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MEAN-VARIANCE ANALYSIS OF SUSTAINABLE NEWSVENDOR PROBLEMS WITH STOCHASTIC DEMAND UNDER CAP-AND-TRADE POLICY*

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This work is dedicated to Prof Fukushima to honor his outstanding contributions in optimization on his 75th birthday.

Abstract: This paper studies the single-product single-period newsvendor problem with sustainability investment, where the manufacturer determines its optimal production quantity and sustainability level under cap-and-trade policy. Specifically, the proposed model addresses the trade-off between the expected profit and its variance under the mean-variance framework. The demand under consideration is stochastic and dependent on the sustainability level. By taking into account two different forms of the underlying demand, that is, additive and multiplicative, we prove the existence and uniqueness of the optimal solution of the corresponding optimization models under some mild conditions. Further, we conduct sensitivity analysis on the optimal expected profit and its variance with respect to the risk-averse. Preliminary numerical experiments are conducted to illustrate the theoretical results and managerial insights.

Key words: *newsvendor model, mean-variance analysis, risk measure, sustainability, cap-and-trade policy*

Mathematics Subject Classification: *90B60, 91A05*

1 Introduction

The newsvendor problem is a well-known model in stochastic inventory management that has received extensive attention since Whitin [33] formulated a newsvendor model. The objective of this problem is to determine an optimal production decision of a perishable product to satisfy a stochastic demand over a single period. Two comprehensive review of the early literature on the newsvendor problem were presented by Khouja [16] and Qin et al. [26]. Currently, the newsvendor problem is extended and studied to consider the price-dependent demand (Petruzzi and Dada [25]), clearance pricing (Mitra [24]), asymmetric information (Güler et al. [14]) and data-driven optimization (Huber et al. [15]). The majority of the literature mentioned above assume that the decision maker is risk-neutral. Experimental studies and the survey show that the decision makers are inclined to have risk-averse attributes in the stochastic demand environment [21, 22]. The mean-variance (MV) approach that is a fundamental theory in portfolio management is to balance both

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the expected profit and the risk, where the risk is measured by the variance of the profit. A growing number of researchers employ the MV approach to formulate the expected utility and study the operational decisions of the risk-averse newsvendor problem [10, 18, 31].

In literature, newsvendor approaches have been proposed to investigate the impacts of carbon emission reduction on the firm's operations [5, 20, 38]. Dong et al. [11] develop a linear function to model a stochastic demand influenced by the sustainability level. The authors use a newsvendor approach to study the sustainable investment decision and coordination of a two-tier supply chain under cap-and-trade regulation. Ma et al. [19] extend a newsvendor model to that for a single-manufacturer and multi-manufacturer supply chain under an emissions trading mechanism. The authors focus on studying how information asymmetry affect the coordination of the regulated supply chain. Meng et al. [23] use a newsvendor approach to study the make-or-buy decision of the manufacturer under carbon tax regulation. Cao and Yu [4] consider the carbon permit as a financing mechanism and develop a supply chain financing model in the context of the newsvendor problem under cap-and-trade regulation. Bai et al. [2] consider a newsvendor model with limited demand information and compare the impacts of carbon tax regulation with those of carbon cap regulation on the operational decision. Bai et al. [3] develop a distributionally robust newsvendor model for a remanufacturing system under cap-and-trade regulation. The aforementioned studies are focused on the development of newsvendor models under different carbon regulations and studying how carbon emission reduction affect the operational decisions.

On the other hand, mean-variance analysis of inventory models optimizes the problem with consideration of both the expected profit and the risk [8, 13]. There has been a rapid growing popularity in studying stochastic inventory operations [6, 7, 18, 32]). Recently, Rubio-Herrero et al. [30] propose the mean-variance approach to study the impacts of the risk aversion on a newsvendor model with price-dependent demand. Some extensions have further investigated by two recent publications [28, 29]. Ray and Jenamani [27] conduct a mean-variance analysis of the order allocation decision for a supply chain with multi-supplier and single-risk-averse-buyer. Chiu et al. [9] consider a luxury fashion firm who is risk-averse and use a mean-variance approach to solve the optimal customer portfolios and budget allocation strategies. Zhang et al. [36] extend the classic newsvendor problem to that considers the preferences for mean, variance, skewness, and kurtosis of the firm's profit. In the framework of the mean-variance theory, the authors study the structural properties of the optimal operational decisions and make some extensions. Yang et al. [34] develop a mean-variance model to conduct risk analysis of the coordination for a single-supplier and capital-constrained-retailer system. Zhang et al. [37] study the effectiveness of coordinating contracts in a two-echelon supply chain under the mean-variance and mean-downside-risk objectives.

In recent years, sustainable operations have become one of the prominent issues in the global business environment because carbon emissions contribute to global warming. The production stage is the main source of emitting carbon emissions in the enterprise operations. Many countries and regions have carried out cap-and-trade policies to reduce carbon emissions. Under a cap-and-trade policy, the government sets a carbon limit to the enterprise, if the enterprise emits more carbon emissions than the carbon limit, the neeeded emissions credits are purchased from the carbon market. Otherwise, the surplus emission credits are sold to other enterprises [1]. Moreover, consumers are increasingly favoring low-carbon products, some manufacturers have to invest in sustainability technology to produce the product with green label while the production cost are increased. In this situation, when a risk-averse manufacturer invests in the sustainability technology under a cap-and-trade policy, the following issues arise: (i) What are the conditions for the existence of the optimal decisions on sustainability level and production quantity ? (ii) How does the risk-averse attribute affect the regulated manufacturer's operations?

To fill in the research gaps above, this paper considers a risk-averse manufacturer under a cap-and-trade policy who invests in the sustainability technology. In the framework of newsvendor model, we study the impacts of the sustainability level on the demand, with specific to additive and multiplicative forms of sustainability-dependent demand. To address the trade-offs between expected profit and its variance due to the ambiguity of demand fluctuation in the market, we employ the mean-variance approach to determining optimal decisions on the sustainability level and production quantity. Further, we investigate sufficient conditions for the existence and uniqueness of the optimal solution of the underlying models. Different from the existing studies in literature, this study considers carbon emission reduction in the newsvendor model, incorporating both cap-and-trade regulation and the investment in the sustainability technology by taking into account both additive and multiplicative demand. We believe the obtained results would greatly enrich and complement the field of green manufacturing and sustainable development.

The remainder of this paper is organized as follows. Section 2 presents a benchmark model of the underlying problem using mean-variance analysis. The corresponding models with an additive demand and with a multiplicative demand are discussed in Sections 3 and 4, respectively. We study the conditions for the existence and uniqueness of the optimal decisions for the latter two models. Section 5 presents numerical examples and sensitivity analysis together with managerial implications of our model. Section 6 concludes the paper.

2 Model Development

2.1 Notation

In this paper, we consider the single-product single-period newsvendor problem with sustainability investment under stochastic demand. The underlying manufacturer seeks to determine the optimal production plan in the framework of mean-variance analysis. Various cost factors are incurred in the manufacturing process, such as cost of the production, inventory holding cost or overproduction cost, investment cost regarding the sustainable development, and cost related to carbon emissions under the cap-to-trade policy. We first establish a benchmark model. In next two sections, we consider two different forms of the demand and study the corresponding models with a focus on the existence and uniqueness of the optimal solution and sensitivity analysis in terms of risk averse factor. In the following, we list the notations of parameters and variables needed in model development.

Parameters:

p: unit selling price of the product in the market,

c: unit cost of production of the manufacturer,

c_h: unit inventory holding cost of the manufacturer for leftover products,

a: the carbon emission quantity per unit when the sustainability level is zero,

b: coefficient of the sustainability effect on the carbon emission reduction,

c_e: unit carbon emission price,

c_I: coefficient of the sustainability investment,

K: total allowable carbon emissions under cap-and-trade policy,

β: coefficient of the sustainability effect on demand,

ϵ: random variable describing demand fluctuations in the market with a support set $[A, B]$, $A \in \mathbb{R}$, $B > 0$,

D: random demand of the product characterized by stochastic market fluctuations ϵ and sustainability level s,

d: fixed base demand of product,

$\mathbb{E}[\tilde{\Pi}]$: expected profit of the manufacturer,

Π: utility of the manufacturer under the mean-variance framework.

Decision variables:

s: sustainability level of the manufacturer,

x: production quantity of the manufacturer.

2.2 A benchmark model

For the demand under consideration, $D(s, \epsilon)$ is sustainability-dependent [11], where s denotes the sustainability level and ϵ is random variable characterizing demand fluctuation in the market. Let $f(\cdot)$, $F(\cdot)$ denote the probability density function and cumulative distribution function of ϵ, respectively. We assume $F(\cdot)$ is twice continuously differentiable. Following the existing literature [12], we denote $h(z)$ as the failure rate of ϵ, that is, $h(z) = f(z)/(1 - F(z))$. In the case when the production quantity x is greater than the demand $D(s, \epsilon)$, the manufacturer will bear the cost of the excess product at cost c_h per unit. In this paper, we assume the manufacturer has adequate production capacity to meet the demand. In short, the manufacturer is more concerned about the optimal production planning to avoid excess production as far as possible, rather than the issue of product shortage.

In alignment with green manufacturing and promoting sustainable development, the manufacturer invests in the sustainability technology to reduce carbon emissions generated at the production stage. Let a represent the basal quantity of carbon emissions in the production process without adoption of any sustainability technology, i.e., the sustainability level is zero. If the manufacturer produces x units of the product, $(a - bs)x$ units of carbon emissions will be emitted, where b denotes the coefficient of the sustainability effect on the emissions reduction. It is evident that the sustainability level s satisfies $0 \leq s < a/b$. Under the cap-and-trade regulation, if the total carbon emissions are higher (or lower) than an allowable quota K, the manufacturer will buy (or sell) a certain amount of carbon credits at the unit price c_e from the carbon trading market. Following Dong et al. [11], the sustainability investment cost is a quadratic function in s in form of $c_I s^2/2$, where c_I denotes the sustainability investment coefficient. In practice, the sustainability level is improved by a large investment. Using similar assumptions in [11], we assume that c_I is sufficiently high, i.e. $c_I > 2c_e b\beta$. Based on the above discussions, the expected profit of the manufacturer is given by

$$\begin{aligned}\mathbb{E}[\tilde{\Pi}(x, s)] \quad = \quad & \mathbb{E}[p \min\{D(s, \epsilon), x\}] - cx - \mathbb{E}[c_h(x - D(s, \epsilon))^+] \\ & -c_e((a - bs)x - K) - \frac{c_I}{2}s^2. \end{aligned} \tag{2.1}$$

In equation (2.1), the first term represents the manufacturer's revenue from selling the products, the second term is the manufacturing cost of the products, the third term refers to the leftover cost or holding cost of the product, the fourth term denotes the cost (or revenue) from trading the emission credits, and the last term is the sustainability investment. The underling manufacturer is risk-averse in decision making in that he/she likes to maximize the expected profit and also seeks to minimize the variance of its profit. Due to this, we then apply the mean-variance method to determine the joint optimal decisions of production quantity and sustainability level by maximizing the utility Π of of the manufacturer, namely

$$\begin{aligned} &\Pi(x,s) = \mathbb{E}[\tilde{\Pi}(x,s)] - \lambda Var[\tilde{\Pi}(x,s)] \\ = \; &(p+c_h)\mathbb{E}[\min\{x, D(s,\epsilon)\}] - (c+c_h)x - c_e((a-bs)x-K) \\ &-\frac{c_I}{2}s^2 - \lambda Var(p+c_h)\min\{x, D(s,\epsilon)\}, \end{aligned} \tag{2.2}$$

where $\lambda > 0$ is a risk-averse parameter, representing the degree of the manufacturer's risk-averse. Thus, we have the mean-variance optimization model of the underlying newsvendor problem as follows.

$$\begin{aligned} \max \quad & \Pi(x,s) \\ \text{s.t.} \quad & 0 \le s < \frac{a}{b}, \\ & x \ge 0. \end{aligned} \tag{2.3}$$

Note that when $\lambda = 0$, model (2.3) will reduce to the usual expected profit maximization problem in risk-neutral decision-making. In general, the objective function $\Pi(x,s)$ is not tractable. Furthermore, it is impossible to derive optimal solution (x^*, s^*) of (2.3) in a closed form even if exists. In this paper, our interest is to investigate the existence and uniqueness of the optimal solution by considering the demand in form of additive and multiplicative expressions in the sustainability level s following discussions in literature, which will be elaborated in subsequent sections.

Before end of this section, we introduce some important notions of newsvendor problems as discussed in [17] to analyze economic significance and managerial insights of our optimization model.

Definition 2.1. For a given sustainability level s and inventory level x, the lost sales rate (LSR) elasticity is defined as

$$\tilde{\xi}(s,x) = \frac{sG_s(s,x)}{1-G(s,x)} \tag{2.4}$$

where $G(s,x) := P(D(s,\epsilon) \le x)$ and $G_s(s,x) := \partial G(s,x)/\partial s$.

The LSR elasticity describes how the probability of fulfilling demand (i.e., $P(D(s,\epsilon) \le x)$) changes as we increase the sustainability level of the product. More specifically, for a given safety stock, the LSR elasticity represents the percentage change in the rate of sales loss relative to the change in the sustainability level.

Definition 2.2. The elasticity of the optimal sustainability level, ρ, measures the percentage change in the optimal sustainability level $s^*(z)$ when there is a one percent change in the safety stock z, defined by

$$\rho(z) := \frac{ds^*(z)}{dz}\frac{z}{s^*(z)}. \tag{2.5}$$

Definition 2.3. The elasticity of the expected safety stock surplus (ESSS elasticity), ω, measures the percentage change in the expected excess of safety stock when there is a one percent change in the safety stock z, defined by

$$\omega(z) := \frac{d\mathbb{E}[(z-\epsilon)^+]}{dz}\frac{z}{\mathbb{E}[(z-\epsilon)^+]}. \tag{2.6}$$

Note that the expected safety stock surplus $\mathbb{E}[(z-\epsilon)^+] \equiv z - \mu(z)$, where $\mu(z) = \mathbb{E}[\min\{z,\epsilon\}]$. By definition, the expected safety stock surplus is positive and an increase in the safety stock will inevitably produce an increment in this expectation. Then, $\omega(\cdot)$ is a nonnegative function.

By incorporating these definitions into our analysis, we could better understand the relevant conditions regarding the existence and uniqueness of the models discussed in Section 3 and 4 from economic perspective, enhancing our analysis in a managerial context, not merely deductions and explanations using mathematical terms.

3 The model with an additive demand

In this section, we consider an additive demand in form of

$$D(s,\epsilon) = y(s) + \epsilon, \tag{3.1}$$

where $y(s) = d + \beta s$ with $d > 0$ and $\beta > 0$, d is the base demand and β is the coefficient of the sustainability effect on demand. To ensure the well-definedness (i.e., the non-negativeness) of the demand, we have $A \geq -d$. In addition, we make the following assumption on the uncertain demand fluctuation ϵ.

Assumption 3.1. (i) ϵ is a random variable with finite variance $Var(\epsilon)$ and compact support interval $[A,B]$, where $A < 0$ and $B > 0$. (ii) $\mathbb{E}(\epsilon) = 0$.

The above assumption is mild and reasonable. Assumption 3 (i) holds because if ϵ is a random variable defined on an open interval, following certain distribution such as an exponential or a normal distribution, we can consider an efficient truncation to capture more information with an closed support interval [A, B]. Assumption 3 (ii) is intuitive concerning random variations (i.e., increase and decrease) in demand fluctuation in the market.

To ease our analysis, in what follows we reformulate the underlying model using a transformation $z = x - y(s)$. Here, z refers to a safety stock factor implying that the difference between the stock quantity x and the risk-free demand with a sustainability level s, as stated in [25, 30]. By Assumption , we have $y(s) + A \leq x \leq y(s) + B$. Applying basic mathematical operations and by replacing x by $z + y(s)$ using the transformation, the utility function $\Pi(x,s)$ can be written as

$$\begin{aligned}
&\Pi(s,z)\\
=\;& (p+c_h)\mathbb{E}[\min\{y(s)+z, y(s)+\epsilon\}] - (c+c_h)(y(s)+z) - c_e((a-bs)(y(s)+z)\\
&+c_eK - \frac{c_I}{2}s^2 - \lambda Var(p+c_h)\min\{y(s)+z, y(s)+\epsilon\}\\
=\;& (p+c_h)(y(s)+\mathbb{E}[\min\{z,\epsilon\}]) - (c+c_h)(y(s)+z) - c_e((a-bs)(y(s)+z)-K)\\
&-\frac{c_I}{2}s^2 - \lambda Var(p+c_h)(y(s)+\min\{z,\epsilon\}) \qquad (3.2)\\
=\;& (p+c_h)(y(s)+\mu(z)) - (c+c_h)(y(s)+z) - c_e((a-bs)(y(s)+z)-K)\\
&-\frac{c_I}{2}s^2 - \lambda((p+c_h))^2\sigma^2(z),
\end{aligned}$$

where

$$\mu(z) = \mathbb{E}[\min\{z,\epsilon\}] = \int_z^B (z-u)f(u)du, \tag{3.3}$$

$$\begin{aligned}\sigma^2(z) &= Var[\min\{z,\epsilon\}] \\ &= Var(\epsilon) + \int_z^B (z^2-u^2)f(u)du - \left[\int_z^B (z-u)f(u)du\right]^2.\end{aligned} \tag{3.4}$$

Note that $\mu(\cdot)$ is an increasing and concave function since $\mu'(z) = 1 - F(z) > 0$ and $\mu''(z) = -f(z) < 0$. In addition, $\sigma^2(\cdot)$ is increasing as its first order derivative $\sigma^2(z)' = 2(1-F(z))(z-\mu(z)) > 0$. Then, we have the optimization model under additive demand as follows.

$$\begin{aligned}\max \quad & \Pi(s,z) := (p+c_h)(y(s)+\mu(z)) - (c+c_h)(y(s)+z) \\ & -c_e((a-bs)(y(s)+z)-K) - \frac{c_I}{2}s^2 - \lambda((p+c_h))^2\sigma^2(z) \\ \text{s.t.} \quad & 0 \le s < \frac{a}{b}, \quad z \in [A,B].\end{aligned} \tag{3.5}$$

In what follows, we use the optimization method in [35] to solve the optimal values of sustainability level and stock factor. The procedures are as follows. First, for any given value of the safety stock z, we solve the optimal sustainability level $s^*(z)$ that maximizes $\Pi(s,z)$. Then, substituting the optimal sustainability level into the objective function of (3.5) and simplifying its expression, we solve the optimal safety stock z^* that maximizes $\Pi(s^*,z)$.

Lemma 3.1. *For any given $z \in [A,B]$, the following holds. (i) $s^*(z)$ is strictly positive in z. (ii) $s^*(z)$ is increasing in z. (iii) $s^*(z)$ are increasing in β and b, and decreasing in c_I.*

Proof. (i) For any given z, take the first order derivative of $\Pi(s,z)$ with respect to s and solve $\frac{\partial \Pi}{\partial s} = 0$, it follows that

$$s^*(z) = \frac{(c_e a + c - p)\beta - c_e b(z+d)}{2c_e b\beta - c_I}. \tag{3.6}$$

Noticing that $\Pi(s,z)$ is a concave function with respect to s because $\partial^2\Pi(s,z)/\partial s^2 = 2c_e b\beta - c_I \le 0$. Then, the optimal sustainability level s^* exists. In addition, $2c_e b\beta - c_I < 0$ and $p > c + c_e a$, we then have $s^*(z) > 0$ as $z \ge A \ge -d$.

(ii) It follows from (3.6) that $\frac{\partial s^*(z)}{\partial z} = \frac{-c_e b}{2c_e b\beta - c_I} > 0$, which implies $s^*(z)$ is monotonically increasing in z.

(iii) With help of equation (3.6), taking the first order derivative of $s^*(z)$ with respect to β, it gives that

$$\frac{\partial s^*(z)}{\partial \beta} = \frac{(c_e a + c - p)(2c_e b\beta - c_I) - 2[(c_e a + c - p)\beta - c_e b(z+d)]c_e b}{(2c_e b\beta - c_I)^2}. \tag{3.7}$$

Since $p > c + c_e a$, $z \ge A \ge -d$, and $2c_e b\beta - c_I < 0$, we then have $\frac{\partial s^*(z)}{\partial \beta} > 0$. Similarly, we have

$$\frac{\partial s^*(z)}{\partial b} = \frac{-c_e(z+d)(2c_e b\beta - c_I) - 2[(c_e a + c - p)\beta - c_e b(z+d)]c_e\beta}{(2c_e b\beta - c_I)^2} > 0, \tag{3.8}$$

and

$$\frac{\partial s^*(z)}{\partial c_I} = \frac{(c_e a + c - p)\beta - c_e b(z+d)}{(2c_e b\beta - c_I)^2} < 0. \tag{3.9}$$

Thus, the desired result follows immediately by virtue of (3.7) – (3.9). □

According to Lemma 3.1, we have the following result, stating an upper bound on the optimal stock z^*.

Lemma 3.2. *For model (3.5) with an additive demand, the optimal safety stock z^* satisfies*

$$z^* < F^{-1}\left(\frac{p - c - c_e(a - bs^*(z))}{p + c_h}\right).$$

Proof. Substituting $s^*(z)$ into the utility function of (3.2), we have

$$\begin{aligned}\Pi(z) = \Pi(s^*(z), z) &= (p + c_h)(y(s^*(z)) + \mu(z)) - (c + c_h)(z + y(s^*(z))) \\ &-c_e((a - bs^*(z))(z + y(s^*(z)) - K) - \frac{c_I}{2}(s^*(z))^2 - \lambda(p + c_h)^2\sigma^2(z).\end{aligned} \tag{3.10}$$

Taking the first order derivative of $\Pi(z)$ with respect to z and from (3.10), we have

$$\frac{\partial \Pi(z)}{\partial z} = (p + c_h)\mu'(z) - (c + c_h) - c_e(a - bs^*(z)) - \lambda(p + c_h)^2{\sigma^2}'(z). \tag{3.11}$$

Solving the equation $\frac{\partial \Pi(z)}{\partial z} = 0$, it gives that

$$\lambda = \frac{(p + c_h)(1 - F(z)) - (c + c_h) - c_e(a - bs^*(z))}{2(p + c_h)^2(1 - F(z))(z - \mu(z))}. \tag{3.12}$$

Since $\lambda > 0$, we have $(p + c_h)(1 - F(z)) - (c + c_h) - c_e(a - bs^*(z)) > 0$. By virtue of (3.6), it follows that

$$z^* < F^{-1}\left(\frac{p - c - c_e(a - bs^*(z)}{p + c_h}\right). \tag{3.13}$$

□

According to Definition 2.1, the LSR elasticity with respect to the additive demand can be rewritten as

$$\tilde{\xi}(s, x) = \frac{sG_s(s, x)}{1 - G(s, x)} = \frac{-s\beta f(z)}{1 - F(z)} =: \xi(s, z). \tag{3.14}$$

Then, $\xi(s^*(z), z) = \xi^*(z) = -\beta s^*(z)h(z)$ holds at the point of the optimal sustainability level $s^*(z)$, where $h(z)$ represents the failure rate of ϵ. Let $\eta(A) = \tilde{c} + c_e(a - bs^*(A)$ and $\tilde{c} = c + c_h$.

According to Lemma 3.2 and (3.14), we have the following result, which is one of main results of this paper.

Theorem 3.3. *For model (3.5) with an additive demand, if the LSR elasticity satisfies*

$$\xi^*(z) < \frac{\beta b c_e}{2c_e b\beta - c_I}\left[\frac{\omega(z)}{\rho(z)} + c_e b\frac{s^*(B)}{\eta(A)}\right], \tag{3.15}$$

then the utility $\Pi(s^*(z), z)$ is quasiconcave in $[A, B]$. Moreover, there exists a unique solution $(z^*, s^*(z^*))$ that maximizes the utility of the manufacturer.

Proof. Since $\Pi(z)$ is a continuous in $[A,B]$, it follows from (3.11) that there is at least one solution on $[A,B]$ satisfying $\Pi'(z)=0$. Since $s^*(z)\in[0,a/b)$, we have $\Pi^{'}(A)=p-c-c_e(a-bs'(A))>0$ and $\Pi'(B)=-\tilde{c}-c_e(a-bs^*(B))<0$.

By the expressions of $\mu'(z)$ and $\sigma^2(z)'$ and equation (3.11), we rewrite the first order equation $\frac{\partial\Pi(z)}{\partial z}=0$ as

$$1-2\lambda\tilde{p}(z-\mu(z))=\tilde{c}+c_e(a-bs^*(z))/\tilde{p}(1-F(z)), \tag{3.16}$$

and

$$2\lambda\tilde{p}^2(1-F(z))=(\tilde{p}(1-F(z))-\tilde{c}-c_e(a-bs^*(z)))/(z-\mu(z)). \tag{3.17}$$

Taking the second order derivative of $\Pi(z)$ with respect to z and with help of equation (3.11), we have

$$\frac{\partial^2\Pi(z)}{\partial z^2}=(p+c_h)(-f(z))-\frac{(c_eb)^2}{2c_eb\beta-c_I}-\lambda(p+c_h)^2\sigma^{2''}(z), \tag{3.18}$$

and

$$\frac{\partial^2\Pi(z)}{\partial z^2}=-\frac{F(z)}{z-\mu(z)}(\tilde{p}(1-F(z))-\tilde{c}-c_e(a-bs^*(z)))+c_ebs^{*\prime}(z)-h(z)(\tilde{c}+c_e(a-bs^*(z))), \tag{3.19}$$

where $\tilde{p}=c+c_h$, $\tilde{c}=c+c_h$.

By assumption, we have $\xi^*(z)<\frac{\beta bc_e}{2c_eb\beta-c_I}\left[\frac{\omega(z)}{\rho(z)}+c_eb\frac{s^*(B)}{\eta(A)}\right]$. By virtue of (3.6) and $s^{*\prime}(z)=c_eb/(c_I-2c_eb\beta)$, we have

$$\begin{aligned}\frac{\xi^*(z)}{\beta s^{*\prime}(z)} &\le -\frac{\omega(z)}{\rho(z)}-\frac{c_ebs^*(B)}{\eta(A)}\\ &\le \frac{\omega(z)}{\rho(z)}(\frac{\tilde{p}(1-F(z))}{\eta(A)}-1)-\frac{c_ebs^*(B)}{\eta(A)}\\ &< \frac{\omega(z)}{\rho(z)}(\frac{\tilde{p}(1-F(z))}{\tilde{c}+c_e(a-bs^*(z))}-1)-\frac{c_ebs^*(z)}{\tilde{c}+c_e(a-bs^*(z))},\end{aligned} \tag{3.20}$$

where $\eta(A)=\tilde{c}+c_e(a-bs^*(A))$. By equation (3.20), it yields that

$$-\frac{\omega(z)}{\rho(z)}(\tilde{p}(1-F(z))-\tilde{c}-c_e(a-bs^*(z)))+c_ebs^*(z)+\frac{\xi^*(z)}{\beta s^{*\prime}(z)}(\tilde{c}+c_e(a-bs^*(z)))<0,$$

from which, we have $\Pi^{''}(z)|_{\Pi'(z)=0}<0$. Hence, $\Pi(z)$ is a concave function of z. Thereby there exists a unique optimal solution $(z^*,s^*(z^*))$ to maximize the utility of the manufacturer as desired. □

4 The model with a multiplicative demand

In this section, we consider the model with a multiplicative demand in form of $D(s,\epsilon)=y(s)\epsilon$, where $y(s)=ds^\beta$ with $d>0$, $\beta>0$. The multiplicative demand under consideration has been widely studied in literature such as [29, 39]. Similarly, we make the following assumption on the random variable ϵ.

Assumption 4.1. $\mathbb{E}(\epsilon)=1$ and $\epsilon\in[A,B]$ with $0<A<1<B$.

With similar arguments as the case of the additive demand previously, we use the transformation of $z = x/y(s)$ to reformulate the underlying model. It is not hard to see that $z \in [A, B]$ and $x \in [y(s)A, y(s)B]$. Accordingly, the utility $\Pi(x, s)$ in (2.2) can be rewritten as

$$\begin{aligned}\Pi(s,z) &= (p+c_h)\mathbb{E}[\min\{y(s)z, y(s)\epsilon\}] - (c+c_h)y(s)z - c_e((a-bs)y(s)z-k) \\ &\quad -\frac{c_I}{2}s^2 - \lambda Var(p+c_h)\min\{y(s)z, y(s)\epsilon\} \\ &= (p+c_h)y(s)\mathbb{E}[\min\{z,\epsilon\}] - (c+c_h)y(s)z - c_e((a-bs)y(s)z-k) \\ &\quad -\frac{c_I}{2}s^2 - \lambda Var(p+c_h)y(s)\min\{z,\epsilon\} \\ &= (p+c_h)y(s)\mu(z) - (c+c_h)y(s)z - c_e((a-bs)y(s)z-k) \\ &\quad -\frac{c_I}{2}s^2 - \lambda((p+c_h)y(s))^2\sigma^2(z), \end{aligned} \tag{4.1}$$

where

$$\mu(z) = \mathbb{E}[\min\{z,\epsilon\}] = \int_z^B (z-u)f(u)du + 1, \tag{4.2}$$

$$\begin{aligned}\sigma^2(z) &= Var[\min\{z,\epsilon\}] \\ &= z^2 - B^2 + 2\int_z^B uF(u)du + Var(\epsilon) + 1 - \mu^2(z). \end{aligned} \tag{4.3}$$

Note that $\mu'(z) = 1 - F(z) > 0$ and $\mu''(z) = -f(z) < 0$, hence $\mu(z)$ is increasing and concave. Again, σ^2 is an increasing function since ${\sigma^2}'(z) = 2(1-F(z))(z-\mu(z)) > 0$. Then, the corresponding optimization model can be reformulated as

$$\begin{aligned}\max \quad & \Pi(s,z) := p+c_h)y(s)\mu(z) - (c+c_h)y(s)z - c_e((a-bs)y(s)z-k) \\ & -\frac{c_I}{2}s^2 - \lambda((p+c_h)y(s))^2\sigma^2(z), \\ \text{s.t.} \quad & 0 \le s < \frac{a}{b}, \ \ z \in [A, B]. \end{aligned} \tag{4.4}$$

With help of equations (4.1) – (4.3), we obtain the following result.

Lemma 4.1. *If $s^*(z)$ is the optimal solution of problem (4.4) with respect to any fixed $z \in [A, B]$, then the following condition holds.*

$$\tilde{p}\mu(z)\beta - \tilde{c}z\beta - c_e az\beta + c_e zb(\beta+1)s - d^{-1}c_I s^{2-\beta} - 2\lambda\tilde{p}^2 d\beta s^\beta\sigma^2(z) = 0, \tag{4.5}$$

where $\tilde{p} = p + c_h$ and $\tilde{c} = c + c_h$.

It can be seen that the case of the solution of Eq. (4.5) with respect to $s^*(z)$ is very complicated and related to β. When $\beta \neq 1$, it is very difficult to obtain a closed-form expression with respect to $s^*(z)$, which is a great hindrance to our subsequent analysis. Therefore, in what follows, we concentrate on the case of the demand with $\beta = 1$, i.e., $D(s,\epsilon) = ds\epsilon$. First, from Lemma 4.1, we have the following results.

Theorem 4.2. *For any given $z \in [A, B]$, if $2c_e bz - d^{-1}c_I - 2\lambda\tilde{p}^2 d\sigma^2(z) < 0$, the utility function $\Pi(s, z)$ is concave in s. Moreover, if $\tilde{p}\mu(z) - \tilde{c}z - c_e az > 0$, then the optimal sustainability level $s^*(z)$ exists and*

$$s^*(z) = \frac{-\tilde{p}\mu(z) + \tilde{c}z + c_e az}{2c_e bz - d^{-1}c_I - 2\lambda\tilde{p}^2 d\sigma^2(z)}.$$

Proof. From (4.1) – (4.3), we have

$$\Pi(s,z) = (\tilde{p}d\mu(z) - \tilde{c}zd - c_e azd)s + c_e K + (c_e bzd - \frac{c_I}{2} - \lambda\tilde{p}^2 d^2\sigma^2(z))s^2. \tag{4.6}$$

Taking the first and second order derivatives of $\Pi(s,z)$ with respect to s, it gives that

$$\frac{\partial\Pi(s,z)}{\partial s} = \tilde{p}d\mu(z) - \tilde{c}zd - c_e azd + 2s(c_e bzd - \frac{c_I}{2} - \lambda\tilde{p}^2 d^2\sigma^2(z)), \tag{4.7}$$

$$\frac{\partial^2\Pi(s,z)}{\partial s^2} = 2(c_e bzd - \frac{c_I}{2} - \lambda\tilde{p}^2 d^2\sigma^2(z)). \tag{4.8}$$

By assumption, $2c_e bz - d^{-1}c_I - 2\lambda\tilde{p}^2 d\sigma^2(z) < 0$. Then, from (4.8), we can easily see that the utility function $\Pi(z,s)$ is concave in s. Since $\tilde{p}\mu(z) - \tilde{c}z - c_e az > 0$, there exists a unique solution $s^*(z)$. By virtue of (4.7) and solving $\frac{\partial\Pi(s,z)}{\partial s} = 0$, we have

$$s^*(z) = \frac{-\tilde{p}\mu(z) + \tilde{c}z + c_e az}{2c_e bz - d^{-1}c_I - 2\lambda\tilde{p}^2 d\sigma^2(z)}. \tag{4.9}$$

Since $0 \le s < \frac{a}{b}$, $s^*(z)$ is the optimal sustainability level when $s^*(z) < \frac{a}{b}$. □

Note that in this case the corresponding LSR elasticity in Definition 2.1 becomes to

$$\tilde{\xi}(s,x) = \frac{sG_s(s,x)}{1 - G(s,x)} = \frac{-z\beta f(z)}{1 - F(z)} =: \xi(z). \tag{4.15}$$

Following the above discussion, we have the following results.

Theorem 4.3. *For the multiplicative demand d with $\beta = 1$, if the LSR elasticity satisfies*

$$\xi(z) > \omega(z) + \frac{(\tilde{p} - \tilde{c} - c_e a)^2}{(\tilde{p}/B - \tilde{c} - c_e a)\tilde{c}}, z \in [A,B], \tag{4.16}$$

then the utility function $\Pi(s^*(z),z)$ is a unimodal function, where $s^*(z)$ is in form of (4.9). Moreover, there exists a unique optimal solution $(s^*(z^*), z^*)$ that maximizes the utility of the manufacturer.

Proof. Substituting $\beta = 1$ and $s^*(z)$ into the utility function $\Pi(s,z)$ in (4.1), it follows that

$$\begin{aligned} &\Pi(z) = \Pi(s^*(z),z) \qquad (4.17)\\ = \;&(\tilde{p}d\mu(z) - \tilde{c}zd - c_e azd)s^*(z) + c_e K + (c_e bzd - \frac{c_I}{2} - \lambda\tilde{p}^2 d^2\sigma^2(z))(s^*(z))^2. \end{aligned}$$

Taking the first order derivative of $\widetilde{\Pi}(z)$ with respect to z, we have

$$\Pi'(z) = (\tilde{p}d\mu'(z) - \tilde{c}d - c_e ad)s^*(z) + (c_e bd - \lambda\tilde{p}^2 d^2\sigma^{2\prime}(z))(s^*(z))^2 = s^*(z)dR(z), \tag{4.18}$$

where

$$R(z) = \tilde{p}\mu'(z) - \tilde{c} - c_e a + (c_e b - \lambda\tilde{p}^2 d\sigma^{2\prime}(z))s^*(z). \tag{4.19}$$

Since $R(z)$ is continuous in $[A,B]$, then there is at least one solution $z \in [A,B]$ such that $\Pi'(z) = 0$ or $R(z) = 0$. Noticing that $s^*(z) \in [0, a/b)$, it follows that $R(A) = \tilde{p} - \tilde{c} - c_e a + c_e bs^*(A) > 0$ and $R(B) = -\tilde{c} - c_e a + c_e bs^*(B) < 0$. In the following, we'll study properties of $R(x)$ so as to prove the concavity of $\Pi(z)$.

We first consider the first order derivative of $R(z)$ as follows

$$\begin{aligned} R'(z) &= \tilde{p}\mu''(z) - \lambda\tilde{p}^2 d\sigma^{2\prime\prime}(z)s^*(z) + (c_e b - \lambda\tilde{p}^2 d\sigma^2(z))s^*(z) \\ &= -\tilde{p}f(z) - 2\lambda\tilde{p}^2 d((1-F(z))F(z) - f(z)(z-\mu(z)))s^*(z) \\ &\quad +(c_e b - 2\lambda\tilde{p}^2 d(1-F(z))(z-\mu(z)))s^{*\prime}(z), \end{aligned} \tag{4.20}$$

from which, it yields

$$\begin{aligned} R'(z)\Big|_{R(z)=0} &= -\tilde{p}f(z) - 2\lambda\tilde{p}^2 d((1-F(z))F(z) - f(z)(z-\mu(z)))s^*(z) \\ &\quad +(-\tilde{p}\mu'(z) + \tilde{c} + c_e a)\frac{s^{*\prime}(z)}{s^*(z)}, \end{aligned} \tag{4.21}$$

and

$$\begin{aligned} R'(z)\Big|_{R(z)=0} &= [1-F(z)]\{h(z)\tilde{p}(2\lambda\tilde{p}d(z-\mu(z))s^*(z) - 1) - 2\lambda\tilde{p}^2 dF(z)s^*(z)\} \\ &\quad - (\tilde{p}(1-F(z)) - \tilde{c} - c_e a)\frac{s^{*\prime}(z)}{s^*(z)}. \end{aligned} \tag{4.22}$$

It follows from $R(z) = 0$, we have

$$2\lambda\tilde{p}d[z-\mu(z)]s^*(z) - 1 = \frac{-\tilde{c} - c_e a + c_e b s^*(z)}{\tilde{p}[1-F(z)]},$$

and

$$2\lambda\tilde{p}^2 dF(z)s^*(z) = \frac{\tilde{p}F(z)[1-F(z)]}{[1-F(z)][z-\mu(z)]} - \frac{\tilde{c} + c_e a - c_e b s^*(z)}{[1-F(z)][z-\mu(z)}.$$

Then, (4.22) can be simplified as

$$\begin{aligned} R'(z)\Big|_{R(z)=0} &= h(z)(-\tilde{c} - c_e a + c_e b s^*(z)) - \frac{F(z)(\tilde{p}(1-F(z)) - \tilde{c} - c_e a + c_e b s^*(z))}{z-\mu(z)} \\ &\quad -(\tilde{p}(1-F(z)) - \tilde{c} - c_e a)\frac{s^{*\prime}(z)}{s^*(z)}. \end{aligned} \tag{4.23}$$

From (4.9), we have

$$s^{*\prime}(z) = s^*(z)\frac{(\tilde{p}(1-F(z)) - \tilde{c} - c_e a) - (2c_e b - 2\lambda\tilde{p}^2 d\sigma^{2\prime}(z))s^*(z)}{s^*(z)(2c_e bz - d^{-1}c_I - 2\lambda\tilde{p}^2 d\sigma^2(z))}.$$

and

$$\begin{aligned} \frac{s^{*\prime}(z)}{s^*(z)}\Big|_{R(z)=0} &= \frac{\tilde{p}(1-F(z)) - \tilde{c} - c_e a}{s^*(z)(2c_e bz - d^{-1}c_I - 2\lambda\tilde{p}^2 d\sigma^2(z))} \\ &= \frac{\tilde{p}(1-F(z)) - \tilde{c} - c_e a}{-\tilde{p}\mu(z) + \tilde{c}z + c_e az}. \end{aligned} \tag{4.24}$$

By assumption, $\xi(z) > \omega(z) + \frac{(\tilde{p}-\tilde{c}-c_e a)^2}{(\tilde{p}/B-\tilde{c}-c_e a)\tilde{c}}$. Then, by virtue of (4.9) and (4.24), it gives that

$$\begin{aligned}
\xi(z) &\geq \omega(z) + \frac{(\tilde{p} - \tilde{c} - c_e a)^2}{(\tilde{p}/B - \tilde{c} - c_e a)\tilde{c}} \qquad (4.25)\\
&\geq \omega(z) + \frac{z(\tilde{p} - \tilde{c} - c_e a)^2}{(\tilde{p}\mu(z) - \tilde{c}z - c_e az)(\tilde{c} + c_e a - c_e bs^*(z))}\\
&> -\omega(z)\left(\frac{\tilde{p}(1 - F(z))}{\tilde{c} + c_e a - c_e bs^*(z)} - 1\right) - \frac{z(\tilde{p}(1 - F(z)) - \tilde{c} - c_e a)^2}{(-\tilde{p}\mu(z) + \tilde{c}z + c_e az)(\tilde{c} + c_e a - c_e bs^*(z))}.
\end{aligned}$$

Recalling $h(z) = \frac{f(z)}{1-F(z)}$, we rearrange (4.25) as

$$h(z) > -\frac{F(z)}{z - \mu(z)}\left(\frac{\tilde{p}(1 - F(z))}{\tilde{c} + c_e a - c_e bs^*(z)} - 1\right) - \frac{\tilde{p}(1 - F(z)) - \tilde{c} - c_e a}{\tilde{c} + c_e a - c_e bs^*(z)} \frac{s^{*\prime}(z)}{s^*(z)}. \qquad (4.26)$$

By virtue of (4.23), (4.24) and (4.26), we then have $R'(z)|_{R(z)=0} < 0$, implying that $\Pi(z)$ is a unimodal function of z. Therefore, there exists a unique optimal solution $(s^*(z^*), z^*)$ that maximizes the utility of the manufacturer, where $(s^*(z^*), z^*)$ is determined by (4.9) and (4.18). This completes the proof. □

5 Numerical Examples and Sensitivity Analysis

5.1 Numerical examples

To illustrate the theoretical properties of optimal solutions derived in the previous section, following the literature [7] and [11], we conduct preliminary numerical experiments on artificial examples with different forms of the demand. These problems are solved using the software MATLAB V9.5.0 in a personal computer with Windows 10 operating system. By setting various values of risk-averse parameter λ, we compare the derived numerical results of interest, such as the manufacturer's total carbon emissions C, optimal utility Π^*, optimal stock level z^*, optimal sustainability level s^*, and optimal production quantity x^*.

1. Example with additive demand

In this example, we consider the underlying problem with the additive demand in form of $D(s, \epsilon) = 90 + s + \epsilon$ where $\epsilon \sim N(0, 5^2)$, $A = -10$ and $B = 10$. The values of other parameter are set as follows. $p = 150$, $c = 50$, $c_h = 0$, $c_e = 8$, $a = 8$, $b = 0.5$, $c_I = 35$, $K = 200$. Table 1 shows numerical results for different values of risk-averse parameter λ.

Table 1: Numerical results with different values of λ under the additive demand

λ	z^*	$s^*(z^*)$	x^*	Π^*	C
0	-9.7616	13.2205	93.4589	8472.8219	129.8843
0.2	-1.6017	14.4294	102.8277	7247.6121	80.7516
0.4	-1.2645	14.4793	103.2149	7111.3428	78.4775
0.6	-1.1496	14.4964	103.3468	7007.8174	77.6982
0.8	-1.0916	14.5049	103.4133	6912.7094	77.3044
1	-1.0567	14.5101	103.4534	6821.0067	77.0667

2. Example with multiplicative demand

In this example, we consider the problem with the multiplicative demand in form of $D(s, \epsilon) = 10^2 s\epsilon$, where $\epsilon \sim U(0.001, 1.999)$, $A = 0.001$, $B = 1.999$. The values of other parameter are set as follows. $p = 300$, $c = 50$, $c_h = 0$, $c_e = 10$, $a = 5$, $b = 0.5$, $c_I = 25$, $K = 200$. Table 2 shows the numerical results for different values of risk-averse parameter λ.

Table 2: Numerical results with different values of λ under the additive demand

λ	z^*	$s^*(z^*)$	x^*	Π^*	C
0	0.5784	0.7812	45.1846	2238.4752	208.2739
0.2	0.5865	0.3998	23.4483	1860.3781	112.5540
0.4	0.5908	0.2024	11.9578	1825.6972	58.5788
0.6	0.5923	0.1355	8.0257	1768.1443	39.5846
0.8	0.5931	0.1018	6.0378	1654.2731	29.8815
1	0.5935	0.0816	4.8430	1308.3635	24.0172

Tables 1 and 2 demonstrate the risk-averse parameter plays an important role for the manufacturer in production planning to address trade-offs involved in green manufacturing. If the manufacturer likes to improve the utility, he/she would generate higher the carbon emissions, which is again regulated by the cap-and-trade policy. On the other hand, if the manufacturer seeks to minimize the carbon emissions alone, this results in the low utility of the manufacturer. It is important to balance this trade-off for the manufacturer in decision making. Interestingly, this can be leveraged using the underlying risk-averse parameter, following in the above tables.

5.2 Sensitivity analysis

In this section, we analyze how the optimal sustainability level varies as a function of the risk parameter λ for a given safety stock z for the problem with multiplicative demand. Let $\tilde{s}^*(\cdot, z)$ be a function of λ, denoting the optimal sustainability level for a given safety stock z.

Proposition 5.1. *Let $\lambda > 0$, $\beta = 1$. For a given safety stock z, the optimal sustainability level is a nonincreasing function with respect to λ.*

Proof. For a given safety stock z, let $\Pi(\lambda, s, z)$ denote the objective function as a function of λ and s. Let $g(\lambda, s) := \partial\Pi(\lambda, s, z)/\partial s$. Then, we have

$$\frac{d\tilde{s}^*(\lambda, z)}{d\lambda} = -\left.\frac{\frac{\partial g(\lambda,s)}{\partial \lambda}}{\frac{g(\lambda,s)}{\partial s}}\right|_{s=\tilde{s}^*(\lambda,z)} = -\frac{-2\tilde{s}^*(\lambda, z)p^2 d^2 \tilde{\sigma}^2(z)}{\left.\frac{\partial^2\Pi(\lambda,s,z)}{\partial s^2}\right|_{s=\tilde{s}^*(\lambda,z)}} \leq 0.$$

Since $\partial^2\Pi(\lambda, s, z)/\partial s^2 < 0$ when $s = \tilde{s}^*(\lambda, z)$ as discussed in Theorem 4.2. Thus, the optimal sustainability level does not increase with λ. □

Proposition 5.1 shows that, given a safety stock z, the optimal sustainability level decreases as the degree of risk-aversion increases, due to the fact that in this case the variance of demand is increasing with respect to the sustainability level. i.e., $Var(D(s, \epsilon)) = Var(\epsilon)y(s)^2$. Thus, when increasing λ in the risk-averse case, the rise in the sustainability level increases the expected demand $y(s)$, which in turn increases the variance of stochastic demand.

Next, we study the relationship between the profit, its variance and the risk-averse parameter λ. Let $\Pi^*(\lambda, z)$ be a random variable denoting the profit, as a function of the risk parameter λ, at a given safety stock z.

Proposition 5.2. *Let $\lambda > 0$, $\beta = 1$. For a given a safety stock z, the variance of profit decreases with the increase of λ.*

Proof. From (4.1), we have $Var(\Pi^*(\lambda, z) = \tilde{p}^2 d^2 s^2 \sigma^2(z)$. According to Proposition 5.1, it follows that

$$\frac{d}{d\lambda} Var(\Pi^*(\lambda, z)) = 2\tilde{p}^2 d^2 s \sigma^2(z) \frac{d\tilde{s}^*(\lambda, z)}{d\lambda} \leq 0.$$

The result as desired follows immediately. □

Proposition 5.3. *Let $\lambda > 0$, $\beta = 1$. For a given safety stock z, the expected profit decreases as λ increases.*

Proof. From (4.1), $\mathbb{E}(\Pi^*(\lambda, z)) = (\tilde{p}d\mu(z) - \tilde{c}zd - c_e azd)s + c_e K + \left(c_e bzd - \frac{c_I}{2}\right) s^2$. Then, we have

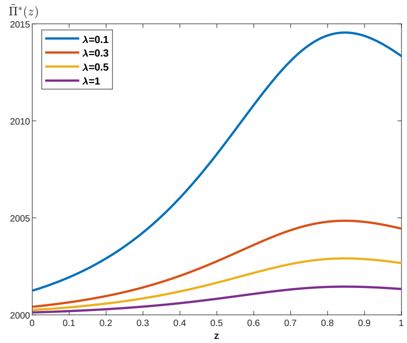
$$\frac{d\mathbb{E}(\Pi^*(\lambda, z))}{d\lambda} = \tilde{s}^{*\prime}(\lambda, z) \left(\tilde{p}d\mu(z) - \tilde{c}zd - c_e azd + (c_e bzd - \frac{c_I}{2})2s\right).$$

Note that the first factor $\tilde{s}^{*\prime}(\lambda, z) \leq 0$ by Proposition 5.1. Per (4.9), $\tilde{s}^*(0, z) = (-\tilde{p}d\mu(z) + \tilde{c}zd + c_e azd)/(2c_e bzd - c_I)$ Since $\tilde{s}^{*\prime}(\lambda, z) \leq 0$, we have $\tilde{s}^*(\lambda, z) \leq \tilde{s}^*(0, z)$. Note that $\lambda = 0$, the second factor above equals to 0 and $\tilde{s}^*(\lambda, z)$ decreases as λ increases. Thereby, the second factor is nonnegative when $\lambda > 0$. Hence, the derivative of $\mathbb{E}(\Pi^*(\lambda, z))$ with respect to λ is negative. The result as desired follows immediately. □

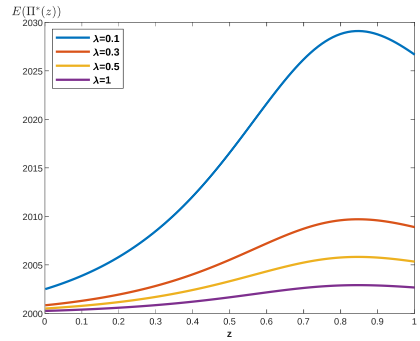
For illustration purpose, we analyze an example for the case of the multiplicative demand with $\beta = 1$. We set $D(s, \epsilon) = 30s\epsilon, \epsilon \sim U[0.001, 1.001]$, $p = 300$, $c = 50$, $c_h = 0$, $c_e = 10$, $a = 5$, $b = 0.5$, $c_I = 25$, $K = 200$. Figure 1 shows the objective function $\Pi^*(\lambda, \cdot)$ as well as $\mathbb{E}(\Pi^*(\lambda, \cdot))$ and $Std(\Pi^*(\lambda, \cdot)) = \sqrt{Var \Pi^*(\lambda, \cdot)}$ for different values of λ for risk-aversion. The behavior stated in Propositions 5.2 and 5.3 can be observed in this figure, that is, for a given safety stock z, the expected profit and the variance of profit decrease with the risk-aversion. Also, as demonstrated in Figure 1, the change rates (decreasing rates) of expected profit and its variance are particularly remarkable as risk-verse parameter λ increases from 0.1 to 0.3, comparing with other scenarios such as from 0.3 to 0.5 and from 0.5 to 1.

6 Conclusions

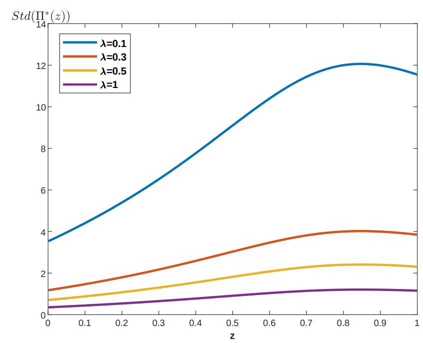
Newsvendor model as a fundamental approach has been widely used to determine the optimal inventory decision with the maximization of the expected profit when the enterprise faces a stochastic demand in a single period. It remains unclear how both carbon emission reduction and risk-averse attribute affect the newsvendor model. This factor motivate us to study a risk-averse newsvendor model with sustainability investment. Specifically, we consider a risk-averse manufacturer under a cap-and-trade policy and investigate the impacts of the sustainability level on the stochastic demand. We have that the investment in the sustainability technology has a positive effects on the market demand. In this scenario, we formulate additive and multiplicative forms of the sustainability-dependent stochastic demand. We first develop a benchmark model and present several definitions and assumptions. In the framework of the mean-variance theory, we establish two optimization models for the cases with additive and multiplicative demand forms. By solving the two models, we have that when the lost sales rate(LSR) elasticity satisfies certain conditions, the existence of the optimal decisions on sustainability level and production quantity are derived. We further provide several numerical examples to illustrate the developed models.



(a) Impact on the objective function



(b) Impact on the expected profit



(c) Impact on the standard deviation of the profit

Figure 1: Objective function, expected profit, and standard deviation of the profit under different risk-averse scenarios

Our work can be extended in the following several research directions. In this paper, we only consider that the stochastic demand is affected by the sustainability level. A possible research direction is to consider the impacts of both the sales price and the sustainability level on the demand. Another research direction is to compare the risk-averse newsvendor model under a cap-and-trade policy with that under other carbon policies. Finally, it may be of interest to consider the risk-averse newsvendor model with multiple products.

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