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CONFIDENCE INTERVAL-VALUED INTUITIONISTIC FUZZY WEIGHTED GEOMETRIC AGGREGATION OPERATOR BASED ON ARCHIMEDEAN T-NORM AND T-CONORM FOR GROUP DECISION MAKING

YE FU ZHENG AND JUN XU*

Abstract. Experts frequently face the presence of vague data with their field familiarity (a.k.a. confidence levels) in multiple attribute group decision-making (MAGDM) problems, such as the review of doctoral dissertation. This work develops a class of new confidence aggregation operators to solve MAGDM problems under interval-valued intuitionistic fuzzy (IVIF) environment. We introduce new operational laws of IVIF numbers (IVIFNs) based on Archimedean t-norm and t-conorm, and investigate their relationships and properties. Some family of confidence IVIF weighted geometric aggregation operators based on Archimedean t-norm and t-conorm are presented as the generalizations of existing ones. Moreover, we study some desirable properties for the aforementioned aggregation operators. We also propose new approaches for MAGDM problems with IVIF information under confidence levels by utilizing these operators. Finally, we show the effectiveness and benefit of the current confidence IVIF aggregation operators using an example of dissertation evaluation and experimental analysis. The results show that the proposed method can meet the group decision-making problems with different confidence levels of experts.

1. Introduction

Fuzzy multi-attribute decision-making (MADM) method can comprehensively consider the fuzziness of multiple attributes, and provide more scientific and reasonable decision support for decision-makers [1,10]. In many cases decisions need to be made in an environment wherein the non-membership and membership degrees in intuitionistic fuzzy (IF) sets are Illustrated in interval form instead of real numbers, due to vague and uncertain evaluation objectives. To overcome the drawbacks, Atanassov [2] proposed interval-valued IF (IVIF) Sets, which have been widely applied to model real-life problems with uncertainties. These problems include hotel location selection, expert systems, computer numerical control, and decision-making problems [3–9, 11–20].

As an important research branch, IVIF aggregation operators have triggered significant theoretical developments and it successfully contributed to the solving of IVIF multi-attribute decision problems. Existing aggregation operators of IVIFSs can be divided into the following categories: (1) IVIF aggregation operators based on

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*Corresponding author.

algebraic operations. Xu *et al.* [18] defined the weighted geometric (IVIFWG) operator, and provided an algorithm for decision-making with IVIF numbers (IVIFNs). Various IVIF aggregation operators were proposed to capture the interrelationships of dependent attributes, including the IVIF power average operators [4] and the Bonferroni mean operators [8]. (2) IVIF aggregation operators based on Einstein operations. Wei and Liu [16] introduced Einstein operations and proposed the IVIF Einstein weighted geometric ($IVIFEWG$) operator utilizing Einstein sum and product for smooth approximations. Jmail [9] further developed the induced generalized Einstein geometric aggregation operators. (3) IVIF aggregation operators based on Hamacher operations. Some IVIF Hamacher ordered weighted operators are applied evaluate new rural development level in China [11]. Zhu *et al.* [20] introduced several linguistic IVIF Hamacher aggregation operators and used them to solve the supplier selection problem.

It is worth noting that the above methods assume that experts have the same level of proficiency in all evaluation criteria. However, different experts have different professional academic backgrounds, and it is unreasonable to assume that all experts have the same level of proficiency in evaluation criteria. To ensure the rationality and impartiality of the ratings provided by decision experts, scholars have introduced several aggregation operators with confidence levels. It is used to reflect the different professional knowledge levels of experts, for the sake of enhancing the reliability of results. To address this issue, Garg [6] investigated several Pythageorean fuzzy aggregation operators with confidence levels. Yu [19] constructed several confidence IF weighted aggregations and applied them to evaluate the dissertation. Later, some new confidence aggregations operators are developed, such as the confidence IF Einstein hybrid aggregations [13], the confidence Pythagorean fuzzy aggregations [12], and confidence IF Dombi aggregations [3]. It is evident from the extant research that confidence levels have not been applied to aggregation operators on IVIFNs. Moreover, the aggregated result obtained by the confidence Pythageorean fuzzy geometric operators [6] and confidence IF geometric operators [19] may decrease as the confidence levels of experts increase from 0 to 1, which is inconsistent with reality. Therefore, the aim of this work is to develop some confidence IVIF weighted geometric ($CIVIFWG$) aggregation operators.

In summary, most IVIF aggregation operators are based on specific t-norm and t-conorm (TTnorm), lacking a generalized IVIF aggregation operator and not considering the differences in expert confidence levels. The Archimedean TTnorm can generalize TTnorm [14] and hence have greatly contributed to aggregation operators with IFSs [17] and Pythagorean fuzzy sets [6]. Therefore, this study first presents the IVIF operations based on Archimedean TTnorm and investigate the operational law theory. Then, we propose the Archimedean TTnorm based confidence interval-valued intuitionistic fuzzy weighted geometric ($ATT\text{-}CIVIFWG$) operator and it desirable properties. Moreover, based on three common additive generator g, the corresponding specific $CIVIFWG$ operators are given to solve decision problem. Finally, a group decision-making method with IVIFNs is also developed based on these operators and applied to deal with dissertation evaluation problem.

The structure of this paper is designed as following: In Section 2, we introduce some concepts and operational laws of IVIFSs based on Archimedean TTnorm. In

Section 3, the confidence aggregation operators for IVIFNs are presented, and three properties of the *ATT-CIVIFWG* operators are investigated. In Section 4, we gave an MAGDM approach with IVIF under confidence levels. In Section 5, a numerical example about doctoral thesis evaluation is provided to verify the effectiveness and practicality of the proposed approach. Finally, we conclude and remark the paper.

2. Preliminaries

2.1. t-norm and t-conorm.

Definition 2.1 ([14])**.** A t-norm function $\tau(p,q)$ is a mapping $\tau : [0,1]^2 \to [0,1]$ satisfying the following properties:

(1) Neutrality: $\tau(1,p) = p$, for $\forall p$.
(2) Commutativity: $\tau(p,q) = \tau(q,p)$.
(3) Monotonicity: $\tau(p,q) = \tau(p',q')$ if $p \leq p'$ and $q \leq q'$.
(4) Associativity: $\tau(p,\tau(q,z)) = \tau(\tau(p,q),z)$.

Definition 2.2 ([14])**.** A t-conorm function $\varsigma(p,q)$ is a mapping $\varsigma : [0,1]^2 \to [0,1]$ satisfying the following properties:

(1) Neutrality: $\varsigma(0,p) = p$, for $\forall p$.
(2) Commutativity: $\varsigma(p,q) = \varsigma(q,p)$.
(3) Monotonicity: $\varsigma(p,q) = \varsigma(p',q')$ if $p \leq p'$ and $q \leq q'$.
(4) Associativity: $\varsigma(p,\varsigma(q,z)) = \varsigma(\varsigma(p,q),z)$.

Theorem 2.3 (Representation Theorem)**.** *Suppose $\tau : [0,1]^2 \to [0,1]$ satisfies the following conditions:*

(1) *$\tau(p,q)$ is a t-norm function,*
(2) *$\tau(p,q)$ is Archimedean, i.e., $p^n < q \ \exists n > 0$,*
(3) *$\tau(p,q)$ is continuous and strictly increasing. Then $\tau(p,q)$ admits the representation*

$$\tau(p,q) = h^{-1}(h(p) + h(q)) \tag{2.1}$$

where additive generator h is a monotone increasing function from $[0,1]$ to $[0,\infty]$, with $h(1) = 0$.

In the same way, its dual t-conorm $\varsigma(p,q)$ allows for representation

$$\varsigma(p,q) = s^{-1}(s(p) + s(q)) \tag{2.2}$$

where $s(t) = h(1-t)$ is a monotone increasing function, and $s(0) = 0$.

2.2. Interval-valued intuitionistic fuzzy set.

Definition 2.4 ([2])**.** The concept of IVIF set Γ in X is presented as $\Gamma = \{< x, \mu_\Gamma(x), v_\Gamma(x) > | x \in X\}$, where $\mu_\Gamma(x) = [\mu_\Gamma^l(x), \mu_\Gamma^h(x)] : X \to [0,1]$ and $v_\Gamma(x) = [v_\Gamma^l(x), v_\Gamma^h(x)] : X \to [0,1]$, such that $\mu_\Gamma^h(x) + v_\Gamma^h(x) \in [0,1]$, for any $x \in X$. The interval numbers $\mu_\Gamma(x)$ and $v_\Gamma(x)$ indicate the membership and non-membership degree of x in Γ.

Definition 2.5 ([18])**.** Let $\tilde{\alpha} = ([\mu^l, \mu^h], [v^l, v^h])$ be an interval-valued intuitionistic fuzzy number (IVIFN), the score and accuracy degrees of $\tilde{\alpha}$ can be formulated as

$$S(\tilde{\alpha}) = \frac{1}{2}(\mu^l + \mu^h - v^l - v^h), \tag{2.3}$$

$$H(\tilde{\alpha}) = \frac{1}{2}(\mu^l + \mu^h + v^l + v^h) \tag{2.4}$$

where $S(\tilde{\alpha}) \in [-1, 1]$, $H(\tilde{\alpha}) \in [0, 1]$. An IVIFN is considered larger if it has a higher score value; in cases of equal scores, the IVIFN with greater accuracy takes precedence.

2.3. **Interval-valued intuitionistic fuzzy operations based on t-norm and t-conorm.** Recently, Beliakov *et al.* [3] utilized TTnorm to build some arithmetic operations on two intuitionistic fuzzy numbers. After that, Wang and liu [16] mentioned some analogous operations on IVIFSs. Through the above-mentioned analysis, some interval-valued intuitionistic fuzzy operations based on TTnorm can be also formulated as follows:

Definition 2.6. Given three IVIFNs denoted as $\alpha_1 = ([a_1, b_1], [c_1, d_1])$, $\alpha_2 = ([a_2, b_2], [c_2, d_2])$, and $\alpha = ([a, b], [c, d])$, the following holds.

(1) $\alpha_1 \oplus \alpha_2 = ([s^{-1}(s(a_1) + s(a_2)), s^{-1}(s(b_1) + s(b_2))], [h^{-1}(h(c_1) + h(c_2)), h^{-1}(h(d_1) + h(d_2))])$.
(2) $\alpha_1 \otimes \alpha_2 = ([h^{-1}(h(a_1) + h(a_2)), h^{-1}(h(b_1) + h(b_2))], [s^{-1}(s(c_1) + s(c_2)), s^{-1}(s(d_1) + s(d_2))])$.
(3) $\lambda\alpha = ([s^{-1}(\lambda s(a)), s^{-1}(\lambda s(b))], [h^{-1}(\lambda h(c)), h^{-1}(\lambda h(d))])$, $\lambda\rangle 0$.
(4) $\alpha^\lambda = ([h^{-1}(\lambda h(a)), h^{-1}(\lambda h(b))], [s^{-1}(\lambda s(c)), s^{-1}(\lambda s(d))])$, $\lambda\rangle 0$.

Theorem 2.7. *Let λ, λ_1, $\lambda_2 \geq 0$. Then*

(1) $\alpha_1 \oplus \alpha_2 = \alpha_2 \oplus \alpha_1$;
(2) $\lambda(\alpha_1 \oplus \alpha_2) = \lambda\alpha_1 \oplus \lambda\alpha_2$;
(3) $\lambda_1\alpha \oplus \lambda_2\alpha = (\lambda_1 + \lambda_2)\alpha$;
(4) $\alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1$;
(5) $(\alpha_1 \otimes \alpha_2)^\lambda = \alpha_1^\lambda \otimes \alpha_2^\lambda$;
(6) $\alpha^{\lambda_1} \otimes \alpha^{\lambda_2} = \alpha^{\lambda_1+\lambda_2}$.

Proof. Eveidently, according to the Commutativity of $\tau(p, q)$ and $\varsigma(p, q)$, (1) and (4) are correct. The others are proved as follows:

For (2):

$$\begin{aligned}\lambda(\alpha_1 \oplus \alpha_2) &= \lambda([s^{-1}(s(a_1) + s(a_2)), s^{-1}(s(b_1) + s(b_2))],\\ &\quad [h^{-1}(h(c_1) + h(c_2)), h^{-1}(h(d_1) + h(d_2))])\\ &= ([s^{-1}(\lambda s(s^{-1}(s(a_1) + s(a_2)))), s^{-1}(\lambda s(s^{-1}(s(b_1) + s(b_2))))],\\ &\quad [h^{-1}(\lambda h(h^{-1}(h(c_1) + h(c_2)))), h^{-1}(\lambda h(h^{-1}(h(d_1) + h(d_2))))])\\ &= ([s^{-1}(\lambda(s(a_1) + s(a_2)))), s^{-1}(\lambda(s(b_1) + s(b_2))))],\\ &\quad [h^{-1}(\lambda(h(c_1) + h(c_2))), h^{-1}(\lambda(h(d_1) + h(d_2)))])\end{aligned}$$

and

$$\lambda\alpha_1 \oplus \lambda\alpha_2 = ([s^{-1}(\lambda s(a_1)), s^{-1}(\lambda s(b_1))],$$

$$
\begin{aligned}
&[h^{-1}(\lambda h(c_1)), h^{-1}(\lambda h(d_1))]) \\
&\quad \oplus ([s^{-1}(\lambda s(a_2)), s^{-1}(\lambda s(b_2))], \\
&[h^{-1}(\lambda h(c_2)), h^{-1}(\lambda h(d_2))]) \\
&= ([s^{-1}(s(s^{-1}(\lambda s(a_1))) + s(s^{-1}(\lambda s(a_2)))), \\
&s^{-1}(s(s^{-1}(\lambda s(b_1))) + s(s^{-1}(\lambda s(b_2))))], \\
&\quad [h^{-1}(h(h^{-1}(\lambda h(c_1))) + h(h^{-1}(\lambda h(c_2)))), \\
&\qquad h^{-1}(h(h^{-1}(\lambda h(d_1))) + h(h^{-1}(\lambda h(d_2))))]) \\
&= ([s^{-1}(\lambda s(a_1) + \lambda s(a_2)), s^{-1}(\lambda s(b_1) + \lambda s(b_2))], \\
&[h^{-1}(\lambda h(c_1) + \lambda h(c_2)),
\end{aligned}
$$

$$
h^{-1}(\lambda h(d_1) + \lambda h(d_2))])
$$

$$
\therefore \lambda\alpha_1 \oplus \lambda\alpha_2 = \lambda(\alpha_1 \oplus \alpha_2)
$$

The proof of (3) is as follows:

$$
\begin{aligned}
\lambda_1\alpha \oplus \lambda_2\alpha &= ([s^{-1}(\lambda_1 s(a)), s^{-1}(\lambda_1 s(b))], [h^{-1}(\lambda_1 h(c)), h^{-1}(\lambda_1 h(d))]) \\
&\quad \oplus ([s^{-1}(\lambda_2 s(a)), s^{-1}(\lambda_2 s(b))], [h^{-1}(\lambda_2 h(c)), h^{-1}(\lambda_2 h(d))]) \\
&= ([s^{-1}(s(s^{-1}(\lambda_1 s(a))) + s(s^{-1}(\lambda_2 s(a)))), \\
&\quad s^{-1}(s(s^{-1}(\lambda_1 s(b))) + s(s^{-1}(\lambda_2 s(b))))], \\
&\qquad [h^{-1}(h(h^{-1}(\lambda_1 h(c))) + h(h^{-1}(\lambda_2 h(c)))), \\
&\qquad\quad h^{-1}(h(h^{-1}(\lambda_1 h(d))) + h(h^{-1}(\lambda_2 h(d))))]) \\
&= ([s^{-1}(\lambda_1 s(a) + \lambda_2 s(a)), s^{-1}(\lambda_1 s(b) + \lambda_2 s(b))], \\
&\quad [h^{-1}(\lambda_1 h(c) + \lambda_2 h(c)), \\
&\qquad h^{-1}(\lambda_1 h(d) + \lambda_2 h(d))]) \\
&= ([s^{-1}((\lambda_1 + \lambda_2)s(a)), s^{-1}((\lambda_1 + \lambda_2)s(b))], \\
&\quad [h^{-1}((\lambda_1 + \lambda_2)h(c)), h^{-1}((\lambda_1 + \lambda_2)h(d))]) \\
&= (\lambda_1 + \lambda_2)\alpha
\end{aligned}
$$

Similarly, it is easy to complete the proof of rules (5) and (6). □

3. Confidence interval-valued intuitionistic fuzzy weighted geometric aggregation operator based on TTnorm

It is common for DMs to specify their expertise in the evaluation domains (also called confidence levels) [12, 15]. This section emphasizes the application of the previous operational laws to aggregate IVIF information with confidence levels.

Definition 3.1. For a set of IVIFNs α_j $(j = 1, 2, \ldots, n)$, $\sigma_j \in [0, 1]$ are the corresponding confidence levels, the *ATT-CIVIFWG* operator can be defined as

$$\begin{aligned} &ATT\text{-}CIVIFWG(\langle\alpha_1, \sigma_1\rangle, \langle\alpha_2, \sigma_2\rangle, \ldots, \langle\alpha_n, \sigma_n\rangle) \\ &= \mathop{\otimes}_{j=1}^{n} (\alpha_j^{(1-\sigma_j)})^{w_j}, \end{aligned} \tag{3.1}$$

where w_j is the weight of α_j.

Specifically, if a_j is IF numbers and $\sigma_j = 0$, for all j, then the *ATT-CIVIFWA* operator will be simplified to the Archimedean TTnorm based IF weighted geometric (*ATT-IFWG*) operator, which is coincident with Xia *et al.* [17].

Theorem 3.2. *For a set of IVIFNs $\alpha_j = ([a_j, b_j], [c_j, d_j])$ $(j = 1, 2, \ldots, n)$, $\sigma_j \in [0, 1]$ and $w_j \in [0, 1]$ are the corresponding confidence level and weight of α_j satisfying $\sum_{j=1}^{n} w_j = 1$. The result obtained through the ATT-CIVIFWG operator remains an IVIFN and*

$$\begin{aligned} &ATT\text{-}CIVIFWG(\langle\alpha_1, \sigma_1\rangle, \langle\alpha_2, \sigma_2\rangle, \ldots, \langle\alpha_n, \sigma_n\rangle) = \mathop{\otimes}_{j=1}^{n} (\alpha_j^{(1-\sigma_j)})^{w_j} \\ &= ([h^{-1}(\textstyle\sum_{j=1}^{n} w_j(1-\sigma_j)h(a_j)), h^{-1}(\sum_{j=1}^{n} w_j(1-\sigma_j)h(b_j))], \\ &\quad [s^{-1}(\textstyle\sum_{j=1}^{n} w_j(1-\sigma_j)s(c_j)), s^{-1}(\sum_{j=1}^{n} w_j(1-\sigma_j)s(d_j))]) \end{aligned} \tag{3.2}$$

Proof. First, since

$$\begin{aligned} \alpha_j^{(1-\sigma_j)} &= ([h^{-1}((1-\sigma_j)h(a_j)), h^{-1}((1-\sigma_j)h(b_j))], \\ &\quad [s^{-1}((1-\sigma_j)s(c_j)), s^{-1}((1-\sigma_j)s(d_j))]) \\ (\alpha_j^{(1-\sigma_j)})^{w_j} &= ([h^{-1}(w_j h(h^{-1}((1-\sigma_j)h(a_j)))), h^{-1}(w_j h(h^{-1}((1-\sigma_j)h(b_j))))], \\ &\quad [s^{-1}(w_j s(s^{-1}((1-\sigma_j)s(c_j)))), s^{-1}(w_j s(s^{-1}((1-\sigma_j)s(d_j))))]) \\ &= ([h^{-1}(w_j(1-\sigma_j)h(a_j)), h^{-1}(w_j(1-\sigma_j)h(b_j))], \\ &\quad [s^{-1}(w_j(1-\sigma_j)s(c_j)), s^{-1}(w_j(1-\sigma_j)s(d_j))]) \end{aligned}$$

And

$$\begin{aligned} (\alpha_j)^{w_j(1-\sigma_j)} &= ([h^{-1}(w_j(1-\sigma_j)h(a_j)), h^{-1}(w_j(1-\sigma_j)h(b_j))], \\ &\quad [s^{-1}(w_j(1-\sigma_j)s(c_j)), s^{-1}(w_j(1-\sigma_j)s(d_j))]) \end{aligned}$$

Thus,

$$(\alpha_j^{(1-\sigma_j)})^{w_j} = (\alpha_j)^{w_j(1-\sigma_j)}.$$

Then we prove

$$\begin{aligned} ATT\text{-}CIVIFWG &= \mathop{\otimes}_{j=1}^{n} (\alpha_j^{(1-\sigma_j)})^{w_j} = \mathop{\otimes}_{j=1}^{n} (\alpha_j)^{w_j(1-\sigma_j)} \\ &= ([h^{-1}(\textstyle\sum_{j=1}^{n} w_j(1-\sigma_j)h(a_j)), h^{-1}(\sum_{j=1}^{n} w_j(1-\sigma_j)h(b_j))], \\ &\quad [s^{-1}(\textstyle\sum_{j=1}^{n} w_j(1-\sigma_j)s(c_j)), s^{-1}(\sum_{j=1}^{n} w_j(1-\sigma_j)s(d_j))]) \end{aligned} \tag{3.3}$$

By using mathematical induction on n:

(1) For $n = 2$, we have

$$ATT\text{-}CIVIFWG(\langle\alpha_1, \sigma_1\rangle, \langle\alpha_2, \sigma_2\rangle) = (\alpha_1)^{w_1(1-\sigma_1)} \otimes (\alpha_2)^{w_2(1-\sigma_2)}$$

$$
\begin{aligned}
&= ([h^{-1}(h(h^{-1}(w_1(1-\sigma_1)(a_1))) + h(h^{-1}(w_2\sigma_2 h(a_2)))),\\
&\quad h^{-1}(h(h^{-1}(w_1(1-\sigma_1)h(b_1))) + h(h^{-1}(w_2\sigma_2 h(b_2))))],\\
&\qquad [s^{-1}(s(s^{-1}(w_1(1-\sigma_1)s(c_1))) + s(s^{-1}(w_2(1-\sigma_2)s(c_2)))),\\
&\qquad\quad s^{-1}(s(s^{-1}(w_1(1-\sigma_1)s(d_1))) + s(s^{-1}(w_2(1-\sigma_2)s(d_2))))])\\
&= ([h^{-1}(w_1(1-\sigma_1)h(a_1) + w_2(1-\sigma_2)h(a_2)),\\
&\quad h^{-1}(w_1(1-\sigma_1)h(b_1) + w_2(1-\sigma_2)h(b_2))],\\
&\qquad [s^{-1}(w_1(1-\sigma_1)s(c_1) + w_2\sigma_2 s(c_2)),\\
&\qquad\quad s^{-1}(w_1\sigma_1 s(d_1) + w_2\sigma_2 s(d_2))])\\
&= ([h^{-1}(\textstyle\sum_{j=1}^{2} w_j(1-\sigma_j)h(a_j)), h^{-1}(\sum_{j=1}^{2} w_j(1-\sigma_j)h(b_j))],\\
&\quad [s^{-1}(\textstyle\sum_{j=1}^{2} w_j(1-\sigma_j)s(c_j)), s^{-1}(\sum_{j=1}^{2} w_j(1-\sigma_j)s(d_j))])
\end{aligned}
$$

Hence, Eq. (3.3) is correct.

(2) Suppose Eq. (3.3) holds for $n = k$, that is

$$
\begin{aligned}
&ATT\text{-}CIVIFWG(\langle\alpha_1,\sigma_1\rangle, \langle\alpha_2,\sigma_2\rangle, \dots, \langle\alpha_k,\sigma_k\rangle)\\
&= \bigotimes_{j=1}^{k} (\alpha_j)^{w_j(1-\sigma_j)} = (\alpha_1)^{w_1(1-\sigma_1)} \otimes (\alpha_2)^{w_2(1-\sigma_2)} \otimes \cdots \otimes (\alpha_k)^{w_k(1-\sigma_k)}\\
&= ([h^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)h(a_j)), h^{-1}(\sum_{j=1}^{k} w_j(1-\sigma_j)h(b_j))],\\
&\quad [s^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)s(c_j)), s^{-1}(\sum_{j=1}^{k} w_j(1-\sigma_j)s(d_j))])
\end{aligned}
$$

then, when $n = k + 1$, we have

$$
\begin{aligned}
&ATT\text{-}CIVIFWG(\langle\alpha_1,\sigma_1\rangle, \langle\alpha_2,\sigma_2\rangle, \dots, \langle\alpha_k,\sigma_k\rangle, \langle\alpha_{k+1},\sigma_{k+1}\rangle)\\
&= \bigotimes_{j=1}^{k} (\alpha_j)^{w_j(1-\sigma_j)} \otimes (\alpha_{k+1})^{w_{k+1}(1-\sigma_{k+1})}\\
&= ([h^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)h(a_i)), h^{-1}(\sum_{j=1}^{k} w_j(1-\sigma_j)h(b_i))],\\
&\quad [s^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)s(c_i)), s^{-1}(\sum_{j=1}^{k} w_j(1-\sigma_j)s(d_i))])\\
&\qquad \otimes ([h^{-1}(w_{k+1}(1-\sigma_{k+1})h(a_{k+1})), h^{-1}(w_{k+1}(1-\sigma_{k+1})h(b_{k+1}))],\\
&\qquad\quad [s^{-1}(w_{k+1}(1-\sigma_{k+1})s(c_{k+1})), s^{-1}(w_{k+1}(1-\sigma_{k+1})s(d_{k+1}))])\\
&= ([h^{-1}(h(h^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)h(a_i))) + h(h^{-1}(w_{k+1}(1-\sigma_{k+1})h(a_{k+1})))),\\
&\quad h^{-1}(h(h^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)h(b_i))) + h(h^{-1}(w_{k+1}(1-\sigma_{k+1})h(b_{k+1}))))],\\
&\qquad [s^{-1}(s(s^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)s(c_i))) + s(s^{-1}(w_{k+1}(1-\sigma_{k+1})s(c_{k+1})))),\\
&\qquad\quad s^{-1}(s(s^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)s(d_i))) + s(s^{-1}(w_{k+1}(1-\sigma_{k+1})s(d_{k+1}))))])\\
&= ([h^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)h(a_i) + w_{k+1}(1-\sigma_{k+1})s(a_{k+1})),\\
&\quad h^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)h(b_i) + w_{k+1}(1-\sigma_{k+1})s(b_{k+1}))],\\
&\qquad [s^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)s(c_i) + w_{k+1}(1-\sigma_{k+1})s(c_{k+1})),\\
&\qquad\quad s^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)s(d_i) + w_{k+1}(1-\sigma_{k+1})s(d_{k+1}))])
\end{aligned}
$$

$$= ([h^{-1}(\textstyle\sum_{j=1}^{k} w_j(1-\sigma_j)h(a_i)), h^{-1}(\sum_{j=1}^{k} w_j(1-\sigma_j)h(b_i))],$$
$$[s^{-1}(\textstyle\sum_{j=1}^{k+1} w_j(1-\sigma_j)s(c_i)), s^{-1}(\sum_{j=1}^{k+1} w_j(1-\sigma_j)s(d_i))])$$

i.e. Eq. (3.3) holds for $n = k+1$. Thus, Eq. (3.3) is correct for all n.

Apparently, the aggregated value by the *ATT-CIVIFWG* operator satisfies the conditions in Definition 2.4 Thus it is also an IVIFN. □

It is easy to prove that *ATT-CIVIFWG* operator has the following properties:

(1) (**Boundness**):

$$\min_j(\alpha_j^{1-\sigma_j}) \le ATT\text{-}CIVIFWG(\langle\alpha_1,\sigma_1\rangle, \langle\alpha_2,\sigma_2\rangle, \dots, \langle\alpha_n,\sigma_n\rangle) \le \max_j(\alpha_j^{1-\sigma_j})$$

(2) (**Monotonicity**):

For two set of IVIFs $\alpha_j = ([a_j, b_j], [c_j, d_j])$ and $\alpha'_j = ([a'_j, b'_j], [c'_j, d'_j])$, if $\alpha_j \le \alpha'_j$, i.e., $a_j \le a'_j$, $b_j \le b'_j$, $c_j \ge c'_j$ and $d_j \ge d'_j$, for all j, then

$$ATT\text{-}CIVIFWG(\langle\alpha_1,\sigma_1\rangle, \langle\alpha_2,\sigma_2\rangle, \dots, \langle\alpha_n,\sigma_n\rangle)$$
$$\le ATT\text{-}CIVIFWG(\langle\alpha'_1,\sigma_1 >, \langle\alpha'_2,\sigma_2\rangle, \dots, \langle\alpha'_n,\sigma_n\rangle)$$

(3) (**Idempotency**): For all j $\alpha_j = \alpha$ and $\sigma_j = \sigma$, then

$$ATT\text{-}CIVIFWG(\langle\alpha_1,\sigma_1\rangle, \langle\alpha_2,\sigma_2\rangle, \dots, \langle\alpha_n,\sigma_n\rangle) = \alpha^{1-\sigma}$$

It is generally known that the additive generator h can be given by different forms. We investigate some specific confidence IVIF weighted geometric operators in the following formula.

Case 1. When $h(t) = -\log(t)$, the Algebraic TTnorm functions are derived as $\tau(p,q) = pq$ and $\varsigma(p,q) = p+q-pq$ [17]. By Definition 2.6 and Eq. (3.2), the *ATT-CIVIFWG* operator will be simplified to the following

$$CIVIFWG = ([\textstyle\prod_{j=1}^{n}(a_j)^{(1-l_j)w_j}, \prod_{j=1}^{n}(b_j)^{(1-l_j)w_j}],$$
$$[1-\textstyle\prod_{j=1}^{n}(1-c_j)^{(1-l_j)w_j}, 1-\prod_{j=1}^{n}(1-d_j)^{(1-l_j)w_j}]) \tag{3.4}$$

which is the *CIVIFWG* operator.

Case 2. When $h(t) = \log\left(\frac{2-t}{t}\right)$, the Einstein TTnorm functions are derived as $\tau(p,q) = \frac{pq}{1+(1-p)(1-q)}$ and $\varsigma(p,q) = \frac{p+q}{1+pq}$ [17]. Similarly, the *ATT-CIVIFWG* operator will be simplified to the following

(3.5)

$$CEIVIFWG = \left(\left[\frac{2\prod_{j=1}^{n} a_j^{\Delta_j}}{\prod_{j=1}^{n}(2-a_j)^{\Delta_j} + \prod_{j=1}^{n} a_j^{\Delta_j}}, \frac{2\prod_{j=1}^{n} b_j^{\Delta_j}}{\prod_{j=1}^{n}(2-b_j)^{\Delta_j} + \prod_{j=1}^{n} b_j^{\Delta_j}}\right],\right.$$
$$\left.\left[\frac{\prod_{j=1}^{n}(1+c_j)^{\Delta_j} - \prod_{j=1}^{n}(1-c_j)^{\Delta_j}}{\prod_{j=1}^{n}(1+c_j)^{\Delta_j} + \prod_{j=1}^{n}(1-c_j)^{\Delta_j}}, \frac{\prod_{j=1}^{n}(1+d_j)^{\Delta_j} - \prod_{j=1}^{n}(1-d_j)^{\Delta_j}}{\prod_{j=1}^{n}(1+d_j)^{\Delta_j} + \prod_{j=1}^{n}(1-d_j)^{\Delta_j}}\right]\right)$$

which is defined the confidence Einstein IVIF weighted geometric (*CEIVIFWG*) operator. Where $\Delta_j = (1-\sigma_j)w_j$. Especially, when $\sigma_j = 0$, it further reduces to the Einstein IVIF weighted geometric (*EIVIFWG*) operator developed in [16].

Case 3. When $h(t) = \log\left(\frac{\epsilon+(1-\epsilon)t}{t}\right)$, $\epsilon\rangle 0$, the Hammer TTnorm functions are derived as $\tau(p,q) = \frac{pq}{\epsilon+(1-\epsilon)(p+q-pq)}$ and $\varsigma(p,q) = \frac{p+q-pq-(1-\epsilon)pq}{1-(1-\epsilon)pq}$ [17]. Similarly, the *ATT-CIVIFWG* operator will be simplified to the following

$$\begin{aligned} &CHIVIFWG \\ &= \left(\left[\frac{\epsilon \prod_{j=1}^{n} a_j^{\Delta_j}}{\prod_{j=1}^{n}(1+(\epsilon-1)(1-a_j))^{\Delta_j} + (\epsilon-1)\prod_{j=1}^{n} a_j^{\Delta_j}}, \right.\right. \\ &\qquad \left. \frac{\epsilon \prod_{j=1}^{n} b_j^{\Delta_j}}{\prod_{j=1}^{n}(1+(\epsilon-1)(1-b_j))^{\Delta_j} + (\epsilon-1)\prod_{j=1}^{n} b_j^{\Delta_j}} \right], \\ &\qquad \left[\frac{\prod_{j=1}^{n}\left((1+(\epsilon-1)c_j)^{(1-\sigma_j)w_j}\right) - \prod_{j=1}^{n}\left((1-c_j)^{(1-\sigma_j)w_j}\right)}{\prod_{j=1}^{n}\left((1+(\epsilon-1)c_j)^{(1-\sigma_j)w_j}\right) + \prod_{j=1}^{n}\left((1-c_j)^{(1-\sigma_j)w_j}\right)}, \right. \\ &\qquad \left.\left. \frac{\prod_{j=1}^{n}\left((1+(\epsilon-1)d_j)^{(1-\sigma_j)w_j}\right) - \prod_{j=1}^{n}\left((1-d_j)^{(1-\sigma_j)w_j}\right)}{\prod_{j=1}^{n}\left((1+(\epsilon-1)d_j)^{(1-\sigma_j)w_j}\right) + \prod_{j=1}^{n}\left((1-d_j)^{(1-\sigma_j)w_j}\right)} \right] \right) \end{aligned} \tag{3.6}$$

which is defined the confidence Hammer IVIF weighted geometric (*CHIVIFWG*) operator. Especially, Setting $\epsilon = 1$, results in the *CIVIFWG* operator, while $\epsilon = 2$ generates the *CEIVIFWG* operator.

4. An approach for interval-value intuitionistic fuzzy MAGDM under confidence levels

Consider a MAGDM problem with confidence levels under interval-value intuitionistic fuzzy environment. Assume $A = \{A_1, A_2, \dots A_n\}$ is a collection of alternatives regard to the attribute set $C = \{C_1, C_2, \dots C_m\}$ whose weighted vector is $\varphi = (\varphi_1, \varphi_2, \dots, \varphi_n)^{\mathrm{T}}$. Let $E = \{e_1, e_2, \dots e_p\}$ be the DMs and their weighted vector is $\boldsymbol{w} = (w_1, w_2, \dots, w_n)^{\mathrm{T}}$. Let $l_k = (l_{k1}, l_{k2}, \dots, l_{km})^{\mathrm{T}}$ be the confidence level vector of the DM e_k, which denotes the degrees that e_k is familiar with the evaluation attribute set C, then the assessment of alternative A_i regarding attribute C_j is characterized by an IVIFN α_{ij}^k. Thus, the MAGDM problem can be expressed as $D^k = (\alpha_{ij}^k)_{m\times n}$.

Step 1. Collect the evaluation information and confidence levels of each expert, and form the GDM matrices $D^k = (\alpha_{ij}^k)_{m\times n}$.

Step 2. Convert the matrices $D^k = (\alpha_{ij}^k)_{m\times n}$ into the Normalized IVIF GDM matrices $R^k = (r_{ij}^k)_{m\times n}$. If C_j is benefit attribute, then $r_{ij}^k = \alpha_{ij}^k$.

Step 3. Aggregate the GDM matrices $R^k = (r_{ij}^k)_{m\times n}$ into a collective matrix $R = (r_{ij})_{m\times n}$ by utilized the *ATT-CIVIFWG* operators.

Step 4. Obtain the comprehensive collective rating r_i of A_i by *IVIFWA* operator.

Step 5. Rank the alternatives by Definition 2.5.

5. An illustrative example

5.1. **Dissertation evaluation example.** We invited three experts with different professional familiarity to evaluate five candidate dissertations D_i $(i = 1, 2, \dots 5)$

according to five evaluation attributes (For more details regarding attributes, refer to [19]): (1) C_1: Topic selection and literature review; (2) C_2: Innovation; (3) C_3: Theory basis and special knowledge; (4) C_4: Capacity of scientific research; (5) C_5: Dissertation standardization. The attribute weight vector is denoted using $\varphi = (0.15, 0.3, 0.2, 0.2, 0.15)^{\mathrm{T}}$. Suppose three experts, denoted as e_k and assigned equal weighting, employ IVIFNs to characterize the aforementioned attributes. The ratings and confidence levels of the three experts are shown in Table 1.

TABLE 1. The ratings and confidence levels of three experts

DM	Dissertations	C_1	C_2	C_3	C_4	C_5
e_1,	d_1	([0.7, 0.8], [0.1, 0.2])	([0.7, 0.8], [0.0, 0.2])	([0.6, 0.8], [0.1, 0.2])	([0.6, 0.7], [0.2, 0.3])	([0.5, 0.5], [0.3, 0.4])
0.7	d_2	([0.6, 0.7], [0.1, 0.3])	([0.5, 0.6], [0.3, 0.4])	([0.7, 0.8], [0.1, 0.2])	([0.7, 0.8], [0.1, 0.2])	([0.7, 0.8], [0.0, 0.2])
	d_3	([0.6, 0.7], [0.2, 0.3])	([0.5, 0.5], [0.3, 0.4])	([0.6, 0.7], [0.2, 0.3])	([0.7, 0.8], [0.1, 0.2])	([0.5, 0.8], [0.0, 0.2])
	d_4	([0.5, 0.8], [0.0, 0.2])	([0.5, 0.6], [0.1, 0.3])	([0.4, 0.6], [0.1, 0.2])	([0.5, 0.6], [0.3, 0.4])	([0.5, 0.8], [0.0, 0.2])
	d_5	([0.5, 0.6], [0.3, 0.4])	([0.4, 0.5], [0.4, 0.5])	([0.5, 0.5], [0.4, 0.5])	([0.3, 0.6], [0.2, 0.4])	([0.6, 0.7], [0.1, 0.2])
e_2	d_1	([0.6, 0.7], [0.2, 0.3])	([0.6, 0.7], [0.2, 0.3])	([0.6, 0.7], [0.2, 0.3])	([0.5, 0.6], [0.3, 0.3])	([0.4, 0.5], [0.4, 0.5])
0.9	d_2	([0.5, 0.6], [0.3, 0.4])	([0.5, 0.5], [0.3, 0.4])	([0.6, 0.8], [0.1, 0.2])	([0.6, 0.8], [0.1, 0.2])	([0.6, 0.7], [0.0, 0.2])
	d_3	([0.5, 0.6], [0.3, 0.4])	([0.4, 0.5], [0.2, 0.4])	([0.5, 0.7], [0.1, 0.2])	([0.5, 0.8], [0.2, 0.2])	([0.4, 0.7], [0.1, 0.3])
	d_4	([0.4, 0.7], [0.1, 0.3])	([0.4, 0.6], [0.1, 0.3])	([0.4, 0.5], [0.1, 0.3])	([0.5, 0.6], [0.2, 0.4])	([0.4, 0.7], [0.1, 0.2])
	d_5	([0.4, 0.6], [0.2, 0.4])	([0.3, 0.5], [0.3, 0.4])	([0.4, 0.5], [0.3, 0.5])	([0.3, 0.5], [0.3, 0.4])	([0.5, 0.6], [0.2, 0.3])
e_3	d_1	([0.7, 0.8], [0.2, 0.2])	([0.7, 0.8], [0.1, 0.2])	([0.5, 0.8], [0.1, 0.2])	([0.5, 0.7], [0.2, 0.3])	([0.4, 0.5], [0.3, 0.4])
0.7	d_2	([0.5, 0.7], [0.1, 0.3])	([0.4, 0.6], [0.3, 0.4])	([0.6, 0.7], [0.1, 0.2])	([0.6, 0.7], [0.1, 0.2])	([0.5, 0.6], [0.0, 0.1])
	d_3	([0.5, 0.7], [0.2, 0.3])	([0.4, 0.5], [0.3, 0.4])	([0.5, 0.7], [0.2, 0.3])	([0.6, 0.8], [0.1, 0.2])	([0.4, 0.8], [0.1, 0.2])
	d_4	([0.5, 0.7], [0.1, 0.3])	([0.5, 0.5], [0.2, 0.3])	([0.3, 0.5], [0.1, 0.2])	([0.4, 0.6], [0.2, 0.4])	([0.5, 0.6], [0.0, 0.2])
	d_5	([0.4, 0.6], [0.3, 0.4])	([0.4, 0.4], [0.4, 0.5])	([0.4, 0.5], [0.4, 0.5])	([0.4, 0.5], [0.2, 0.4])	([0.5, 0.7], [0.2, 0.2])

Step 1. The GDM matrices $D^k = (\alpha_{ij}^k)_{5\times 5}$ $(k = 1, 2, 3)$ can be constructed in Table 1.

Step 2. Given that C_j corresponds to advantageous characteristics, we can get the normalized GDM matrices $R^k == (r_{ij}^k)_{5\times 5} = (\alpha_{ij}^k)_{5\times 5}(k = 1, 2, 3)$.

Step 3. We employ the special case *CEIVIFWG* operator among *ATT-CIVIFWG* operators to aggregate R^k containing confidence levels to a collective matrix R.

Step 4. Through the *IVIFWA* operator, the overall collective evaluation v_i of dissertations can be derived as follows: $v_1 = ([0.89, 0.93], [0.04, 0.07])$, $v_2 = ([0.87, 0.91], [0.03, 0.07])$, $v_3 = ([0.85, 0.91], [0.04, 0.08])$, $v_4 = ([0.71, 0.86], [0.03, 0.10])$ and $v_5 = ([0.77, 0.82], [0.1, 0.14])$.

Step 5. According to Eq. (2.3), we have $S(v_1) = 0.851$, $S(v_2) = 0.838$, $S(v_3) = 0.821$, $S(v_4) = 0.725$ and $S(v_5) = 0.677$. Therefore, the ranking order of dissertations is $d_2 \succ d_1 \succ d_3 \succ d_4 \succ d_5$ and the best one is d_2.

5.2. **Sensitivity analysis.** We conduct a sensitivity analysis with the confidence level of e_2, shown as in Table 2. When $\sigma_j = 0$, the *CEIVIFWG* operator reduces to the traditional methods [16]. From Table 2, if $\sigma_j = 0, 1, 3$, then the ranking order of dissertations is $d_2 \succ d_1 \succ d_3 \succ d_4 \succ d_5$, aligning with the computational outcomes reported in [16]. The analysis reveals that the proposed method is a generalized form of traditional methods. If $\sigma_j = 5, 7, 9$, the ranking order of dissertations is $d_1 \succ d_2 \succ d_3 \succ d_4 \succ d_5$. Obviously, when the expert's confidence level is different, the corresponding ranking of dissertations are not quite identical. This suggests that the confidence level of experts influences the decision-making outcomes for the alternatives. From the above-mentioned computing process, the key features of confidence IVIF aggregation operators over traditional ones are due not only to the adaptability of IVIF environment but also to the consideration of DMs' confidence levels. This makes the proposed MAGDM approaches more practical & feasible.

TABLE 2. The ranking with different confidence levels of experts

σ_2	$S(v_1)$	$S(v_2)$	$S(v_3)$	$S(v_4)$	$S(v_5)$	Ranking	Best
0	0.663	0.683	0.634	0.481	0.246	$d_2 \succ d_1 \succ d_3 \succ d_4 \succ d_5$	d_2
0.1	0.686	0.702	0.657	0.509	0.294	$d_2 \succ d_1 \succ d_3 \succ d_4 \succ d_5$	d_2
0.3	0.732	0.740	0.683	0.666	0.562	$d_2 \succ d_1 \succ d_3 \succ d_4 \succ d_5$	d_2
0.5	0.779	0.778	0.749	0.628	0.519	$d_1 \succ d_2 \succ d_3 \succ d_4 \succ d_5$	d_1
0.7	0.830	0.810	0.796	0.692	0.618	$d_1 \succ d_2 \succ d_3 \succ d_4 \succ d_5$	d_1
0.9	0.873	0.851	0.845	0.759	0.738	$d_1 \succ d_2 \succ d_3 \succ d_4 \succ d_5$	d_1

5.3. **Comparison with existing methods.** This section conducts a comparative analysis between our proposed MAGDM methods and two similarity approaches [19] [6]. The main merits of our technique are presented below.

(1) When $a_j = b_j$ and $c_j = d_j$ are satisfied for every j, the proposed aggregation operator will reduce to confidence IF geometric operator, which can be applied to evaluate the doctoral dissertation presented in [19]. It can be seen that the scores of five dissertations increase with the confidence level increase which is consistent with reality. However, when the confidence levels of experts increase from 0 to 1, the scores of the five dissertations decrease by the confidence IF geometric operators [12], which is inconsistent with reality. Therefore, the proposed CWG-GDM is more reasonable than the confidence IF geometric operators based GDM in [19].

(2) The proposed confidence IVIF aggregation operators are based on Archimedean TTnorm, which include algebraic, Einstein and Hamacher TTnorms etc. Whereas the confidence Pythageorean fuzzy aggregations [6] and confidence IF aggregations [19] are both based on the algebraic norm operations, a specialized form of Archimedean TTnorm. Hence, the developed aggregations are more general and flexible for decision situation.

6. Conclusion

Motivated by the characteristics of Archimedean TTnorm, this study establishes some new operational laws for IVIFNs and discusses the properties. The confidence IVIF weighted geometric aggregation operator is developed based on Archimedean TTnorm. Some key conclusions are also drawn. A series of specific confidence IVIF aggregation operators are deduced by assigning different additive generators, such as confidence algebraic, Einstein, and Hamacher IVIF aggregation operators. Additionally, the proposed aggregation operators generalize existing confidence IF aggregation techniques, making them suitable for addressing IVIF MAGDM problems with diverse confidence levels. An example centered on doctoral dissertation evaluation highlights the method's effectiveness and practical applicability.

But the expert confidence level of the proposed method is given subjectively, which is not suitable for solving online Intelligent MAGDM problems [7]. In the future, we can objectively calculate the confidence level of experts according to their reputation and the proportion of successful review papers.

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Y. F. Zheng
School of Economics and Management, East China JiaoTong University Nanchang 330013, China
E-mail address: dondon105@163.com

J. Xu
School of Statistics and Data Science, Key Laboratory of Data Science in Finance and Economics, Jiangxi University of Finance and Economics Nanchang 330013, China
E-mail address: xujun1028@126.com