



PREFACE: ANALYSIS AND PDE

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Special issue on Analysis and PDE dedicated to
Professor Vladimir Maz'ya on the occasion of his 85th birthday

This special issue on Analysis and PDE is dedicated to Professor Vladimir Maz'ya on the occasion of his 85th birthday.

Vladimir Maz'ya, a highly esteemed Russian-born Swedish mathematician, has recently celebrated his 85th birthday. This significant milestone provides an excellent occasion to contemplate the extensive and diverse body of his scientific work, particularly in the field of mathematical analysis and the theory of partial differential equations, and to assess its influence on contemporary mathematics.

By any metric, Vladimir Maz'ya has been extraordinarily prolific, as his 65 years of research activities have culminated in about 40 research monographs, and more than 550 articles, containing fundamental results and powerful novel techniques. In addition to its remarkable depth and innovation, his work displays incredible diversity, and here we venture only as far as listing some of the highlights from his impressive array of achievements.

Vladimir Maz'ya has discovered the equivalence between Sobolev and isoperimetric/isocapacitary inequalities (1960), constructed counterexamples related to Hilbert's 19th and Hilbert's 20th problems (1968), solved, together with Yuri Burago, a problem in harmonic potential theory (1967) posed by Riesz and Szökefalvi-Nagy (1955), extended the Wiener regularity test to the p -Laplacian and proved its sufficiency for boundary regularity, solved Vladimir Arnol'd's oblique derivative boundary value problem (1970), and Fritz John's problem on the oscillations of a fluid in the presence of an immersed body (1977). More recently, Vladimir Maz'ya proved a Wiener-type criterion for higher-order elliptic equations, solved together with Mikhail Shubin a problem in the spectral theory of the Schrödinger operator formulated by Israel Gelfand in 1953, found necessary and sufficient conditions for the validity of maximum principles for elliptic and parabolic systems of PDE, and introduced the so-called approximate approximations. He also contributed to the development of the theory of capacities, nonlinear potential theory, the asymptotic and qualitative theory of arbitrary order elliptic equations, the theory of ill-posed problems, the theory of water waves, the theory of multipliers between spaces of differentiable functions, and the field of boundary value problems in domains with piecewise smooth boundary.

An array of awards and professional distinctions have acknowledged the impact of his influential work. In 1962 Vladimir Maz'ya was awarded the "Young Mathematician" prize by the Leningrad Mathematical Society for his results on Sobolev spaces. In 1990 he was awarded an honorary doctorate from Rostock University. In 1999 Maz'ya received the Humboldt Prize. He was elected member of the Royal Society of Edinburgh in 2000, and of the Swedish Academy of Science in 2002. In March 2003 he, jointly with Tatyana Shaposhnikova, was awarded the Verdaguer Prize by the French Academy of Sciences. In 2004 he was awarded the Celsius Gold Medal, the top award of the Royal Society of Sciences in Uppsala, "for his outstanding research on partial differential equations and hydrodynamics." He was awarded the Senior Whitehead Prize by the London Mathematical Society in 2009, and in 2012 he was elected a Fellow of the American Mathematical Society. In 2013 he was elected a foreign member of the Georgian National Academy of Sciences.

Currently, Vladimir Maz'ya is an honorary Senior Fellow at Liverpool University, and Professor Emeritus at Linköping University.

In this special issue, paying homage to Vladimir Maz'ya's professional achievements, we have collected nineteen papers authored by a selected group of experts in the areas of Analysis and PDE from Canada, China, Finland, France, Germany, India, Israel, Italy, Portugal, Russia, Sweden, United Kingdom, and the USA.

These articles cover a wide spectrum of important problems and topics of current research interest, including uniqueness for a class of nonlinear elliptic equations with lower-order terms, entropic isoperimetric inequalities for generalized Fisher information, $L_*^{1 \leq q < \infty}$ -lift of L^1 -space, a characterization of $W^{1,p}(R^d)$, extended power weighted Rellich-type inequalities with logarithmic refinements, composition operators on Sobolev spaces and Ball's classes, mean value inequality and generalized capacity on doubling spaces, Hölder regularity for domains of fractional powers of elliptic operators, discrete Riesz transforms on rearrangement-invariant Banach sequence space, the extension property for domains with one singular point, potential theory and quasisymmetric maps, the fundamental solution of an elliptic equation with singular drift, qualitative questions for mixed local-nonlocal elliptic operators, regularity of weighted Sobolev homeomorphisms, multidimensional examples of the Metropolis algorithm, Gauss-Green formulas on domains with non-rectifiable boundaries, nonlinear potential estimates for sublinear problems, solvability in the sense of sequences for some non-Fredholm operators, and the turnpike phenomenon for perturbed dynamical systems.

We close by expressing our belief that this special issue will hold significant importance for mathematicians with an interest in the latest developments in Analysis and PDE, along with their various applications.

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