximal linearized algorithm is analyzed. Finally, numerical experiments on hand-eye calibration problems and dual quaternion-based regressions illustrate the effectiveness of the proposed algorithm.

C. Ling, H. He, L. Qi and T. Feng [3] first address the concept of spectral norm of dual quaternion matrices. Then, they introduce a von Neumann type trace inequality and a Hoffman-Wielandt type inequality for general dual quaternion matrices, where the latter characterizes a simultaneous perturbation bound on all singular values of a dual quaternion matrix. In particular, they also present two variants of the above two inequalities expressed by eigenvalues of dual quaternion Hermitian matrices.

H. Wang, P. Huang and C. Cui [4] give characterizations of dual quaternion Moore-Penrose generalized inverse (DQMPGI), and get necessary and sufficient conditions for the existence of DQMPGI. By applying DQMPGI and positive semi-defnite dual quaternion matrix, they consider properties and characterizations of perfect Hermitian matrix. At last, they introduce two partial orders: Lönwer-S and Lönwer-P partial orders, and discuss their properties, characterizations and relationships.

S.T. Ling, Z.H. Hu, B. Yang and Y.D. Li [5] propose a color linear discriminant analysis (color LDA) method for color face reconstruction and color face recognition. In order to circumvent the singularity problem of within-class scatter matrix, the quaternion generalized singular value decomposition (QGSVD) is used to obtain the whole Fisher projection axes, and an alternative criterion is adopted to select the optimal set of Fisher projection axes such that the extracted features are statistically uncorrelated. The structure-preserving strategy is adopted to accelerate the color LDA method, and the fast color LDA based on randomization is proposed. Numerical results based on famous color face databases are provided to validate the effectiveness of the proposed color LDA-like methods.

W. Hu, Y. Wang and X. Wang [6] present a Lyapunov-based framework for the analysis of an input-constrained coupled rotational and translational tracking control problem. Using the unit dual quaternion (UDQ) parameterization, they develop logarithm based proportional control law for velocity saturated rigid body kinematics via an amplitude limiting function on the twist of dual quaternions. Furthermore, the proposed translational and rotational coupled control law performs more smooth when tracking a moving target confguration compared to a decoupled control law. Additionally, battery energy consumption is discussed as a potential application advantage for the proposed framework. According to the record of volt and current, UDQ-logarithm based method has less energy consumption than the traditional decoupled method.

W. Chen, C. Song and M. Xu [7] consider the nonhomogeneous Yakubovich-(conjugate) quaternion matrix equations $MX - \tilde{X}N = GY + R$, where $\tilde{X}$ is X or the $\{i, j, k\}$-conjugate of X. The STP method for solving the minimal norm least squares lower(upper) triangular Toeplitz solution and the minimal norm least squares lower(upper) triangular Toeplitz $\{i, j, k\}$-conjugate solutions of the above equation are given, and the expressions of the above solutions of these equations. In addition, they also give the necessary and sufficient conditions and expressions for the existence of (anti)self-conjugate solutions of the corresponding conjugate matrix equations.

D. Wang and H. Ma [8] study the properties and perturbations of group inverses of third-order quaternion tensors under the QT-product (called QT-group inverse). First, they give the definition of the index of quaternion tensors under the QT-product, and define drazin inverses and group inverses of quaternion tensors and prove their existence and uniqueness. Secondly, the core nilpotent decomposition and Jordan decomposition of quaternion tensors under the QT-product are given. They give the expression of the group inverses of quaternion tensors, and the limit expression for QT-Drazin inverses of quaternion tensors. Finally, the perturbation analysis of the one-sided and twosided conditions of the quaternion tensor is carried out. Therefore, the expression and the perturbation bounds of the QT-group inverse of the perturbed quaternion tensor are obtained.

S. Chen and H. Hu [9] investigate unitarily invariant norms of dual quaternion matrices. They first introduce symmetric gauge function on dual numbers and establish a one-to-one correspondence between unitarily invariant norms of dual quaternion matrices and the symmetric gauge function of their singular values. Next, they introduce Schatten p-norm and Fan k-norm on dual quaternion matrices. These are two important types of unitarily invariant norms. They also present several inequalities related to the Schatten p-norm and Fan k-norm.

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References

[1] Renjie Xu and Yimin Wei, "Randomized quaternion matrix UTV decomposition and its applications in quaternion matrix optimization".

[2] Can Li, Yannan Chen and Donghui Li, "A proximal linearized algorithm for dual quaternion optimization with applications in hand-eye calibration".

[3] Chen Ling, Hongjin He, Liqun Qi and Tingting Feng, "Spectral norm and von Neumann type trace inequality for dual quaternion matrices".

[4] Hongxing Wang, Pei Huang and Chong Cui, "Perfect Hermitian matrix and dual quaternion Löwner partial orders".

[5] Si-Tao Ling, Zhe-Han Hu, Bing Yang, Yi-Ding Li, "Color linear discriminant analysis for face recognition based on quaternion model".

[6] Wenxin Hu, Yajing Wang and Xiangke Wang, "Unit dual quaternion based tracking control with velocity saturations applied on quadrotor".

[7] Weihua Chen, Caiqin Song and Meirong Xu, "STP method for solving the minimal norm Toeplitz solutions of $MX - \tilde{X}N = GY + R$".

[8] Dandan Wang and Haifeng Ma, "Perturbations of group inverses of quaternion tensors under the QT-product".

[9] Sheng Chen and Haofei Hu, "Unitarily invariant norms on dual quaternion matrices".

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