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OPTIMAL CONTROL FOR NONLINEAR TIME-DELAYED DYNAMICAL SYSTEM OF MICROBIAL CONTINUOUS CULTURE*

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Abstract: In this paper, a nonlinear time-delayed dynamical system is investigated to describe continuous culture of glycerol bioconversion to 1,3-propanediol(1,3-PD) by *Klebsiella pneumoniae*. The aim of this paper is to maximize the concentration of 1,3-PD at the terminal time by controlling the dilution rate and the feeding glycerol concentration. Thus, taking the concentration of 1,3-PD at terminal time as the objective function and the dilution rate, the feeding glycerol concentration as control variables, we propose an optimal control model subjected to the time-delay system and continuous state inequality constraints. Based on the control vector parameterization approach, we developed a sensitivity-based adaptive refinement method to find the optimal control variables, in which the number of stages and the length of each stage's interval are both self-adapted. Moreover, in order to avoid falling into local optimum, we combined improved particle swarm with sensitivity-based adaptive refinement optimization algorithm to solve the problem. The numerical results illustrate the validity of the optimization algorithm.

Key words: *microbial continuous culture, nonlinear time-delay system, optimal control, control vector parameterization, adaptive grid refinement, particle swarm optimization*

Mathematics Subject Classification: *49J15; 49M37; 65K10*

1 Introduction

1,3-propanediol(1,3-PD) is a kind of important material for chemical industry, which can be used to synthesize a variety of polymers with excellent properties [1]. Currently, the process of producing 1,3-PD can be divided into two types: microbial conversion and chemical synthesis [2]. Compared with chemical synthesis approach, microbial conversion method has been given much attention because it is a choice for renewable resources, low cost, high production and no pollution to environment [25, 38]. At present, there are three microbial conversion methods for producing 1,3-PD: batch, continuous and fed-batch fermentations [24, 26]. Among all kinds of microbial conversions, the continuous fermentation method has the advantages of convenient adjustment and control, and can obtain the highest production concentration [21,36]. Consequently, the continuous fermentation process for dissimilation of glycerol to 1,3-PD by klebsiella pneumoniae has important research value and practical significance.

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In recent years, some nonlinear dynamical systems have been proposed to describe the above mentioned microbial process. A widely cited model was proposed by Zeng et al. [38], in which the concentrations of biomass, glycerol and products in reactor were considered. Xiu et al. altered the model by leading in an excess term to describe the continuous culture and fed-batch fermentation in [30]. Yuan et al. consider a nonlinear time-delay dynamic system with uncertain system parameters [34] and a nonlinear enzyme-catalytic time-delayed switched dynamical system [35]to characterize the process of batch fermentation, respectively. Based on the literature [30], Gao et al. studied the parameters identification problem of the nonlinear dynamical system in microbial continuous cultures in [4] and proposed a nonlinear impulsive system of fed-batch culture in [5]; Li et al. [9] proved the existence of optimal solution in continuous cultures; Ma et al. [18] established a model to research Hopf bifurcation and chaos analysis; Ye et al. [32] analyzed the stability of the nonlinear dynamical system in microbial continuous cultures; Li et al. [10] studied an algorithm of the optimal control model for the continuous cultures which the step-size is determined by Armijio line search and the direction is found by the gradient method. On the other hand, based on the parameter identification in [39], Zhai et al. [37] further studied the optimal control for a microbial continuous culture based on the biology robustness which used a parallel particle swam optimization algorithm. The fermentation of glycerol by *K. pneumoniae* under anaerobic conditions is a complex bioprocess and time delays exist in the process. Liu et al. [14] proposed a novel mathematical model including nonlinear time-delay system to describe the batch fermentations of glycerol by *K. pneumoniae*, and investigated some important properties. And in 2018 Liu et al. [15] develop gradient-based optimization algorithms to determine the unknown time-delays and system parameters. Lian et al. [12] considered the oscillatory behavior in microbial continuous cultures with time delay. Li et al. [11] account a finite time delay between the biomass formation and the operating conditions in the kinetic system, studied the stability and Hopf bifurcation of a delay differential system. Wang et al. [29] proposed a modified particle swarm algorithm to solve the stochastic optimal control problem based on the theory of swarm intelligence algorithm. Nonetheless, there are few papers dedicated to the optimal control problem of time-delay dynamic system in microbial continuous fermentation.

In this article we consider a nonlinear time-delay dynamic system to describe the process of continuous fermentation of glycerol bioconversion to 1,3-PD induced by *K. pneumoniae*. Our aim is that how to get the highest concentration of 1,3-PD by controlling the operating conditions. Therefore, we propose an optimal control model, taking the dilution rate and the feeding glycerol concentration as control variables, the concentration of 1,3-PD at the fixed terminal time as objective function. The optimal control problem governed by a nonlinear time-delay dynamic system, is subject to continuous state inequality constraints for ensuring that the concentrations of biomass, glycerol, and reaction products lie in specified limits. In the literatures [10] and [37], the dilution rate and the feeding glycerol concentration are selected as fixed constants during the process of continuous fermentation, which can't be adjusted in the culture process according to the actual fermentation conditions. Since the continuous culture is a-long-time process control problem, we choosing the dilution rate and the feeding glycerol concentration as a continuous function of time t. In order to solve this kind of problem, we use control parameterization method [13,31,33], so as to make the whole process control based on time is transformed into a multi-stage control [19]. Next, we adopt an sensitivity-based adaptive refinement control vector parameterization approach proposed by Liu et al. [27]. Starting from a coarse discretization grid, if the corresponding sensitivities are high, the new points are inserted, and the points are eliminated if the sensitivities are low. In this algorithm, the number of stages and the length of each stage's interval are

both self-adapted. Moreover, different initial values may have different effects on the results in this algorithm. In order to solving this problem, we embed the above algorithm into an improved particle swarm optimization algorithm [3]. In this way, a high-quality global solution can be obtained. Numerical results are presented to show the effectiveness of the algorithm.

The rest of this paper is organized as follows. Section 2 introduces the nonlinear time-delayed dynamic system in continuous fermentation. Section 3 gives the optimal control model and its approximation problem. Section 4 constructs a sensitivity-base adaptive control vector parameterization approach and a particle swarm optimization algorithm to solve the optimal control problem, while Section 5 illustrates the numerical results. And the conclusions are presented in Section 6.

2 Nonlinear Time-Delayed Dynamical System of Continuous Culture in Microbial Fermentation

In continuous fermentation, glycerol is added to the reactor continuously and the broth in reactor pours out at the same rate. During the process of the culture, the volume of the fermentation broth remains constant. According to the fermentation process, we assume that

H1 The concentrations of reactants are uniform in reactor, nonuniform space distribution are ignored.

H2 During the process of continuous culture, the substrate added to the reactor only includes glycerol and the fermentation broth is exported by the dilution rate D.

Under the above assumptions H1 and H2, considering that the growth rate of microorganisms is not only related to the concentration of microorganisms at this time, but also related to the concentration at the previous time. So, a time delay should be taken into account in modelling the fermentation process. In continuous microbial cultures mass balance of biomass, substrate and products can be described as the following nonlinear time-delayed dynamical system [11] :

$$\begin{cases} \dot{x_1}(t) &= \mu x_1(t-\tau) - Dx_1(t), \\ \dot{x_2}(t) &= D(c_{s0} - x_2(t)) - q_2 x_1(t-\tau), t \in [0, t_f], \\ \dot{x_i}(t) &= q_i x_1(t-\tau) - Dx_i(t), i = 3, 4, 5, \end{cases} \tag{2.1}$$

with initial conditions $x(t) = x_0$, for $t \in [-\tau, 0]$. x_0 is the initial value for the state variable, which $x_0 := (x_{10}, x_{20}, 0, 0, 0)^T \in R^5$, x_{10}, x_{20} are known values selected by experience. Among them,$x(t) := (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t))^T$ is the state vector whose components are, respectively, the concentrations of biomass, glycerol, 1,3-PD, acetate and ethanol in the reactor at time t; t_f is the terminal moment; τ is the time-delay argument of the fermentation process, with the constant value 0.26. The elements of the control variable $u = (D, c_{s0})^T \in R^2$ are dilution rate, glycerol concentration in feed. The specific growth rate of cells μ, specific consumption rate of substrate q_2 and specific formation rate of product $q_i, i = 3, 4, 5$, are expressed by the following equations on the basis of previous work [38].

$$\mu = \mu_m \frac{x_2(t)}{x_2(t) + k_s} \prod (1 - \frac{x_i(t)}{x_i^*}), \tag{2.2}$$

$$q_2 = m_2 + \frac{\mu}{Y_2}\Delta q_2 \frac{x_2(t)}{x_2(t)+k_2}, \tag{2.3}$$

$$q_3 = m_3 + \mu Y_3 + \Delta q_3 \frac{x_2(t)}{x_2(t)+k_3}, \tag{2.4}$$

$$q_4 = m_4 + \mu Y_4 + \Delta q_4 \frac{x_2(t)}{x_2(t)+k_4}, \tag{2.5}$$

$$q_5 = q_2\left(\frac{b_1}{c_1 + Dx_2(t)} + \frac{b_2}{c_2 + Dx_2(t)}\right). \tag{2.6}$$

Under anaerobic conditions at 37^0C and pH 7.0, the maximum specific growth rate μ_m and Monod saturation constant k_s are $0.67h^{-1}$ and $0.28mmol/L$, respectively. Saturation constants for substrate and products in kinetic equations with excess terms are $k_2 = 11.43mmol/L, k_3 = 15.50mmol/L, k_4 = 86.71mmol/L$. $c_1 = 0.06, c_2 = 50.45$.

It should be noted that there exist critical concentrations, and out of range will cease to grow of biomass, glycerol, 1,3-PD, acetate and ethanol. As a result it is biologically meaningful to restrict the concentrations of biomass, glycerol and products in a set W defined as

$$x(t) \in W := [x_*, x^*] = \prod_{i=1}^{5}[x_{i*}, x_i^*] \subset R_+^5,$$

with $x_* = [0.001, 100, 0, 0, 0]^T, x^* = [10, 2039, 939.5, 1026, 360.9]$.

Table 1: The parameters in Eqs.(3)-(5)

Substrate/Products	m_i	Y_i	Δq_i
$i = 2$	2.1854	0.0082	31.2328
$i = 3$	-2.2942	75.477	24.2336
$i = 4$	-1.1345	30.8599	5.0099

Since the continuous culture is a-long-time process control problem, the control variable u are considered as a continuous function of time t, that is $u(t)$. The control function $u(t)$ to be identified range in

$$u(t) = [D(t), c_{s0}(t)]^T \in U_{ad} = [0.05, 0.67] \times [100, 1800],$$

m_i, Y_i and Δq_i are parameters whose values can be referred to the literature [4,28] as shown in Table1. $b_1 = 0.03, b_2 = 4.56$. Let

$$\begin{aligned} f(t, u(t), x(t), x(t-\tau)) : &= (f_1(t, u(t), x(t), x(t-\tau)), ..., f_5(t, u(t), x(t), x(t-\tau)))^T \\ &= (\mu x_1(t-\tau) - D(t)x_1(t), D(t)(c_{s0}(t) - x_2(t)) \\ &\quad -q_2 x_1(t-\tau), q_3 x_1(t-\tau) - D(t)x_3(t), q_4 x_1(t-\tau) \\ &\quad -D(t)x_4(t), q_5 x_1(t-\tau) - D(t)x_5(t))^T. \end{aligned} \tag{2.7}$$

Thus, we can formulate the continuous cultures as follows:

$$\begin{cases} \dot{x}(t) &= f(t, u(t), x(t), x(t-\tau)), t \in [0, t_f], \\ x(t) &= x_0, t \in [-\tau, 0]. \end{cases} \tag{2.8}$$

In view of the mechanism of bio-dissimilation of glycerol to 1,3-PD, we assume that

H3 $x_0 \in W$, and for $u \in U_{ad}$, there exists at least one steady state of the system (2.8) and the number of steady states is finite.

Under the assumption (H3), we can easily obtain the following properties of the system (2.8).

Property 1. *Suppose $u \in U_{ad}$, then the system (2.8) has a unique solution, denoted by $x(t;u)$, and $x(t;u)$ is continuous in u on U_{ad}.*

Property 2. *Suppose $u \in U_{ad}$, then the function f given by (2.8) satisfies that $f \in C(0,t_f;R^5)$ and f is a locally Lipschitz continuous in x on W.*

3 Optimal Control Models

3.1 Optimal control problem

In continuous fermentation, it is desired that the concentration of 1,3-PD should be maximized at fixed terminal time. This is achieved by manipulating the dilution rate and the glycerol concentration of glycerol in feed. The optimal control problem is to choose an optimal control strategy such that the concentration of 1,3-PD at the terminal time is maximized. Thus, the optimal control problem(P1) can be formulated as:

$$\begin{array}{ll} min & J_0(u) = -x_3(t_f) \\ s.t. & \dot{x}(t) = f(t,u(t),x(t),x(t-\tau)), t \in [0,t_f] \\ & x(t) = x_0, t \in [-\tau, 0] \\ & x(t) \in W, u \in U_{ad}. \end{array}$$

Theorem 3.1. *There exists an optimal solution of (P1), that is, $u^* \in U_{ad}$ so that*

$$J_0(u^*) \leq J_0(u), \forall u \in U_{ad}.$$

Proof. see [4] Theorem 1. □

3.2 Problem approximation

Since the constraint in (P1) is a continuous state inequality constraint [7, 8], (P1) can be viewed as a semi-infinite programming problem. An efficient algorithm for solving optimization problem of this type is to use the constraints transcript technology [6, 16, 20, 22]. We briefly introduce the application of this algorithm to the problem (P1).

Let

$$\begin{aligned} &g_i(t,u(t),x(t)) = x_i(t) - x_i^*, \\ &g_{5+i}(t,u(t),x(t)) = x_{*i} - x_i(t), i = 1,2,...,5. \end{aligned}$$

The condition $x(t) \in W$ is equivalently transformed to

$$G(u) = 0, \tag{3.1}$$

where

$$G(u) = \int_0^{t_f} \sum_{i=1}^{10} max\{0, g_i(t,u(t),x(t))\}dt.$$

Then, the objective function in (P1) is augmented as follows:

$$J_1 = J_0 + \rho \int_0^{t_f} \sum_{i=1}^{10} max\{0, g_i(t, u(t), x(t))\}dt, \tag{3.2}$$

where $\rho > 0$ is the given penalty parameter. However,the equality constraint (3.1) is non-differentiable at the points when $g_i = 0$. Consequently, standard optimization routines would have difficulties in dealing with this type problem. The following smooth function is therefore introduced to approximate the non-smooth max operator [27],

$$\varphi_\delta(x) = \frac{1}{2}(x + \sqrt{x^2 + 4\delta^2}), \tag{3.3}$$

where the smoothing paratemeter δ is a very small positive number. For any $x \in R$, $\varphi_\delta(x)$ has the following properties [27]:

$$\lim_{\delta \to 0} \varphi_\delta(x) = max\{x, 0\}, \tag{3.4}$$

$$0 < \varphi_\delta(x) - max\{x, 0\} < \delta. \tag{3.5}$$

Then the augmented objective function (3.2) can be reformulated as

$$J_2 = J_0 + \rho \int_0^{t_f} \sum_{i=1}^{10} \varphi_\delta(g_i(t, u(t), x(t)))dt, \tag{3.6}$$

Then (P1) can be approximated by the following problem (P2):

$$\begin{array}{ll} min & J_2 = -x_3(t_f) + \rho \int_0^{t_f} \sum\limits_{i=1}^{10} \varphi_\delta(g_i(t, u(t), x(t)))dt \\ s.t. & \dot{x} = f(t, u(t), x(t), x(t-\tau)), t \in [0, t_f] \\ & x(t) = x_0, t \in [-\tau, 0] \\ & u(t) \in U_{ad}, \\ & t \in [0, t_f]. \end{array}$$

Theorem 3.2. *Let u_δ^* be the optimal solution of the approximate problem (P2). Suppose that there exists an optimal solution u^* of the original problem (P1). Then*

$$\lim_{\delta \to 0} J_2(u_\delta^*) = J_0(u^*).$$

Proof. By the equations (3.4)(3.5), we can get that

$$\lim_{\delta \to 0} \varphi_\delta(x) = max\{x, 0\},$$

so

$$\lim_{\delta \to 0} J_2(u_\delta^*) = J_1(u^*),$$

The objective function J_1 is equivalent to that of problem (P1). Thus,we have

$$\lim_{\delta \to 0} J_2(u_\delta^*) = J_0(u^*).$$

□

4 Computational Approaches

4.1 Control vector parameterization

In order to solve problem (P2), we adopt the method of control vector parameterization [13, 31, 33], in which the control variables $u_i(t)(i = 1, 2)$ are discretized. For each $i = 1, 2$, the ith component u_i of the control u is a piecewise constant function over the interval $[0, t_f]$ with jump points at $t_0, t_1, ..., t_p$. In other words, u_i takes a constant until the next switching time is reached, at this jump point, u_i changes instantaneously to another constant, the whole time-based process control is transformed into a multi-stage control. Mathematically, u_i may be expressed as:

$$u_i(t) \approx \hat{u}_i(t) = \sum_{k=1}^{p} \sigma_{i,k}\chi_k(t), \tag{4.1}$$

where $\sigma_{i,k}$ is the value of $u_i(t)$ in the kth subinterval $[t_{k-1}, t_k)$,$\hat{u} = [\hat{u}_1, \hat{u}_2]$ and χ_k is defined as

$$\chi_k(t) = \begin{cases} 1, & \text{if } t \in [t_{k-1}, t_k), \\ 0, & otherwise, \end{cases} \tag{4.2}$$

the time t_k, $k = 1, .., p$ are such that $0 = t_0 < t_1 < \cdots < t_{p-1} < t_p = t_f$. Let $\sigma = [\sigma_1, \sigma_2]^T$, in which $\sigma_1 = [\sigma_{1,1}, \sigma_{1,2}, ..., \sigma_{1,p}]$, $\sigma_2 = [\sigma_{2,1}, \sigma_{2,2}, ..., \sigma_{2,p}]$. Define $\Xi_{ad} = \{\sigma = [\sigma_{1,1}, ..., \sigma_{1,p}, \sigma_{2,1}, ..., \sigma_{2,p}]^T : 0.05 \leqslant \sigma_{1,i} \leqslant 0.67, 100 \leqslant \sigma_{2,i} \leqslant 1800, i = 1, ..., p\}$. $u \in U_{ad}$ is equivalent to $\sigma \in \Xi_{ad}$ [22]. In this way, (P3) can be reformulated as an NLP problem, in which σ is regarded as the decision vector.

We may now specify the approximate problem (P3) as follows:

Problem (P3) *Find a control parameter vector $\sigma \in U_{ad}$ to minimize the cost function $J(\sigma)$.*

$$\begin{aligned} min \quad & J = -x_3(t_f \mid \sigma) + \rho \int_0^{t_f} \sum_{i=1}^{10} \varphi_\delta(g_i(t, \sigma, x(t)))dt \\ s.t. \quad & \dot{x} = \tilde{f}(t, \sigma, x(t), x(t-\tau)), t \in [0, t_f] \\ & x(t) = x_0, t \in [-\tau, 0] \\ & \sigma \in \Xi_{ad}, \\ & t \in [0, t_f]. \end{aligned}$$

where

$$\tilde{f}(t, \sigma, x(t), x(t-\tau)) = f(t, \sum_{k=1}^{p} \sigma_{i,k}\chi_k(t), x(t), x(t-\tau))$$

Theorem 4.1. *Let $\hat{u}^*$ be the optimal control of the approximate problem (P3). Suppose that the original problem (P1) has an optimal control u^*. Then,*

$$\lim_{p \to \infty} J(\hat{u}^*) = J(u^*)$$

Proof. see [22] Theorem 6.5.1. □

To solve the problem(P) as a mathematical programming problem, we require the gradient formulae for the function J. We shall derive the required formulae as follows [23, 31]:

Let the corresponding Hamiltonian function for the cost function be defined by

$$H(t, x(t|\tau), \sigma, \lambda) = \pounds(t, \sigma, x(t)) + \lambda^T \tilde{f}(t, \sigma, x(t), x(t-\tau)) \tag{4.3}$$

where

$$\pounds(t,\sigma,x(t)) = \rho\sum_{i=1}^{10}\varphi_\delta(g_i(t,\sigma,x(t))),$$

$$\tilde{f}(t,\sigma,x(t),x(t-\tau)) = f(t,\sum_{k=1}^{p}\sigma_{i,k}\chi_k(t),x(t),x(t-\tau))$$

and

$$\lambda(t) = (\lambda_1(t),\lambda_2(t),\lambda_3(t),\lambda_4(t),\lambda_5(t))^T \tag{4.4}$$

is the solution of the costate system

$$\dot{\lambda(t)} = -\frac{\partial H(t,x(t|\tau),\sigma,\lambda)^T}{\partial x} \tag{4.5}$$

with the boundary continuous

$$\lambda(t_f) = (0,0,0,0,0)^T. \tag{4.6}$$

The gradient of J is computed form

$$\frac{\partial J}{\partial\sigma} = \int_0^{t_f}\frac{\partial H(t,x(t|\tau),\sigma,\lambda)}{\partial\sigma}dt \tag{4.7}$$

During actual computation, very often the control parametrization is carried out on an partition of the interval $[0,t_f]$, i.e.,

$$\hat{u}_i(t) = \sum_{k=1}^{p}\sigma_{i,k}\chi_k(t),\ i=1,2$$

As such, each component of (4.7) can be written in a more specific form:

$$\frac{\partial J}{\partial\sigma_{i,j}} = \int_{t_{j-1}}^{t_j}\frac{\partial H(t,x(t|\tau),\sigma,\lambda)}{\partial\hat{u}_i}dt,\ j=1,...,p. \tag{4.8}$$

4.2 Sensitivity-based adaptive refinement strategy

In the process of control vector parameterization, in order to select a suitable discretization level of time grids, we adopt a sensitivity-based adaptive refinement method.

Let J^{*l}, $\hat{u}_i^{*l} = [\sigma_{i,1}^{*l},...,\sigma_{i,p}^{*l}],(i=1,2)$, $\Delta^l = [t_0^l,...,t_p^l]^T$ as the optimal objective function value, the optimal solution and the corresponding discretization time grid found in iteration l. $\Delta^{l'} := [t_0^{l'},...,t_{2p}^{l'}]^T$ is obtained by bisecting each subinterval in Δ^l with initial control variable $\hat{u}_1^{l'} = [\sigma_{1,1}^{*l},\sigma_{1,1}^{*l},..,\sigma_{1,p}^{*l},\sigma_{1,p}^{*l}]^T, \hat{u}_2^{l'} = [\sigma_{2,1}^{*l},\sigma_{2,1}^{*l},..,\sigma_{2,p}^{*l},\sigma_{2,p}^{*l}]^T$. Suppose $J^{*l'}, \hat{u}_1^{*l'} = [\sigma_{1,1}^{*l'},...,\sigma_{1,2p}^{*l'}], \hat{u}_2^{*l'} = [\sigma_{2,1}^{*l'},...,\sigma_{2,2p}^{*l'}]$ are the optimal objective function value and the optimal solution in iteration l', respectively. The refinement strategy is to find a new discretization grid to make it better adapted to the solution.

Let the sensitivity of $\sigma_{i,j}^{l'}$ as follows:

$$s_{i,j} = \left|\frac{\partial J}{\partial\sigma_{i,j}^{l'}}\right|,\quad which\ \ \sigma_{i,j}^{l'} = \sigma_{i,\lfloor(j+1)/2\rfloor}^{*l}, \tag{4.9}$$

where $\lfloor \frac{j+1}{2} \rfloor$ denotes the maximum integer that does not exceed $\frac{j+1}{2}$. Suppose $\sigma_{i,K}^{*l-1}$ and $\sigma_{i,K}^{*l}$ are the optimal control values in time interval $K := [t_{2k-2}^{l'}, t_{2k}^{l'}]$ in iteration $l-1$ and iteration l, respectively.

For a given value $\varepsilon_1 > 0$, if

$$\left|\sigma_{i.K}^{*l} - \sigma_{i,K}^{*l-1}\right| < \varepsilon_1, \tag{4.10}$$

then let

$$s_{i,2k-1} = 0 \quad and \quad s_{i,2k} = 0. \tag{4.11}$$

If the following conditions

$$s_{i,2k-1} > \lambda_1 \bar{s_i} \quad or \quad s_{i,2k} > \lambda_1 \bar{s_i} \tag{4.12}$$

are established, where

$$\bar{s_i} = \frac{1}{2p} \sum_{j=1}^{2p} s_{i,j}, \tag{4.13}$$

then the grid point $t_{2k-1}^{l'}$ in $\Delta^{l'}$ is reserved; otherwise, eliminate it. When both $t_{2k-1}^{l'}$ and $t_{2(k+1)-1}^{l'}$ are removed, the grid point $t_{2k}^{l'}$ is also eliminated if

$$s_{i,2k-1} < \lambda_2 \bar{s_i},\ s_{i,2k} < \lambda_2 \bar{s_i},\ s_{i,2k+1} < \lambda_2 \bar{s_i},\ s_{i,2(k+1)} < \lambda_2 \bar{s_i},\ \ and\ \ \left|\sigma_{i,k+1}^{*l} - \sigma_{i,k}^{*l}\right| < \varepsilon_2, \tag{4.14}$$

where λ_1, λ_2 and ε_2 are given constants, and $\lambda_1 > 0, \lambda_2 \in (0, \lambda_1], \varepsilon_2 > 0$.

The main steps of the sensitivity-based adaptive control vector parameterization algorithm are as follows:

Algorithm A

Step 0. Given the initial values $\hat{u}^0 = [\hat{u}_{01}, \hat{u}_{02}]^T$, $\hat{u}^0$ is the initial value for all subintervals, given time grids Δ^0, the maximum number of iterations $l^{max} \geq 1$, error tolerance $\xi > 0$, constants $\rho > 0$, $\delta > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\lambda_1 > 0$, $\lambda_2 \in (0, \lambda_1]$.

Step 1. Set $l = 0$.

Step 2. Let $\hat{u}^l$ as the starting point and Δ^l as the starting time grids. By using the quadratic sequence programming algorithm to solve the NLP to get the optimal objective function value J^{*l} and the optimal solution $\hat{u}^{*l}$.

Step 3. If $l = l^{max}$ or $|\frac{J^{*l} - J^{*l-1}}{J^{*l}}| < \xi (l > 0)$, stop; otherwise, go to Step4.

Step 4. Refine time grids.

Step 4.1. Bisecting each subinterval in Δ^l to get the temporary grids $\Delta^{l'}$ and the corresponding control variables $\hat{u}^{l'}$,

Step 4.2. Compute the sensitivity according to (4.9), (4.10) and (4.11).

Step 4.3. Eliminate unnecessary grid points according to (4.12) (4.13) and (4.14).

Step 4.4. Let $\hat{u}^{l+1} = \hat{u}^{l'}$, $\Delta^{l+1} = \Delta^{l'}$.

Step 5. Set $l = l + 1$. If $l = l^{max}$, stop; otherwise, go to Step 2.

4.3 Particle swarm optimization algorithm

In the algorithm A, different initial values will produce different results, thus this algorithm will likely get trapped at a local solution. In order to overcome this difficulty, we embed algorithm A into an improved particle swarm optimization (PSO) algorithm [3] to solve thee problem(P2). The conventional PSO algorithm converges quickly in the initial stages, but slows down when approaching the optimal solution [17]. So we adopt the improved PSO algorithm proposed in the literature [3], which can quickly converge to the optimal solution.

The parameters in Algorithm B are defined below:

- Let N denote the total number of particles in the swarm.
- c_1 and c_2 are the cognitive and social scaling parameters.
- ω_{max} and ω_{min} are the maximum and minimum inertia weights.
- V_{max} and V_{min} are vectors containing the maximum and minimum particle velocities.
- K_{max} is the maximum number of iteration.
- d_1 and d_2 are control factors.
- k is the iteration index.

Algorithm B

Step 0. Initialize the parameters $N, l, c_1, c_2, d_1, d_2, \omega_{max}, \omega_{min}, V_{max}, V_{min}, K_{max}$.

Step 1. Randomly generate N particles with uniform distribution on U_{ad}. Denote the position and velocity of particles by $\hat{u}_n^0 = [\hat{u}_{01}^n, \hat{u}_{02}^n] \in U_{ad}$ and $v^n = [v_1^n, v_2^n]$, respectively, where $v_i^n \in [V_{min}^i, V_{max}^i], i = 1, 2$, V_{min}^i and V_{max}^i denote the ith components of the V_{min} and V_{max}. Set the J_{pbest}^n is the best objective value found by the nth individual particle, $\hat{u}_{n*}^0 = [\hat{u}_{01}^{n*}, \hat{u}_{02}^{n*}]$ is the best position found by the nth individual particle. Let J_{gbest} denote the best objective value found by any member of the swarms, $\hat{u}_*^0 = [\hat{u}_{01}^*, \hat{u}_{02}^*]$ denote the best position found by any member of the swarms.

Step 2. Let $k = 1$, $J_{pbest}^n \to +\infty$, $J_{gbest} \to +\infty$

Step 3. For each $n = 1, 2, ..., N$, put $\hat{u}_n^0$ into the algorithm A to calculate the corresponding objective function values $J(\hat{u}_n^0)$

Step 4. If $J(\hat{u}_n^0) < J_{pbest}^n$, then set $J_{pbest}^n = J(\hat{u}_n^0)$ and $\hat{u}_{n*}^0 = \hat{u}_n^0$.

Step 5. If $J_{pbest}^n < J_{gbest}$, then set $J_{gbest} = J_{pbest}^n$ and $\hat{u}_*^0 = \hat{u}_{n*}^0$.

Step 6. If $k \le K_{max}$, then go to Step 7; otherwise, stop.

Step 7. Update the inertia term according to the following formula:

$$\omega = (\omega_{max} - \omega_{min} - d_1)\ exp\{\frac{1}{K_{max} + d_2(k-1)}\}.$$

Step 8. For each $n = 1, ..., N$,$i = 1, 2$ compute

$$v_i^n = \omega v_i^n + c_1 r_1(\hat{u}_{0i}^{n*} - \hat{u}_{0i}^n) + c_2 r_2(\hat{u}_{0i}^* - \hat{u}_{0i}^n),$$

where r_1, r_2 obey the uniform distribution on $[0, 1]$.

Step 9. For each $n = 1, 2, ...N$ update the velocity of the nth particle according to the following formula:

$$v_i^n = \begin{cases} V_{min}^i, & \text{if } v_i^n < V_{min}^i, \\ v_i^n, & \text{if } v_i^n \in [V_{min}^i, V_{max}^i], \\ V_{max}^i, & \text{if } v_i^n > V_{max}^i, \end{cases}$$

where $i = 1, 2$.

Step 10. For each $n = 1, 2, ..., N$,$i = 1, 2$ compute

$$\hat{u}_{0i}^n = \hat{u}_{0i}^n + v_i^n.$$

Step 11. For each $n = 1, 2, ..., N$, update the position of the nth particle according to the following formula:

$$\hat{u}_{0i}^n = \begin{cases} U_{min}^i, & \text{if } \hat{u}_{0i}^n < U_{min}^i, \\ \hat{u}_{0i}^n, & \text{if } \hat{u}_{0i}^n \in [U_{min}^i, U_{max}^i], \\ U_{max}^i, & \text{if } \hat{u}_{0i}^n > U_{max}^i, \end{cases}$$

where $\hat{u}_{0i}^n$, U_{min}^i and U_{max}^i denote the ith components of the $\hat{u}_n^0$, U_{min} and U_{max}, respectively.

Step 12. Set $k = k + 1$, and return go to Step 3.

5 Numerical Results

For the parameters in the algorithm A, we choose the following values:

$$\rho = 10^4,\ \delta = 10^{-10},\ \varepsilon_1 = 10^{-6},\ \varepsilon_2 = 10^{-3},\ \lambda_1 = 0.25,\ \lambda_2 = 0.2,\ \xi = 10^{-4},\ l^{max} = 20.$$

And the parameters of the algorithm B are selected as follows:

$$c_1 = 2,\ c_2 = 2,\ d_1 = 0.2,\ d_2 = 0.7,\ \omega_{max} = 0.7,\ \omega_{min} = 0.4,\ \ N = 8,\ K_{max} = 20.$$

In this paper, the initial state $x_0 = (0.1, 400, 0, 0, 0)^T$. The whole continuous fermentation was implemented with enough substrate.The total fermentation time is taken as $100h$. Table 2 gives a few examples for different initial values produce different results by algorithm A.

The first column in Table 2 is the initial value $\hat{u}^0$ in the algorithm A, the second column is the computing concentration of 1,3-PD at the terminal time. As it can be seen, the results produced by different initial values have great difference. In order to avoid falling into local minimum, we embed algorithm A into the algorithm B to obtain a global optimal solution. Owing to the algorithm we adopt is a combination algorithm of improved particle swarm optimization and algorithm A, taking into account the complexity of the gradient computational in the algorithm A, we choose the number of particles in the swarm as $N = 8$. Too small number of particles make the result not good enough. Too large number of particles make the calculation speed slow. So we take the number as $N = 8$. Using this algorithm, we get the concentration of 1,3-PD at the terminal time is 871.7830 mmol/L. By this algorithm, the detailed evolution of time grids is presented in Fig 1, we can see that the optimal time point partition can be found with twice iteration, a high-quality solution can be obtained with low computational cost. The optimal dilution rate and feeding glycerol concentration strategy are shown in Fig 2 and Fig 3, respectively. Moreover, the changes of the trajectories of each substance concentrations are shown in Fig 4. The computational results verify the validity of this optimization algorithm.

Table 2: The results using algorithm A

$\hat{u}^0$	value
$(0.574, 1074.1)$	497.7442
$(0.594, 982.29)$	561.8418
$(0.49, 569.243)$	280.9148
$(0.08, 152)$	781.7031
$(0.639, 1067.8)$	539.4966
$(0.5, 861)$	483.0262
$(0.1, 1287)$	762.8610
$(0.45, 607)$	319.3546
$(0.47, 435)$	210.4031
$(0.2, 1300)$	764.5957

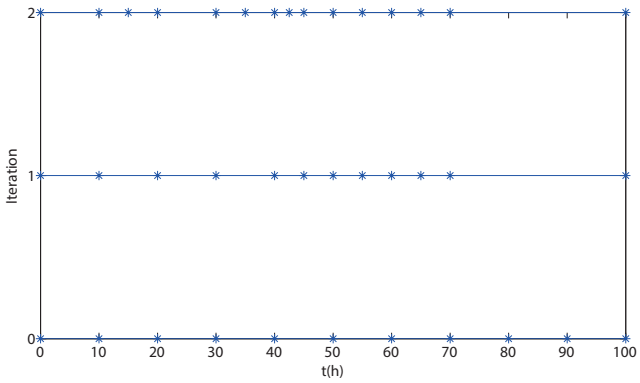

Figure 1: Evolution of the time grids

6 Conclusions

In this paper, we study the optimal control problem of a nonlinear time-delayed dynamical system in microbial continuous fermentation. In order to obtain the higher concentration, we embed a sensitivity-based control vector parameterization adaptive refinement method into an improved particle swarm optimization algorithm to find the optimal dilution rate and feeding glycerol concentration. Through numerical calculation, we obtain the concentration of 1,3-PD at the terminal time and corresponding optimal control stagety, which illustrates the validity and the effectiveness of this optimization algorithm.

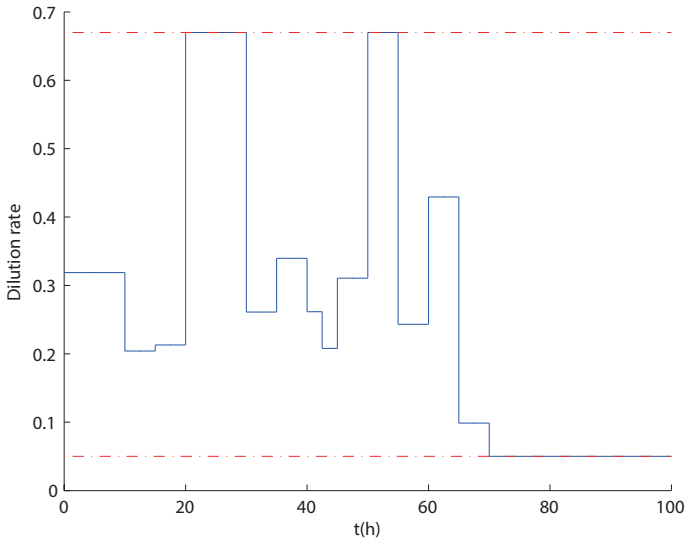

Figure 2: Dilution rate

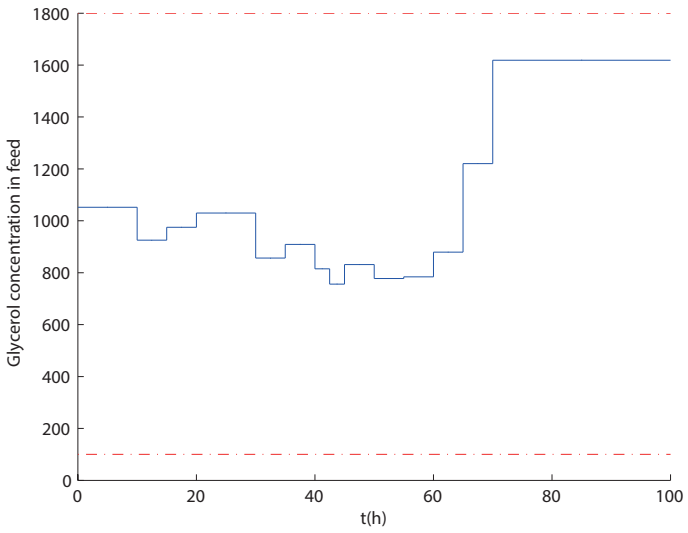

Figure 3: Feeding glycerol concentration

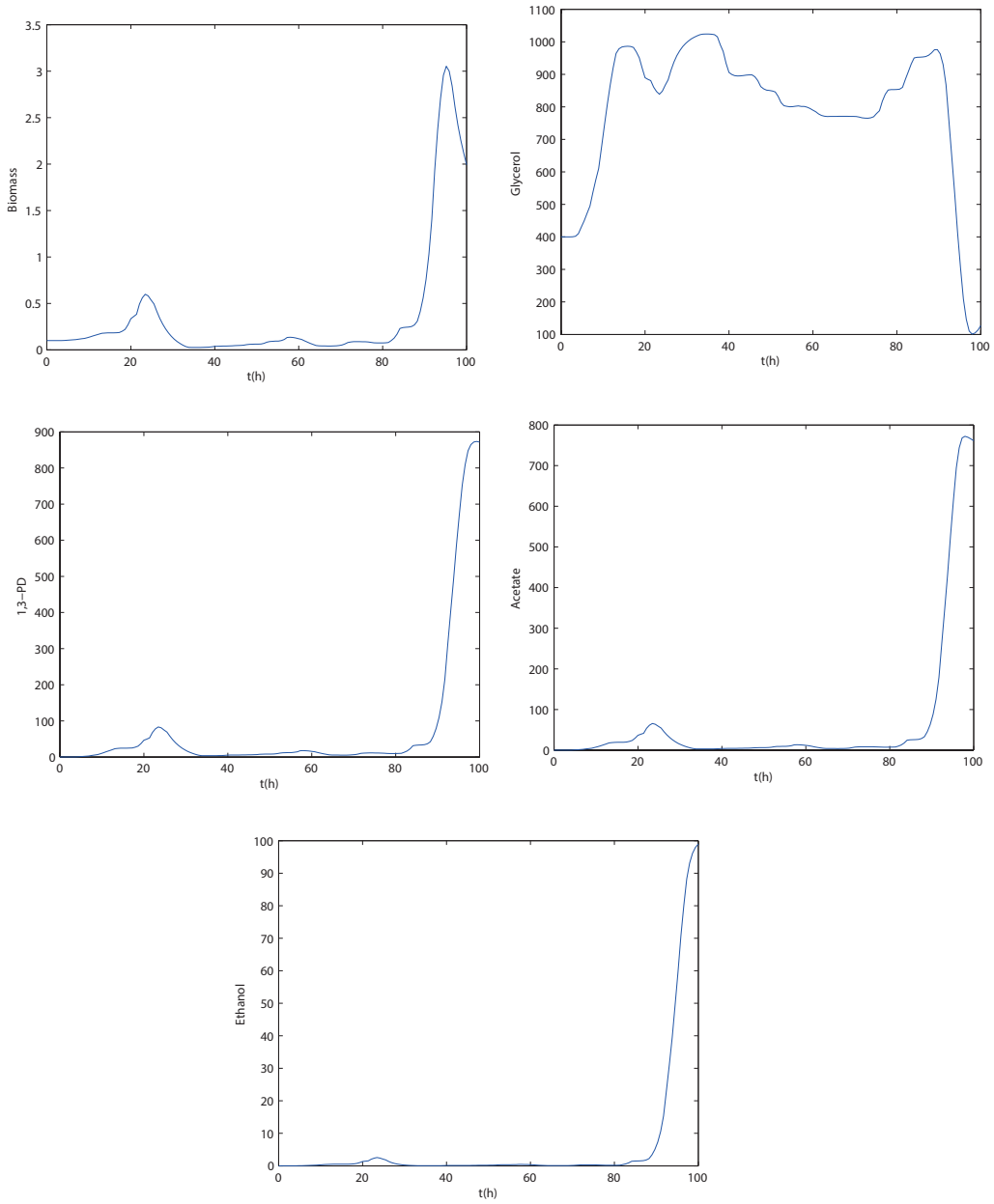

Figure 4: The trajectories of each substance concentrations

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