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PREFACE: SPECIAL ISSUE ON NUMERICAL OPTIMIZATION IN DATA SCIENCE

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With the development of science and technology and the internet, various industries are producing a large amount of data every day, and their effective processing constitutes the basis of realizing intelligence. The main purpose of big data analysis and processing is to dig out useful information and discover basic rules from massive and complex data, so as to maximize the benefits of data resources. In this sense, optimization provides a foundation in big data analysis and processing. In particular, the development of optimal theory and effective method plays an important role in big data analysis and processing.

In recent years, many optimization methods have been used to solve data-driven optimization problems, and great progress has been made. However, due to the explosive growth of data and other reasons, some new structured data processing problems continue to emerge. This field is still facing great challenges.

This special issue aims to reflect the latest advances in numerical optimization methods in data science. A total of 25 papers accounted below, which were reviewed according to the usual high standards of the journal, cover a wide range of theoretical, practical, and applied topics in numerical optimization methods in data science. This special issue is published in two parts.

S. J. Bi, T. Tao and S. H. Pan [1] concern with the factorization form of the rank regularized loss minimization problem, which composes with an $\ell_{2,0}$-norm regularized term, a balanced term and the factored loss function. For the least squares loss, under a restricted condition number assumption on the sampling operator, they establish the KL property of exponent 1/2 of the nonsmooth factored composite function and its equivalent DC regularized surrogates in the set of their global minimizers. They also confirm the theoretical findings by applying a proximal linearized alternating minimization method to the regularized factorizations.

S. Y. Chang and Y. M. Wei [2] study the Sherman-Morrison-Woodbury identity for tensors with Moore-Penrose generalized inverse by utilizing orthogonal projection of the correction tensor part into the original tensor and its Hermitian tensor. Based on the new established identity, the sensitivity analysis for multilinear systems can be performed by deriving the normalized upper bound for the solution of a multilinear system. Furthermore,

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the authors present several numerical examples to demonstrate how the normalized error upper bounds are affected by perturbation degree of tensor coefficients.

Y. N. Chen, L. Q. Qi and X. Z. Zhang [3] propose a lower rank quaternion matrix approximation algorithm and apply it to color image completion. They also introduce a concise form for the gradient of a real function in quaternion matrix variables. The optimality conditions of our quaternion least squares problem have a simple expression with this form. The convergence and convergence rate of the algorithm are established with this tool. Numerical tests on a set of color images demonstrate the efficiency of the algorithm.

L. B. Cui and Q. Hu [4] first present a Zhang neural network model for solving the absolute value equation and then establish a chord-Zhang neural network model. The convergence of the proposed models is studied. Some numerical experiments are presented to illustrate the efficiency of the proposed schemes.

M. Y. Deng, W. Y. Ding and X. P. Guo [5] prove the existence and uniqueness of the positive solution to a non-homogenous $\mathcal{M}$-tensor equation with a positive right-hand side, and extend the classical splitting methods, including the Jacobi-like, Gauss-Seidel-like, simplified Gauss-Seidel-like, and SOR-like methods, to solve this class of tensor equations and show their convergence. Finally, a large number of numerical results show the performance of these methods.

W. X. Du and S. L. Hu [6] present an algorithm for solving the problem of the symmetric low rank orthogonal tensor approximation for a given symmetric tensor. Proximality technique and shifted power technique are tailored into this algorithm. In particular, the authors show that this algorithm converges globally without any assumption once the parameters are chosen appropriately, and moreover the convergence rate is sublinear with an explicitly given rate and it is better than the usual $\mathcal{O}(1/p)$ of first order methods in optimization.

C. J. Fang, R. R. Zhang and S. L. Chen [7] introduce an inertial Tseng's extragradient method for solving multi-valued variational inequalities, in which only one projection is needed at each iteration. They also obtain the strong convergence results of the proposed algorithm, provided that the multi-valued mapping is continuous and pseudomonotone with nonempty compact convex values. Moreover, numerical simulation results illustrate the efficiency of the method when compared to existing methods.

R. X. Feng, P. W. Zhong and Y. T. Xu [8] introduce a support matrix machine based on squared hinge loss (L_2-SMM), which employs a smoothed substitution formula. They construct a subspace elimination strategy for L_2-SMM (SES-L_2-SMM) to expedite the speed of its training process. At each step, an active subspace is chosen so that they can solve a lower-dimensional optimization problem. Later, they use the alternating direction method of multipliers algorithm to resolve the problem. Extensive comparative experiments on multiple real-world datasets demonstrate the efficacy of SES-L_2-SMM.

B. D. Huang and P. P. Shen [9] consider the problem of sum of linear ratios on a polyhedron. the authors show that the problem can be transformed into a two-layer optimization problem. By using the special structure of the transformed problem, a novel convex relaxation technique is proposed to design a new branch and bound algorithm. Furthermore, the convergence analysis and computational complexity of the presented algorithm are given. Finally, numerical experimental results illustrate the effectiveness of the algorithm.

H. W. Jiao, J. Q. Ma and Y. L. Shang [10] present an image space branch-and-bound algorithm for a minimax linear fractional programming problem. Based on an equivalent transformation and a new linearizing technique, the original problem can be approximated by a linear relaxation programming problem. By subsequently refining the initial image space region and successively solving a series of linear relaxation problems, the proposed algorithm converges globally to an optimal solution of the original problem. Numerical experimental results demonstrate the feasibility and effectiveness of the algorithm.

L. J. Kong, A. L. Yan, Y. Li and J. Fan [11] introduce a novel method which consists of Least absolute deviation loss function and an $L_{1/2}$ regularizer. It is a nonconvex, nonsmooth, and non-Lipschitz optimization problem. They design an efficient alternating direction method of multipliers to solve the problem and establish its convergence. Extensive numerical experiments demonstrate that the proposed method can recover sparse signal with less measurements and is robust to dense bounded noise as well as Laplace noise.

M. Z. Li, H. B. Chen and G. L. Zhou [12] give a survey on recent advances of fourth-order partially symmetric tensors such as M-eigenvalue inclusion intervals, M-positive definiteness, strong ellipticity condition and algorithms for computing the largest M-eigenvalue. Some further research questions are also released.

H. W. Li, H. B. Zhang and Y. H. Xiao [13] use an accelerated gradient method (AGM) to solve the sparse subspace clustering problem, and establish its convergence in the sense of converging to a critical point with a certain stepsize policy. They show that closed-form solutions are enjoyed for each subproblem by taking full use of the constraints' structure. Finally, they do numerical experiments by the using of two real datasets, which illustrate that AGM performs better than BDR and ABCGD evidently.

K. P. Liu, and H. T. Che [14] establish some new eigenvalue localization sets for fourth-order partially symmetric tensor. It is revealed that the new eigenvalue localization sets are tighter than some existing results. Numerical examples demonstrate the effectiveness of the results obtained. As applications, some bound estimations for the M-spectral radius and some checkable sufficient conditions for the positive definiteness of the fourth-order partially symmetric tensor are obtained based on the new eigenvalue localization sets.

Y. J. Liu and Q. X. Zhu [15] propose a semismooth Newton based augmented Lagrangian (Ssnal) algorithm for solving Weber problem, which is an important problem arising in the facility location problems. The global convergence and locally asymptotically superlinear convergence of the Ssnal algorithm are characterized under mild conditions. Numerical experiments conducted on synthetic data sets demonstrate that the Ssnal algorithm outperforms several state-of-the-art algorithms in terms of efficiency and robustness.

Z. Y. Peng, X. J. Long, J. Zeng and Z. Lin [16] concern with the stability for a generalized Ky Fan inequality when it is perturbed by vector-valued bifunction sequence and constrain set sequence. By removing the assumptions of the strictly proper quasi C-convexity and the continuous convergence, they establish the Painlevé-Kuratowski convergence of the approximate solution mapping of a family for perturbed problems to the corresponding solution mapping of the original problem. The obtained results are new and improve the corresponding ones in the literatures. Some examples are also given to illustrate the results.

Y. F. Shao, Q. S. Wang and D. R. Han [17] derive several new BB step sizes formulae based on a more general distance measure, and apply them to the mirror descent method and the Frank-Wolfe method. Compared with the two algorithms with traditional BB step sizes, the preliminary numerical experiments demonstrate that the algorithms with the proposed step sizes can achieve better performance.

J. Sun, P. Shang, Q. Y. Xu and B. Z. Chen [18] propose a new matrix regression model which can induce a low-rank and sparse estimator in the sense of row-group with the help of nuclear norm and $\|\cdot\|_{2,1}$ norm. In order to obtain such an estimator, the authors develop a linearized alternating direction method of multipliers and prove its global convergence. Numerical experiments are carried out to demonstrate the properties of the new model and the accuracy of the proposed algorithm.

C. M. Tang, H. Y. Chen, J. B. Jian and S. Liu [19] propose a bundle-type quasi-Newton method for minimizing a nonconvex nonsmooth function. Global convergence of the algorithm is established in the sense that there exists an accumulation point of the serious iterations such that it is a stationary point of the function. Superlinear convergence is proved under suitable assumptions. Preliminary numerical results are reported to illustrate that their method is efficient and has advantages over the redistributed bundle method.

J. Wang [20] presents some generalizations of Von Neumann's trace inequality for matrices to the content of tensors. The focus is on completely orthogonal decomposable tensors, and the angle between two completely orthogonal decomposable tensors is taken into Von Neumann's trace inequality. Moreover, the properties of spectral functions in the case of asymmetrically and symmetrically completely orthogonally decomposable tensors are discussed , respectively.

T. X. Wang, X. J. Cai, Y. Z. Song and X. Gao [21] study a class of difference-of-convex programming. Based on the primal-dual reformulation of the problem, by adopting the framework of the double-proximal gradient algorithm (DPGA) and the inertial technique for accelerating the first-order algorithms, the authors propose a double-inertial proximal gradient algorithm. Under the suitable assumptions, each bounded sequence generated by the algorithm is shown to converge globally to a critical point of the objective function. Finally, the authors apply the algorithm to image processing model and compare it with DPGA to show its efficiency.

Y. X. Wei, Z. Y. Luo and Y. Chen [22] present a momentum accelerated version of the block-randomized stochastic gradient descent algorithm for low-rank tensor CP decomposition. Under mild conditions, the authors show the global convergence to the stationary point. Compared with the algorithms without momentum, the preliminary numerical experiments demonstrate that the proposed accelerated algorithm is efficient.

Y.R. Xu and S. J. Li [23] consider a class of parametric set optimization problems, where both objective functions and constraint functions are perturbed by different parameters. Some upper and lower set orderings with respect to improvement sets are introduced and used to define solution mappings. Then, some assumptions including strong domination properties are proposed to study the Hölder continuity of solution mappings and corresponding optimal value mappings. The obtained results generalize the upper Hölder continuity of efficient solution mappings for parametric vector optimization problems.

J. W. Zhang and Y. N. Yang [24] consider modified versions of the alternating least-squares (ALS) and the regularized alternating least-squares (RALS) algorithms for tensor decomposition. They propose two hybrid alternating methods by combining the extra-gradient method with Newton's method. Theoretically, the step-size of the correction step can be possibly chosen in a wide range. The global convergence of the algorithms are discussed under certain assumptions. Preliminary numerical experiments show the effectiveness of the proposed methods, compared to the standard ALS and RALS algorithms.

D. Q. Zhou, X. K. Chang and J. F. Yang [25] propose and analyze a golden ratio primal-dual algorithm for solving structured optimization problems involving the sum of three convex terms. The proposed algorithm is of primal-dual and full-splitting type as it solves the primal and the dual problems simultaneously and does not rely on solving any subproblems or linear system of equations iteratively. The convergence rates $\mathcal{O}(1/N)$ and $\mathcal{O}(1/N^2)$ are established for convex and strongly convex cases, respectively, which differentiate themselves from existing results in terms of the adopted optimality measures. Finally, preliminary numerical results are presented to demonstrate the efficiency of the algorithm.

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