Pure and Applied Functional Analysis
Volume 8, Number 6, 2023, 1781–1789
Yokohama Publishers
ISSN 2189-3764 ONLINE JOURNAL

A POROSITY RESULT REGARDING UNIFORMLY LOCALLY CONTRACTIVE MAPPINGS

SIMEON REICH* AND ALEXANDER J. ZASLAVSKI

Abstract. In a 1961 paper by E. Rakotch it was shown that a uniformly locally contractive mapping on a complete metric space has a fixed point. In a recent paper of ours we have shown that for such a mapping, the fixed point problem is well posed and that inexact iterates of such a mapping converge to its unique fixed point, uniformly on bounded sets. In the present paper we show that the complement of the set of uniformly locally contractive mappings in the complete metric space of uniformly locally nonexpansive mappings is σ-porous.

1. Introduction

For more than sixty years now, there has been considerable research activity regarding the fixed point theory of various classes of nonexpansive (that is, 1-Lipschitz) mappings. See, for instance, [2–4, 6–10, 13, 14, 16–18, 20, 22, 24–29, 35, 36] and references cited therein. This activity stems from Banach's classical theorem [1] concerning the existence of a unique fixed point for a strict contraction. It also concerns the convergence of (inexact) orbits of a nonexpansive mapping to one of its fixed points. Since that seminal result, many developments have taken place in this field including, in particular, studies of feasibility, common fixed points and optimization problems, which find important applications in engineering, medical and the natural sciences [5, 11, 12, 32, 33, 35, 36].

In his 1961 paper, E. Rakotch [21] showed that a uniformly locally contractive mapping on a complete metric space has a fixed point. This result was later improved in [23]. In our recent paper [30], we have shown that for a uniformly locally contractive mapping, the fixed point problem is well posed and that inexact iterates of such a mapping converge to its unique fixed point, uniformly on bounded sets. In the present paper we show that the complement of the set of uniformly locally contractive mappings in the complete metric space of uniformly locally nonexpansive mappings is σ-porous.

2020 *Mathematics Subject Classification.* 47H09, 47H10, 54E50.

Key words and phrases. Complete metric space, contractive mapping, fixed point, porous set.

*The first author was partially supported by the Israel Science Foundation (Grant No. 820/17), by the Fund for the Promotion of Research at the Technion and by the Technion General Research Fund.

2. Existence of a fixed point, convergence and well-posedness properties

Assume that (X, ρ) is a complete metric space, $\Delta > 0$, and that the following assumption holds:

(A) For each $x, y \in X$, there exist an integer $q \geq 1$ and points $x_i \in X$, $i = 0, \dots, q$, such that

$$x_0 = x,\ x_q = y,\ \rho(x_i, x_{i+1}) \leq \Delta,\ i \in \{0, \dots, q\} \setminus \{q\}.$$

For each $x, y \in X$, define

$$\rho_1(x, y) := \inf \Big\{ \sum_{i=0}^{q-1} \rho(x_i, x_{i+1}) :\ q \geq 1 \text{ is an integer},$$

$$x_i \in X,\ i = 0, \dots, q,\ x_0 = x,\ x_q = y,\ \rho(x_i, x_{i+1}) \leq \Delta,\ i \in \{0, \dots, q\} \setminus \{q\} \Big\}. \tag{2.1}$$

It follows from assumption (A) and equation (2.1) that for each $x, y, z \in X$, $\rho_1(x, y)$ is finite,

$$\rho(x, y) \leq \rho_1(x, y) < \infty,$$

$$\text{if } \rho(x, y) \leq \Delta, \text{ then } \rho_1(x, y) = \rho(x, y),$$

$$\rho_1(x, y) = \rho_1(y, x),$$

$$\rho_1(x, x) = 0,$$

$$\text{if } \rho_1(x, y) = 0, \text{ then } \rho(x, y) = 0 \text{ and } x = y,$$

and

$$\rho_1(x, z) \leq \rho_1(x, y) + \rho_1(y, z).$$

Clearly, ρ_1 is a metric on X.

Assume that $K \subset X$ is a nonempty closed set and that the following assumption holds:

(B) For each $x, y \in K$,

$$\rho_1(x, y) := \inf \Big\{ \sum_{i=0}^{q-1} \rho(x_i, x_{i+1}) :\ q \geq 1 \text{ is an integer},$$

$$x_i \in K,\ i = 0, \dots, q,\ x_0 = x,\ x_q = y,\ \rho(x_i, x_{i+1}) \leq \Delta,\ i \in \{0, \dots, q\} \setminus \{q\} \Big\}.$$

Let $T : K \to X$ be a mapping. We assume that for each $x, y \in X$ satisfying $\rho(x, y) \leq \Delta$, the inequality

$$\rho(T(x), T(y)) \leq \rho(x, y)$$

holds. In view of (B) and the above assumption, for each $x, y \in X$, we have

$$\rho_1(T(x), T(y)) \leq \rho_1(x, y).$$

Such a mapping T is said to be uniformly locally nonexpansive.

We remark in passing that the smaller class of uniformly local (strict) contractions was introduced in [9], while the larger class of locally nonexpansive mappings was studied in [3]. The work [9] also contains an example of a uniformly local (strict) contraction which is not nonexpansive.

Assume that $\phi : [0, \Delta] \to [0, 1]$ is a decreasing function,

$$\phi(t) < 1 \text{ for all } t \in (0, \Delta],$$

and that for each $x, y \in K$ satisfying $\rho(x, y) \le \Delta$, we have

$$\rho(T(x), T(y)) \le \phi(\rho(x, y))\rho(x, y).$$

The following theorem is established in [31]. It shows the well-posedness of the fixed point problem for the mapping T.

Theorem 2.1. *Given $\epsilon > 0$, there exists $\delta > 0$ such that for each $x, y \in K$ satisfying*

$$\max\{\rho(T(x), x),\ \rho(T(y), y)\} \le \delta,$$

the inequality

$$\rho(x, y) \le \epsilon$$

holds.

Our next theorem was also proved in [31].

Theorem 2.2. *Let $\epsilon, M > 0$ be given. Then there exist a number $\delta \in (0, \epsilon)$ and an integer $n_0 \ge 1$ such that for each integer $n > n_0$ and each finite sequence $\{x_i\}_{i=0}^n \subset K$ which satisfies*

$$\rho_1(x_0, x_1) \le M$$

and

$$\rho(x_{i+1}, T(x_i)) \le \delta,\ i = 0, \dots, n-1,$$

the inequality

$$\rho(x_i, x_{i+1}) \le \epsilon$$

holds for all $i = n_0, \dots, n-1$.

Theorems 2.1 and 2.2 easily imply the following two results regarding the existence of a unique fixed point and the convergence of inexact iterates.

Theorem 2.3. *Assume that for each integer $n \ge 1$, there exists a finite sequence $\{x^{(n)}\}_{i=0}^n \subset K$ such that*

$$\sup\{\rho_1(x_0^{(n)}, x_1^{(n)}) :\ n = 1, 2, \dots\} < \infty$$

and

$$\lim_{n\to\infty} \sup\{\rho(x_{i+1}^{(n)}, Tx_i^{(n)}) :\ i = 0, \dots, n-1\} = 0.$$

Then there exists a unique point $x_* \in K$ such that $T(x_*) = x_*$.

Theorem 2.4. *Let the point $x_* \in K$ satisfy*

$$T(x_*) = x_*$$

and let $\epsilon, M > 0$ be given. Then there exist a number $\delta > 0$ and an integer $n_0 \ge 1$ such that for each integer $n > n_0$ and each finite sequence $\{x_i\}_{i=0}^n \subset K$ which satisfies

$$\rho_1(x_0, x_1) \le M$$

and

$$\rho(x_{i+1}, T(x_i)) \le \delta,\ i = 0, \dots, n-1,$$

the inequality

$$\rho(x_i, x_*) \le \epsilon$$

holds for all $i = n_0, \dots, n$.

Note that in [30] the results of this section were obtained in the case where $K = X$.

3. Hyperbolic spaces and porosity

In the present paper we consider a complete metric space of uniformly locally nonexpansive self-mappings of a bounded and closed subset of a complete hyperbolic space.

Let (X, ρ) be a metric space and let R^1 denote the real line. We say that a mapping $c : R^1 \to X$ is a metric embedding of R^1 into X if $\rho(c(s), c(t)) = |s - t|$ for all real s and t. The image of R^1 under a metric embedding is called a metric line. The image of a real interval $[a, b] = \{t \in R^1 : a \le t \le b\}$ under such a mapping is called a metric segment.

Assume that (X, ρ) contains a family M of metric lines such that for each pair of distinct points x and y in X, there is a unique metric line in M which passes through x and y. This metric line determines a unique metric segment joining x and y. We denote this segment by $[x, y]$. For each $0 \le t \le 1$, there is a unique point z in $[x, y]$ such that

$$\rho(x, z) = t\rho(x, y) \text{ and } \rho(z, y) = (1 - t)\rho(x, y).$$

This point is denoted by $(1 - t)x \oplus ty$. We say that X, or more precisely (X, ρ, M), is a hyperbolic space if

$$\rho\left(\frac{1}{2}x \oplus \frac{1}{2}y, \frac{1}{2}x \oplus \frac{1}{2}z\right) \le \frac{1}{2}\rho(y, z)$$

for all x, y and z in X. An equivalent requirement is that

$$\rho\left(\frac{1}{2}x \oplus \frac{1}{2}y, \frac{1}{2}w \oplus \frac{1}{2}z\right) \le \frac{1}{2}(\rho(x, w) + \rho(y, z))$$

for all x, y, z and w in X. A set $K \subset X$ is called ρ-convex if $[x, y] \subset K$ for all x and y in K.

It is clear that all normed linear spaces are hyperbolic. A discussion of more examples of hyperbolic spaces and, in particular, of the Hilbert ball can be found, for example, in [14, 24].

We now recall the notion of porosity. Let (Y, D) be a complete metric space. We denote by $B(y, r)$ the closed ball of center $y \in Y$ and radius $r > 0$. A subset $E \subset Y$ is called porous in (Y, D) [29] if there exist $\alpha \in (0, 1)$ and $r_0 > 0$ such that for each $r \in (0, r_0]$ and each $y \in Y$, there exists a point $z \in Y$ for which

$$B(z, \alpha r) \subset B(y, r) \setminus E.$$

A subset of the space Y is called σ-porous in (Y, D) if it is a countable union of porous subsets of (Y, D).

Since porous sets are nowhere dense, all σ-porous sets are of the first Baire category. If Y is a finite-dimensional Euclidean space, then σ-porous sets are of

Lebesgue measure 0. In fact, the class of σ-porous sets in such a space is much smaller than the class of sets which have Lebesgue measure 0 and are of the first Baire category. The porosity notion has been used in analysis and in optimization theory [7, 15, 19, 29, 34].

4. The porosity result

Let (X, ρ, M) be a complete hyperbolic space and let K be a nonempty, closed and bounded subset of (X, ρ). Assume that

$$\Delta > 0,\ \theta \in K$$

and that for each $\gamma \in (0, 1)$ and each $x \in K$,

$$\gamma\theta \oplus (1 - \gamma)x \in K. \tag{4.1}$$

Set

$$\operatorname{diam}(K) := \sup\{\rho(x, y) :\ x, y \in K\}. \tag{4.2}$$

It follows from (4.1) and the boundedness of K in (X, ρ) that

$$\sup\{\rho_1(x, y) :\ x, y \in K\} < \infty.$$

Denote by $\mathcal{A}$ the set of all mappings $T : K \to X$ such that

$$\rho(T(x), T(y)) \le \rho(x, y) \text{ for each } x, y \in K \text{ satisfying } \rho(x, y) \le \Delta. \tag{4.3}$$

For each $A, B \in \mathcal{A}$, define

$$d(A, B) := \sup\{\rho(A(x), B(x)) :\ x \in K\}.$$

Since K is bounded in (X, ρ_1), $d(A, B)$ is finite for each $A, B \in \mathcal{A}$. It is not difficult to see that $(\mathcal{A}, d)$ is a complete metric space.

A mapping $T \in \mathcal{A}$ is said to be uniformly locally contractive if there exists a decreasing function $\phi : [0, \Delta] \to [0, 1]$ such that

$$\phi(t) < 1 \text{ for all } t \in (0, \Delta]$$

and for each $x, y \in K$ satisfying $\rho(x, y) \le \Delta$, we have

$$\rho(T(x), T(y)) \le \phi(\rho(x, y))\rho(x, y).$$

Denote by $\mathcal{F}$ the set of all uniformly locally contractive mappings $A \in \mathcal{A}$.

We are now ready to first state and then establish our main result.

Theorem 4.1. *The set $\mathcal{A} \setminus \mathcal{F}$ is σ-porous.*

We may assume that

$$\Delta \le \operatorname{diam}(K).$$

Given $A \in \mathcal{A}$ and $\gamma \in (0, 1)$, define

$$A_\gamma(x) := (1 - \gamma)A(x) \oplus \gamma\theta,\ x \in K. \tag{4.4}$$

In view of (4.4), for each $x, y \in K$ satisfying $\rho(x, y) \le \Delta$,

$$\rho(A_\gamma(x), A_\gamma(y)) \le (1 - \gamma)\rho(A(x), A(y)) \le (1 - \gamma)\rho(x, y), \tag{4.5}$$

and for each $x \in K$,

$$\rho(A(x), A_\gamma(x)) = \rho(A(x), (1-\gamma)A(x) \oplus \gamma\theta) \le \gamma\rho(A(x), \theta) \le \gamma \mathrm{diam}(K). \tag{4.6}$$

By (4.6),

$$d(A, A_\gamma) \le \gamma \mathrm{diam}(K). \tag{4.7}$$

In the sequel we use the convention that the supremum over the empty set is zero.

Theorem 4.1 can easily be deduced from the following two propositions.

For each integer $n \ge 1$, denote by $\mathcal{F}_n$ the set of all $A \in \mathcal{A}$ such that

$$\sup\{\rho(A(x), A(y))\rho(x,y)^{-1} : x, y \in K \text{ and } \Delta \ge \rho(x,y) \ge \Delta n^{-1}(\mathrm{diam}(K)+1)^{-1}\} < 1. \tag{4.8}$$

Set

$$\mathcal{F} := \cap_{n=1}^{\infty} \mathcal{F}_n. \tag{4.9}$$

Proposition 4.2. *Assume that $A \in \mathcal{F}$. Then there exists a decreasing function $\phi : [0, \Delta] \to [0, 1]$ such that*

$$\phi(t) < 1 \textit{ for all } t \in (0, \Delta]$$

and for each $x, y \in K$ satisfying $\rho(x, y) \le \Delta$, we have

$$\rho(A(x), A(y)) \le \phi(\rho(x, y))\rho(x, y).$$

Proof. Set $\phi(0) = 1$. For each $t \in (0, \Delta]$, define

$$\phi(t) := \sup\{\rho(A(x), A(y))\rho(x,y)^{-1} : x, y \in K \text{ and } \Delta \ge \rho(x,y) \ge t\}. \tag{4.10}$$

By (4.8)–(4.10),

$$\phi(t) < 1, \ t \in (0, \Delta].$$

It is clear that $\phi : [0, \Delta] \to [0, 1]$ is a decreasing function. Let $x, y \in K$ satisfy

$$0 < \rho(x, y) \le \Delta.$$

Set $t = \rho(x, y)$. By (4.10),

$$\phi(\rho(x, y)) = \phi(t) \ge \rho(A(x), A(y))\rho(x, y)^{-1}.$$

This completes the proof of Proposition 4.2. □

Proposition 4.3. *Let n be a natural number. Then $\mathcal{A} \setminus \mathcal{F}_n$ is a porous set in $(\mathcal{A}, d)$.*

Proof. Set

$$\alpha = \Delta(8n)^{-1}(\mathrm{diam}(K)+1)^{-2}. \tag{4.11}$$

Assume that

$$A \in \mathcal{A}, \ r \in (0, 1].$$

Set

$$\gamma = 2^{-1}(\mathrm{diam}(K)+1)^{-1}r. \tag{4.12}$$

Define

$$A_\gamma(x) := (1-\gamma)A(x) \oplus \gamma\theta,\ x \in K.$$

In view of (4.7) and (4.12), we have

$$d(A_\gamma, A) \le \gamma \mathrm{diam}(K) \le r/2. \tag{4.13}$$

Assume now that $B \in \mathcal{A}$ satisfies

$$d(B, A_\gamma) \le \alpha r. \tag{4.14}$$

It follows from (4.11), (4.13) and (4.14) that

$$\begin{aligned} d(A,B) &\le d(A, A_\gamma) + d(A_\gamma, B) \\ &\le \alpha r + r/2 \le r. \end{aligned} \tag{4.15}$$

By (4.4), (4.5) and (4.14), for each $x, y \in K$ satisfying

$$\Delta \ge \rho(x,y) \ge \Delta n^{-1}(\mathrm{diam}(K)+1)^{-1} \tag{4.16}$$

we have

$$\begin{aligned} \rho(B(x), B(y)) &\le \rho(A_\gamma(x), A_\gamma(y)) + 2\alpha r \\ &\le (1-\gamma)\rho(x,y) + 2\alpha r \end{aligned}$$

and in view of (4.11)–(4.12) and (4.16),

$$\begin{aligned} \rho(B(x), B(y))\rho(x,y)^{-1} &\le 1 - \gamma + 2\alpha r \rho(x,y)^{-1} \\ &\le 1 - \gamma + 2\alpha r \Delta^{-1} n(\mathrm{diam}(K)+1) \\ &\le 1 - 2^{-1}(\mathrm{diam}(K)+1)^{-1} r + 2r\alpha n(\mathrm{diam}(K)+1)\Delta^{-1} \\ &\le 1 - 2^{-1}(\mathrm{diam}(K)+1)^{-1} r + 4^{-1}(\mathrm{diam}(K)+1)^{-1} r \\ &= 1 - 4^{-1}(\mathrm{diam}(K)+1)^{-1} r. \end{aligned}$$

Thus

$$B \in \mathcal{F}_n$$

and

$$\{B \in \mathcal{A}:\ d(B, A_\gamma) \le \alpha r\} \subset \{B \in \mathcal{A}:\ d(A,B) \le r\} \cap \mathcal{F}_n.$$

Therefore $\mathcal{A} \setminus \mathcal{F}_n$ is a porous set. This completes the proof of Proposition 4.3. □

References

[1] S. Banach, *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales*, Fund. Math. **3** (1922), 133–181.

[2] A. Betiuk-Pilarska and T. Domínguez Benavides, *Fixed points for nonexpansive mappings and generalized nonexpansive mappings on Banach lattices*, Pure Appl. Func. Anal. **1** (2016), 343–359.

[3] R. E. Bruck, W. A. Kirk and S. Reich, *Strong and weak convergence theorems for locally nonexpansive mappings in Banach spaces*, Nonlinear Aalysis **6** (1982), 151–155.

[4] D. Butnariu, S. Reich and A. J. Zaslavski, *Convergence to fixed points of inexact orbits of Bregman-monotone and of nonexpansive operators in Banach spaces*, in: Fixed Point Theory and Its Applications, Yokohama Publishers, Yokohama, 2006, pp. 11–32.

[5] Y. Censor and M. Zaknoon, *Algorithms and convergence results of projection methods for inconsistent feasibility problems: a review*, Pure Appl. Func. Anal. **3** (2018), 565–586.
[6] F. S. de Blasi and J. Myjak, *Sur la convergence des approximations successives pour les contractions non linéaires dans un espace de Banach*, C. R. Acad. Sci. Paris **283** (1976), 185–187.
[7] F. S. De Blasi and J. Myjak, *On the porosity of the set of contractions without fixed points*, C.R. Acad. Sci. Paris **308** (1989), 51–54.
[8] F. S. de Blasi, J. Myjak, S. Reich and A. J. Zaslavski, *Generic existence and approximation of fixed points for nonexpansive set-valued maps*, Set-Valued and Variational Analysis **17** (2009), 97–112.
[9] M. Edelstein, *An extension of Banach's contraction principle*, Proc. Amer. Math. Soc. **12** (1961), 7–10.
[10] M. Gabour, S. Reich and A. J. Zaslavski, *A generic fixed point theorem*, Indian J. Math. **56** (2014), 25–32.
[11] A. Gibali, *A new split inverse problem and an application to least intensity feasible solutions*, Pure Appl. Funct. Anal. **2** (2017), 243–258.
[12] A. Gibali, S. Reich and R. Zalas, *Outer approximation methods for solving variational inequalities in Hilbert space*, Optimization **66** (2017), 417–437.
[13] K. Goebel and W. A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge University Press, Cambridge, 1990.
[14] K. Goebel and S. Reich, *Uniform Convexity, Hyperbolic Geometry, and Nonexpansive Mappings*, Marcel Dekker, New York and Basel, 1984.
[15] P. D. Humke and D. Preiss, *Measures for which sigma-porous sets are null*, J. London Math. Soc. **32** (1985), 236–244.
[16] J. Jachymski, *Extensions of the Dugundji-Granas and Nadler's theorems on the continuity of fixed points*, Pure Appl. Funct. Anal. **2** (2017), 657–666.
[17] W. A. Kirk, *Contraction Mappings and Extensions*, Handbook of Metric Fixed Point Theory, Kluwer, Dordrecht, 2001, pp. 1–34.
[18] R. Kubota, W. Takahashi and Y. Takeuchi, *Extensions of Browder's demiclosedness principle and Reich's lemma and their applications*, Pure Appl. Func. Anal. **1** (2016), 63–84.
[19] D. Preiss and L. Zajiček, *Sigma-porous sets in metric spaces and sigma-directionally porous sets in Banach spaces*, Real. Anal. Exchange, **24** (1998-99), 295–313.
[20] E. Pustylnyk, S. Reich and A. J. Zaslavski, *onvergence to compact sets of inexact orbits of nonexpansive mappings in Banach and metric spaces*, Fixed Point Theory and Applications **2008** (2008), 1–10.
[21] E. Rakotch, *On ϵ-contractive mappings*, Bull. Res. Council Israel **10F** (1961), 53–58.
[22] E. Rakotch, *A note on contractive mappings*, Proc. Amer. Math. Soc. **13** (1962), 459–465.
[23] S. Reich, *Fixed points of contractive functions*, Boll. Un. Mat. Ital. **5** (1972), 26–42.
[24] S. Reich and I. Shafrir, *Nonexpansive iterations in hyperbolic spaces*, Nonlinear Analysis **15** (1990), 537–558.
[25] S. Reich and A. J. Zaslavski, *Well-posedness of fixed point problems*, Far East J. Math. Sci., Special Volume (Functional Analysis and Its Applications), Part III (2001), 393–401.
[26] S. Reich and A. J. Zaslavski, *Generic aspects of metric fixed point theory*, Handbook of Metric Fixed Point Theory, Kluwer, Dordrecht, 2001, pp. 557–575.
[27] S. Reich and A. J. Zaslavski, *The set of noncontractive mappings is σ-porous in the space of all nonexpansive mappings*, C. R. Acad. Sci. Paris **333** (2001), 539–544.
[28] S. Reich and A. J. Zaslavski, *Convergence to attractors under perturbations*, Commun. Math. Anal. **10** (2011), 57–63.
[29] S. Reich and A. J. Zaslavski, *Genericity in Nonlinear Analysis*, Developments in Mathematics, vol. 34, Springer, New York, 2014.
[30] S. Reich and A. J. Zaslavski, *Convergence and well-posedness properties of uniformly locally contractive mappings*, Topological Methods in Nonlinear Analysis **61** (2023), 761–773.
[31] S. Reich and A. J. Zaslavski, Fixed point and well-posedness properties of uniformly locally contractive mappings, J. Nonlinear Convex Anal. accepted for publication.

[32] W. Takahashi, *The split common fixed point problem and the shrinking projection method for new nonlinear mappings in two Banach spaces*, Pure Appl. Funct. Anal. **2** (2017), 685–699.
[33] W. Takahashi, *A general iterative method for split common fixed point problems in Hilbert spaces and applications*, Pure Appl. Funct. Anal. **3** (2018), 349–369.
[34] A. J. Zaslavski, *Optimization on Metric and Normed Spaces*, Springer Optimization and Its Applications. Springer, New York, 2010.
[35] A. J. Zaslavski, *Approximate solutions of common fixed point problems*, Springer Optimization and Its Applications, Springer, Cham, 2016.
[36] A. J. Zaslavski, *Algorithms for solving common fixed point problems*, Springer Optimization and Its Applications, Springer, Cham, 2018.

Manuscript received January 10 2023
revised February 23 2023

S. Reich
Department of Mathematics, The Technion – Israel Institute of Technology, 32000 Haifa, Israel
E-mail address: sreich@technion.ac.il

A. J. Zaslavski
Department of Mathematics, The Technion – Israel Institute of Technology, 32000 Haifa, Israel
E-mail address: ajzasl@technion.ac.il