

PREFACE: CONTEMPORARY TREND OF VECTOR OPTIMIZATION AND APPLICATIONS

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Optimization plays a crucial role in various fields, providing powerful tools to find the best possible solutions to complex problems. It enables decision-making processes that are essential in economics, engineering, and the sciences, facilitating the efficient use of resources and improving outcomes.

In particular, the study of vector optimization and its applications is of significant importance. Vector optimization deals with optimization problems involving multiple objective functions, requiring a nuanced approach to balance and satisfy competing criteria. Applications of these optimization techniques can be found in numerous areas, including finance, logistics, and machine learning. Understanding the underlying mathematical principles and developing robust methods is essential for advancing these fields and addressing practical challenges.

This special issue contains seven papers on contemporary trends in vector optimization and its applications. The contributions explore various aspects, including second-order optimality conditions in group sparsity multiobjective optimization, optimality and duality for approximate solutions in nonsmooth interval-valued multiobjective semi-infinite programming, robust nonsmooth multiobjective semi-infinite optimization on Hadamard manifolds, interior-point algorithms for Fisher market equilibrium problems, extended monotonic results related to fractional derivatives, dynamic pricing and coordination strategies in supply chains considering consumers' expected regret, and novel smooth-type algorithms for second-order cone linear complementarity problems.

The first paper investigates the group sparsity multiobjective optimization problem (GSMOP) using advanced variational analysis. The authors present characterizations of various tangent sets and cones related to group sparse sets. They establish first-order and second-order optimality conditions, utilizing these tangent sets and Dini directional derivatives, providing a comprehensive understanding of the optimality criteria for GSMOP.

The second paper focuses on interval-valued multiobjective semi-infinite programming (IVMOSIP), introducing approximate constraint qualifications and deriving necessary optimality conditions for approximate quasi-efficient solutions. The authors explore the KKT-type necessary optimality conditions under approximate convexity and develop a dual model of Mond-Weir type. The use of Clarke subdifferential tools and examples demonstrates the practical relevance of the results.

In the third paper, the authors examine uncertain nonsmooth multiobjective semi-infinite programming problems on Hadamard manifolds, proposing a robust counterpart known as RNMSIP. They introduce the Abadie constraint qualification (ACQ) and explore the KKT-type necessary optimality criteria for robust weak Pareto and Benson-proper solutions. The paper extends these criteria to sufficient

conditions under geodesic convexity assumptions and offers dual models, backed by significant examples.

The fourth paper presents a feasible interior-point algorithm with full Newton steps for solving the general Fisher market equilibrium problem. By employing a kernel function, the authors derive an equivalent system of equations that facilitates the full-Newton step feasible IPM. The paper provides proofs of global convergence and polynomial complexity, supported by numerical examples, demonstrating the algorithm's effectiveness.

The fifth paper discusses monotonous results with fractional derivatives in the interval $(1,2)$. The authors explore α -order Riemann-Liouville and Caputo fractional derivatives, developing theoretical results supported by numerical examples. The study extends these results to generalized intervals, providing a broader understanding of fractional derivatives and their applications.

In the sixth paper, the authors develop a two-stage dynamic game model for a supply chain involving manufacturers, retailers, and strategic consumers, considering consumers' expected regret. They analyze the effects of consumers' strategic degree, high price regret, and out-of-stock regret on equilibrium results. They propose a two-stage revenue-sharing contract for supply chain coordination, offering insights into strategies for managing consumer behavior and inventory levels.

The seventh paper introduces two novel smooth-type functions for solving second-order cone linear complementarity problems. The authors use smooth Newton methods to demonstrate that the solution sequence converges to the problem's solution under specific assumptions. A performance comparison with original smooth functions reveals improved numerical performance, extending the application of smooth functions in optimization.

We hope that this collection of papers provides valuable insights and stimulates further research in the field of vector optimization and its diverse applications.

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