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FUZZY COMPREHENSIVE EVALUATION WITH MULTIPLE-CHOICE EVALUATION SET AND ITS APPLICATION IN CHINESE COLLEGE STUDENTS' CORE COMPETENCIES

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Abstract. The core competencies of Chinese college students comprehensively take into account aspects such as moral education literacy, intellectual education literacy, physical education literacy, aesthetic education literacy, and labor education literacy. Evaluating these aspects of core competencies is of great significance for university educators in China. The traditional fuzzy comprehensive evaluation method is suitable for the single-choice evaluation set. In this paper, we extend the previous theory of the single-choice evaluation set to the so-called multiple-choice evaluation set. Subsequently, based on interval value and its relevant operation rules, we theoretically propose a fuzzy comprehensive evaluation model to scientifically evaluate the core competencies of college students. The effectiveness and validity of this model are verified through a numerical example. In order to better guide practice, the target audience of this study includes ideological and political educators, university administrators, and researchers interested in the evaluation of college students' core competencies.

1. Introduction

1.1. **The concept and significance of core competencies.** The core competencies of college students in the 21st century plays a crucial role in the growth and success of young people [17]. In the Chinese context, it encompass various aspects, including moral education literacy, intellectual education literacy, physical education literacy, aesthetic education literacy, and labor education literacy [11].

Generally, moral education literacy focuses on cultivating students' moral values, ethics, and social responsibility. It includes qualities such as integrity, honesty, respect for others, and a sense of justice. Students with high moral education literacy are expected to make ethical decisions and contribute positively to society [7]. Intellectual education literacy emphasizes the acquisition of knowledge, critical thinking skills, and the ability to learn and innovate. This includes proficiency in various academic subjects, as well as the ability to analyze and solve problems, think creatively, and communicate effectively. Physical education literacy aims to develop students'

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physical health, fitness, and sports skills [5]. It promotes a healthy lifestyle, including regular exercise, good nutrition, and proper rest. Additionally, it fosters qualities such as teamwork, competition, and perseverance through sports and physical activities [20]. Aesthetic education literacy cultivates students' appreciation and creation of beauty. It includes an understanding of art, music, literature, and other forms of cultural expression. Through aesthetic education, students develop their creativity, imagination, and emotional intelligence. Labor education literacy emphasizes the importance of work and practical skills. It involves teaching students about the value of labor, developing their work ethic, and providing them with hands-on experience in various types of work. This includes skills such as manual labor, technical skills, and the ability to manage and organize work.

Overall, the core competencies of college students in China are designed to nurture well-rounded individuals who possess not only academic knowledge but also the moral, physical, aesthetic, and practical skills needed to succeed in life and contribute to society [9]. These competencies are interrelated and mutually reinforcing, and their development is essential for the holistic growth and development of students, as they not only contribute to their academic success but also play a vital role in shaping their values, outlook on life, and sense of social responsibility [2].

1.2. **Previous studies and their limitations.** Accurately assessing the core competencies of college students is essential for fostering their comprehensive development and enhancing their adaptability to the ever-changing social environment. By identifying and nurturing these competencies, universities can better fulfill their mission of cultivating talents and contributing to the progress of society [4].

Recent studies have shown that the core competencies of college students include not only academic skills but also non-cognitive skills such as critical thinking, creativity, and emotional intelligence [10]. Since the core competencies have complex structures and distinct characteristics, they are implicit and difficult to directly observe, making evaluation challenging [4]. Currently, some scholars have explored the evaluation of college students' core competencies by theoretical construction of the evaluation index system and it mainly adopts the model of Analytic Hierarchy Process(AHP) [16]. Although different scholars have constructed different evaluation index systems, the core problem is that how to figure out the quantitative evaluation results through the evaluation model. Currently, the main evaluation methods include fuzzy comprehensive evaluation method [19], Principal Component Analysis(PCA) method [15], the BP algorithm [3], etc.

However, the existing literature on the evaluation of college students' core competencies lacks a comprehensive and systematic approach [12]. Previous theoretical frameworks, such as the AHP [16], have their limitations in handling the uncertainty and subjectivity in the evaluation process. Additionally, many studies [12] fail to consider the multiple dimensions and characteristics of these competencies, and the evaluation methods used are often simplistic and unable to capture the complexities of the real-world situations. Take the classical fuzzy comprehensive evaluation method as an example, the evaluation experts can only make a single choice based on the evaluation indicators [18]. But due to the subjectivity of the evaluation itself, some experts may have difficulty determining which indicator the evaluated object belongs to, and may hesitate between two or more indicators during the evaluation. This situation is called a multiple-choice evaluation set and the classical fuzzy comprehensive evaluation method is no longer applicable to this situation.

1.3. **Contribution and organization of this work.** Our research problem is to accurately evaluate the core competencies of college students in the 21st century, considering the complexity and uncertainty of these competencies. Unlike previous studies that mainly focus on the theoretical construction of the evaluation index system and overlook the situation where experts may have difficulty making a clear choice between evaluation indicators, this paper proposes a practical fuzzy comprehensive evaluation method based on the multiple-choice evaluation set to address the uncertainty and subjectivity in the evaluation process.

Our study will extend the previous theory of the single-choice evaluation set to the multiple-choice one, providing a more flexible and realistic approach to evaluate college students' core competencies. The proposed model aims to overcome these limitations stated above by using the interval value and its operation rules. We hope that our research target audience of this study includes ideological and political educators, university administrators, and researchers interested in the evaluation of college students' core competencies.

The rest of this paper is organized as follows. In Section 2, we briefly overview the general theory of classical fuzzy comprehensive evaluation method using the single-choice evaluation set. The main results are presented in Section 3. In this section, we present the main results of our methodology, including the multiple-choice evaluation set and its fuzzy evaluation matrix and the comprehensive evaluation model with the multiple-choice evaluation set. To scientifically evaluate the core competencies of college students, the effectiveness and validity of the proposed model are verified through a numerical example in Section 4. Some discussions of our proposed method are provided in Section 5. Finally, we summarize our conclusions in Section 6.

2. Preliminary

The fuzzy comprehensive evaluation method is an evaluation method that combines fuzzy mathematics theory and evaluation techniques to solve complex problems with uncertainty and subjectivity. In this section, we briefly overview the general theory and steps of classical fuzzy comprehensive evaluation method using the single-choice evaluation set [6, 14].

Step 1: Determine the factor set and evaluation set.

According to the real-world problem, clearly define the factor set and possible evaluation set that need to be considered in the evaluation. We denote the factor set $S = \{s_1, s_2, \ldots, s_n\}$ containing different influencing factors (indicators) and the evaluation set $T = \{t_1, t_2, \ldots, t_m\}$ containing different evaluation indicators.

Step 2: Construct the fuzzy evaluation matrix.

Based on the factor set and evaluation set, for the evaluation of one specific thing, people usually do not have absolute affirmation or negation of m evaluation indicators, so a fuzzy number between 0 and 1 can be adopted to represent it. In traditional fuzzy comprehensive evaluation methods, experts can usually only select

TABLE 1. Experts' voting results for the specific evaluated thing with the single-choice evaluation set.

Evaluation item	t_1	t_2	$\dots$	t_m
s_1	g_{11}	g_{12}	$\cdots$	g_{1m}
s_2	g_{21}	g_{22}	$\cdots$	g_{2m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_n	g_{n1}	g_{n2}	$\cdots$	g_{nm}

a single evaluation indicator for one specific evaluated thing. In this situation, we call this as the single-choice evaluation set.

Definition 2.1 (Single-choice evaluation set)**.** Suppose for the evaluation of one specific thing, we call the evaluation set $T = \{t_1, t_2, \dots, t_m\}$ as the single-choice evaluation set, if each expert in the evaluation group can only choose one certain indicator in the evaluation set.

Assuming that the evaluation result of the i-th influencing factor s_i on the j-th evaluation indicator t_j is a fuzzy number $d_{ij} \in [0, 1]$, $i \in \mathbb{I} = \{1, 2, \dots, n\}$, $j \in \mathbb{J} = \{1, 2, \dots, m\}$, then the evaluation result of the experts can be represented by a fuzzy matrix as follow,

$$D = (d_{ij})_{n\times m} = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1m} \\ d_{21} & d_{22} & \dots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{nm} \end{pmatrix}. \tag{2.1}$$

Here, D is called the fuzzy evaluation matrix. The calculation of D has a certain degree of subjectivity. In fact, d_{ij} can be regarded as the degree of membership of the element s_i in the factor set to the element t_j in the evaluation set. In actual evaluation, adopting the "*expert voting method*" is a commonly used method to determine the matrix D. Without loss of generality, based on this method, we show the calculation of the fuzzy evaluation matrix D for the single-choice evaluation set.

Regarding to the factor set $S = \{s_1, s_2, \dots, s_n\}$ and the evaluation set $T = \{t_1, t_2, \dots, t_m\}$, suppose that the evaluation group is composed of g experts, each expert selects only one certain evaluation indicator in the evaluation set through voting, indicating their evaluation results for the specific evaluated thing. The voting result for selecting only one single indicator is generally referred to as the "deterministic voting result".

If the number of votes in favor of the i-th factor indicator s_i as the j-th evaluation indicator t_j is g_{ij}, $i \in \mathbb{I}$, $j \in \mathbb{J}$, then the evaluation results can be summarized as Table 1. Assuming that there is no abstention from voting of all the experts, then it must be satisfied with

$$\sum_{j=1}^{m} g_{ij} = g_{i1} + g_{i2} + \cdots + g_{im} = g, \ \forall i \in \mathbb{I}. \tag{2.2}$$

Based on the voting results in Table 1, the fuzzy evaluation matrix can be calculated accordingly as $D = (d_{ij})_{n\times m}$, where

$$d_{ij} = \frac{g_{ij}}{g_{i1} + g_{i2} + \cdots + g_{im}} = \frac{g_{ij}}{g}, \ \forall i \in \mathbb{I},\ j \in \mathbb{J}. \tag{2.3}$$

Step 3: Determine the weight vector of the factor set.

Due to the different effects of influencing factors on the evaluated thing, in general, the indicators in the factor set also have different weights. Assuming that the weight of the ith indicator s_i on the evaluated thing is $\alpha_i \in [0,1]$, $i \in \mathbb{I} = \{1, 2, \ldots, n\}$, then the weight vector of the influencing factors can be expressed as an n-dimensional fuzzy vector as follows,

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in [0,1]^n, \tag{2.4}$$

where $\sum_{i=1}^{n} \alpha_i = 1$.

Step 4: Determine the comprehensive evaluation model and calculate the comprehensive evaluation vector.

After obtaining the fuzzy evaluation matrix, based on the weight vectors of the influencing factors, the comprehensive evaluation vector β can be calculated as $\beta = (\beta_1, \beta_2, \ldots, \beta_m)$, and

$$\beta = \alpha \circ D, \tag{2.5}$$

where $\beta_j, j \in \mathbb{J}$, represents the quantitative evaluation result of the evaluated object regarding the j-th evaluation indicator. In Eq. (2.5),"$\circ$" represents the fuzzy synthesis operator and generally, it can take the operators such as $(\wedge, \vee)$, $(\cdot, +)$ and $(\cdot, \vee)$. Obviously, the comprehensive evaluation vector $\beta \in [0,1]^m$ is a fuzzy vector.

Step 5: Make a comprehensive evaluation conclusion.

After obtaining the vector β, subsequently, the problem is that how to determine the result of the comprehensive evaluation. Here are two methods commonly employed.

Method 1: Let the maximum component of the vector β be β_{j^*} , i.e. $\beta_{j^*} = \max\{\beta_1, \beta_2, \ldots, \beta_m\}$, $j \in \mathbb{J}$, then the result of the evaluated thing is the indicator t_{j^*}.

Method 2: Considering that different indicators in the evaluation set contribute differently to the evaluation result, it can be assumed that the score component corresponding to the j-th evaluation indicator is t_j, and the weight vector of the evaluation set can be denoted as $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_m)^T$. Therefore, the total score for evaluating the thing, denoted as $Score$, can be obtained as follows,

$$Score = \beta \cdot \gamma = (\beta_1, \beta_2, \ldots, \beta_m) \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{pmatrix} = \beta_1\gamma_1 + \beta_2\gamma_2 + \cdots + \beta_m\gamma_m. \tag{2.6}$$

If the same criteria are used for evaluation of different evaluated things, then Eq. (2.6) can be used to quantitatively score each object comprehensively. Generally, the higher the quantitative evaluation score, the better the evaluation result of the evaluated thing.

3. Methodology

3.1. **The multiple-choice evaluation set and its fuzzy evaluation matrix.**

In the real world, for evaluating one specific thing, regarding to the elements in the evaluation set, the experts are allowed to choose two or more evaluation indicators when voting. For example, for one of the indicators in the factor set, such as s_1, is evaluated. If the expert determines that its evaluation result is t_1, then the only indicator t_1 is selected by voting. Obviously, this is the case of "single-choice evaluation set". If an expert determines that their evaluation result is t_1 or t_2, but cannot determine whether it is t_1 or t_2, then the expert can choose both t_1 and t_2 for voting at the same time. As stated in Section 2, the voting result for selecting only one single indicator is referred to as the "deterministic voting result", while the voting result for selecting two or more indicators is referred to as the "*hesitant voting result*". To distinguish between these two types of voting results, for the latter situation, we call the evaluation set as "*multiple-choice evaluation set*".

Definition 3.1. (*Multiple-choice evaluation set*) Suppose for the evaluation of one specific thing, we call the evaluation set $T = \{t_1, t_2, \ldots, t_m\}$ as the multiple-choice evaluation set, if each expert in the evaluation group is allowed to choose two or more evaluation indicators in the evaluation set.

Assuming the evaluation of the indicator s_i in the factor set of one certain thing, there are g_{ij} experts who vote as t_j based on a unique indicator, and $g^{'}_{ij}$ experts whose votes contain two or more indicators, including the indicator t_j, in the evaluation set. Then, for the indicator factor s_i, the number of votes for the single-choice t_j is g_{ij}, and the number of votes for the multiple-choice t_j is $g^{'}_{ij}$.

Obviously, the vote of the single-choice indicator shows that experts are 100% certain of their evaluation results, while the multiple-choice one means that experts are currently uncertain and have chosen the evaluation indicator t_j with a hesitant attitude. Therefore, it is reasonable the result of the votes can be given a greater weight for the single-choice evaluation indicator t_j than for the multiple-choice one. Here, we might as well set the weight of the result of the votes for the single-choice indicator t_j to 1 and the weight of the multiple-choice one to 0.5. Then the number of votes for the factor t_j can be equivalently expressed as $[g_{ij}, g_{ij} + 0.5g^{'}_{ij}]$, which is represented in the form of the so-called *interval value*. In this way, the left endpoint of the interval represents the number of confirmed votes of the experts, while the right endpoint contains the equivalent total number of confirmed and hesitant votes. In summary, the expert voting results with the multiple-choice evaluation set can be described as Table 2 below.

If the number of experts is still recorded as g, then the voting results in Table 2 satisfy

$$g_{i1} + g_{i2} + \cdots + g_{im} \leq g \leq g_{i1} + g^{'}_{i1} + g_{i2} + g^{'}_{i2} + \cdots + g_{im} + g^{'}_{im}, \ \forall i \in \mathbb{I}. \tag{3.1}$$

For the case of the single-choice evaluation set, we employ a fuzzy number $d_{ij} \in [0, 1]$ to represent the evaluation result of the i-th influencing factor s_i on the j-th evaluation indicator t_j, $i \in \mathbb{I}$, $j \in \mathbb{J}$. Hence, we can generate the the fuzzy evaluation matrix. But for the case of the multiple-choice evaluation set, in order

TABLE 2. Experts' voting results for the specific evaluated thing with the multiple-choice evaluation set.

Evaluation item	t_1	t_2	$\ldots$	t_m
s_1	$[g_{11}, g_{11} + 0.5g'_{11}]$	$[g_{12}, g_{12} + 0.5g'_{12}]$	$\ldots$	$[g_{1m}, g_{1m} + 0.5g'_{1m}]$
s_2	$[g_{21}, g_{21} + 0.5g'_{21}]$	$[g_{22}, g_{22} + 0.5g'_{22}]$	$\ldots$	$[g_{2m}, g_{2m} + 0.5g'_{2m}]$
$\vdots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
s_n	$[g_{n1}, g_{n1} + 0.5g'_{n1}]$	$[g_{n2}, g_{n2} + 0.5g'_{n2}]$	$\ldots$	$[g_{nm}, g_{nm} + 0.5g'_{nm}]$

to generate this kind of matrix, we define the so-called fuzzy evaluation matrix with the multiple-choice evaluation set in the following.

Definition 3.2. (*Fuzzy evaluation matrix with multiple-choice evaluation set*) For the multiple-choice evaluation set, the experts' voting results are described as Table 2, we call matrix G as the fuzzy evaluation matrix with the multiple-choice evaluation set, where

$$G = (G_{ij})_{n\times m} = ([g_{ij}, g_{ij} + 0.5g'_{ij}])_{n\times m}, i \in \mathbb{I},\ j \in \mathbb{J}. \tag{3.2}$$

Remark 3.3. As is shown in Definition 3.2, we can see that each element in the fuzzy matrix G may not be one number, but in the form of the interval value. Moreover, the left point and the right point of the interval may not fall in $[0, 1]$, this is different from the fuzzy number in matrix D.

3.2. **Interval value and its operation rule.** In order to better construct the fuzzy comprehensive evaluation model based on the interval value and its operation rules, here we show the general theory and properties of interval value.

Definition 3.4 ([6, 13, 14])**.** Numbers in the form of $[a, b]$ are called interval value or interval number, where a and b are both real numbers and $a \le b$.

Definition 3.5 ([6, 13, 14])**.** Let $[a, b]$, $[c, d]$ be any two interval values, and k be any real number. The operation rules for interval values are as follows:

(1) $[a, b] + [c, d] = [a + c, b + d]$;
(2) $[a, b] - [c, d] = [a - c, b - d]$;
(3) $k[a, b] = \begin{cases} [ka, kb], & if\ k \ge 0, \\ [kb, ka], & if\ k < 0; \end{cases}$
(4) $[a, b] \wedge [c, d] = [a \wedge c, b \wedge d]$;
(5) $[a, b] \vee [c, d] = [a \vee c, b \vee d]$.

Suppose the set consisting of all interval values is denoted as IV. Next, we define the relationship "$\le$" on the set of IV in the following.

Definition 3.6. If $[a, b]$, $[c, d] \in IV$, then $[a, b]$ is said to be less than $[c, d]$, denoted as $[a, b] < [c, d]$, if

(1) $\frac{a+b}{2} < \frac{c+d}{2}$; or
(2) $\frac{a+b}{2} = \frac{c+d}{2}$ and $b - a > d - c$.

(3) Additionally, it is called $[a,b]$ equal to $[c,d]$, denoted as $[a,b]=[c,d]$, if $a=c, b=d$.

Remark 3.7. It is obviously not difficult to verify $[a,b]\leq[c,d]$ if and only if $[a,b]<[c,d]$ or $[a,b]=[c,d]$. The dual symbols of "$<$"and "$\leq$" are respectively referred to as "$>$" and "$\geq$".

According to the above definitions, the following theorem is obvious.

Theorem 3.8. *Assuming that $[a,b]$, $[c,d]$, $[e,f]\in IV$, then it holds that*

(1) $[a,b]\leq[a,b]$.
(2) *if $[a,b]\leq[c,d]$ and $[c,d]\leq[a,b]$ are satisfied, then $[a,b]=[c,d]$.*
(3) *if $[a,b]\leq[c,d]$ and $[c,d]\leq[e,f]$, then $[a,b]\leq[e,f]$.*
(4) *for any $[a,b]\leq[c,d]$ or $[c,d]\leq[a,b]$, there must be one formula that holds.*

Proof. The proof is evident. □

Remark 3.9. According to the above theorem, the relationship "$\leq$" is a fully ordered relationship on IV. Under this fully ordered relationship, any two interval values are comparable in size.

3.3. **The comprehensive evaluation model with the multiple-choice evaluation set.** In this subsection, similar to the situation of the single-choice evaluation set, we provide the general steps to construct the fuzzy comprehensive evaluation model adopting the multiple-choice one in the following.

Step 1: Determine the factor set and its weight vector. Determine the factor set $S=\{s_1,s_2,\ldots,s_n\}$ and the weight vector of the influencing factors as $\alpha=(\alpha_1,\alpha_2,\ldots,\alpha_n)$.

Step 2: Determine the evaluation set and its quantization vector Determine the evaluation set $T=\{t_1,t_2,\ldots,t_m\}$ and its quantization vector $\gamma=(\gamma_1,\gamma_2,\ldots,\gamma_m)^T$.

Step 3: Determine the fuzzy evaluation matrix with the multiple-choice evaluation set. Based on the factor set and evaluation set, the experts conduct a voting on the evaluated object. They are allowed to choose two or more indicators for voting, and the voting results are presented in the form of Table 2, which leads to the formation of the fuzzy evaluation matrix G with the multiple-choice evaluation set as shown in (3.2).

Step 4: Calculate the weighted evaluation vector. Based on the weight vector of influencing factors α and the fuzzy matrix G, we use the formula

$$\begin{aligned}\beta&=(\beta_1,\beta_2,\ldots,\beta_m)=\alpha\odot G\\&=(\alpha_1,\alpha_2,\ldots,\alpha_n)\odot\begin{pmatrix}G_{11}&G_{12}&\ldots&G_{1m}\\G_{21}&G_{22}&\ldots&G_{24}\\\vdots&\vdots&\vdots&\vdots\\G_{n1}&G_{n2}&\ldots&G_{nm}\end{pmatrix}\end{aligned}\tag{3.3}$$

to calculate the weighted evaluation vector β. Here, "$\odot$" represents the synthesis operation "$(\cdot,\vee)$". Adopting the operation rules of interval value, we can see it holds that

$$\beta_j = \alpha_1 G_{1j} \vee \alpha_2 G_{2j} \vee \cdots \vee \alpha_n G_{nj} = \bigvee_{i=1}^{n} \alpha_i [g_{ij}, g_{ij} + 0.5g_{ij}^{'}],\ j = 1, 2, \ldots, m. \tag{3.4}$$

Step 5: Figure out the total score of the comprehensive evaluation. Based on the weighted evaluation vector β and the quantization vector of the evaluation set, i.e. γ, use the Eq. (2.6) to calculate the total score of the comprehensive evaluation for the evaluated thing.

Step 6: Make a comprehensive evaluation conclusion. Based on the total score, conduct a fuzzy comprehensive evaluation on the evaluated object. The higher the total score, the better the evaluation result.

4. Numerical example

In this section, we take the evaluation of core competencies of Chinese college students in the 21st century as an example to illustrate the application of the fuzzy comprehensive evaluation method with the multiple-choice evaluation set we proposed in this paper. At the same time, for the purpose of giving the comparisons of the current findings with the findings of previous research, we will also provide the case of the fuzzy comprehensive evaluation method with the single-choice evaluation set.

Example 4.1. Suppose there are three classes (Class I, Class II, and Class III) in one certain university of China. Considering that the core competencies of college students play a significant role in the growth process of contemporary Chinese college students, the educators of this university plan to conduct a reasonable assessment of the core competencies of the college students in these three classes. To achieve this task, firstly, based on the past practices of the university, the core literacy of college students is divided into five components: Moral education literacy, Intellectual education literacy, Physical education literacy, Aesthetic education literacy, and Labor education literacy. Ten experts were also invited to vote on each evaluation indicator in the questionnaire [1, 8]. When conducting the expert voting, each expert was provided with a clear description of the evaluation criteria and indicators. They were asked to carefully consider the characteristics and performance of the students being evaluated and make their voting decisions based on their professional judgment. In the case of the multiple-choice evaluation set, experts were allowed to select two or more indicators if they were unsure about the exact evaluation result. In this example, we aim to evaluating the core competencies of these college students based on fuzzy comprehensive evaluation method with the single-choice and the multiple-choice evaluation set, respectively.

Solution:

Step 1: According to the previous section, it is necessary to first determine the factor set and evaluation set in the evaluation process. As for this example, the factor set in this model can be determined as $S = \{s_1, s_2, s_3, s_4, s_5\}$, where s_1 represents "Moral education literacy", s_2 represents "Intellectual education literacy", s_3 represents "Physical education literacy", s_4 represents "Aesthetic education literacy", s_5 represents "Labor education literacy". The determination of

TABLE 3. Experts voting results on factor s_1 of Class I based on the single-choice evaluation set.

Experts' ID	Excellent	Good	Medium	Poor
Expert 1	✓	⊗	⊗	⊗
Expert 2	✓	⊗	⊗	⊗
Expert 3	⊗	✓	⊗	⊗
Expert 4	⊗	✓	⊗	⊗
Expert 5	✓	⊗	⊗	⊗
Expert 6	⊗	⊗	✓	⊗
Expert 7	⊗	⊗	✓	⊗
Expert 8	⊗	⊗	✓	⊗
Expert 9	✓	⊗	⊗	⊗
Expert 10	⊗	✓	⊗	⊗

the weight vector of the influencing factors was based on the suggestions of senior education experts in this field. These experts took into account the relative importance of each factor in the development of college students' core competencies and assigned appropriate weights accordingly. This ensured that the weights were reasonable and reflected the actual situation. Here, this vector is employed as $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_5) = (0.15, 0.55, 0.1, 0.1, 0.1)$ based on the suggestions of the senior education experts in this field.

Step 2: Secondly, according to the questionnaire items [1, 8], we determine the evaluation set as $T = \{t_1, t_2, t_3, t_4\}$, where t_1 represents "Excellent", t_2 represents "Good", t_3 represents "Medium", t_4 represents "Poor". Meanwhile, determine the quantization vector of the evaluation set T as $\gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)^T = (1, 0.75, 0.5, 0.25)^T$ based on the suggestions of the senior education experts.

Step 3: Based on the factor set and evaluation set, 10 experts were invited to vote and evaluate Class I, Class II, and Class III. When evaluating each influencing factor, for the case of the single-choice evaluation set, the experts were allowed to choose only one of the indicators for voting, and for the case of multiple-choice one, the experts were allowed to choose two or more indicators. For example, regarding the influencing factor s_1 (i.e. "Moral education literacy") of Class I, the voting results of the experts associated with the single-choice and multiple-choice evaluation set are shown in Table 3 and 4, respectively. Here, "✓" indicates that the expert voted in favor of the corresponding evaluation indicators, and "⊗" indicates that the expert did not vote on the corresponding evaluation indicators.

Due to space limitations, the tables of voting results for all evaluation items of all classes will not be displayed one by one. After collecting the voting data, to ensure the reliability and validity of the study, we conducted a pre-test to validate the questionnaire items and the evaluation indicators. We also calculated the inter-rater reliability to ensure the consistency of the experts' voting results. Additionally, we compared the results of our study with those of previous studies to verify the validity of our model. The voting counts for three classes are shown in Table 5, 6,

TABLE 4. Experts voting results on factor s_1 of Class I based on the multiple-choice evaluation set.

Expert ID	Excellent	Good	Medium	Poor
Expert 1	✓	⊗	⊗	⊗
Expert 2	✓	✓	⊗	⊗
Expert 3	⊗	✓	✓	⊗
Expert 4	⊗	✓	⊗	⊗
Expert 5	✓	✓	⊗	⊗
Expert 6	⊗	⊗	✓	⊗
Expert 7	⊗	✓	✓	⊗
Expert 8	⊗	⊗	✓	⊗
Expert 9	✓	⊗	⊗	⊗
Expert 10	✓	✓	⊗	⊗

TABLE 5. Experts' voting counts of evaluation items for Class I with single-choice evaluation set.

Evaluation item	Excellent	Good	Medium	Poor
Moral education literacy	4	3	3	0
Intellectual education literacy	3	4	3	0
Physical education literacy	1	3	5	1
Aesthetic education literacy	2	4	3	1
Labor education literacy	4	5	1	0

TABLE 6. Experts' voting counts of evaluation items for Class II with single-choice evaluation set.

Evaluation item	Excellent	Good	Medium	Poor
Moral education literacy	1	5	4	0
Intellectual education literacy	3	5	2	0
Physical education literacy	2	7	1	0
Aesthetic education literacy	0	3	7	0
Labor education literacy	1	5	3	1

and 7 based on the single-choice evaluation set, and Table 8, 9, and 10 based on the multiple-choice evaluation set, respectively.

According to Table 5, 6, and 7, after simple calculations, the fuzzy evaluation matrices of the experts' voting results for each class can be represented as D^1, D^2, D^3 associated with the single-choice evaluation set, and G^1, G^2, G^3 associated with

TABLE 7. Experts' voting counts of evaluation items for Class III with single-choice evaluation set.

Evaluation item	Excellent	Good	Medium	Poor
Moral education literacy	3	4	3	0
Intellectual education literacy	1	6	2	1
Physical education literacy	0	2	7	1
Aesthetic education literacy	1	3	4	2
Labor education literacy	2	6	2	0

TABLE 8. Experts' voting counts of evaluation items for Class I with multiple-choice evaluation set.

Evaluation item	Excellent	Good	Medium	Poor
Moral education literacy	[2,3.5]	[1, 3.5]	[2, 3]	[0, 0]
Intellectual education literacy	[1,1.5]	[3,5]	[2,3.5]	[0,0]
Physical education literacy	[0,0.5]	[1,2.5]	[4,6]	[0,1]
Aesthetic education literacy	[1,2]	[3,5]	[0,1.5]	[1,1.5]
Labor education literacy	[2,3.5]	[3,5]	[1,1.5]	[0,0]

TABLE 9. Experts' voting counts of evaluation items for Class II with multiple-choice evaluation set.

Evaluation item	Excellent	Good	Medium	Poor
Moral education literacy	[1,1.5]	[3,4.5]	[3,4]	[0,0]
Intellectual education literacy	[3,4.5]	[2,4.5]	[0,1]	[0,0]
Physical education literacy	[2,2.5]	[6,7]	[0,0.5]	[0,0]
Aesthetic education literacy	[0,0]	[2,3]	[6,7]	[0,0]
Labor education literacy	[0,0.5]	[4,5.5]	[2,3.5]	[0,0.5]

TABLE 10. Experts' voting counts of evaluation items for Class III with multiple-choice evaluation set.

Evaluation item	Excellent	Good	Medium	Poor
Moral education literacy	[2,3]	[3,4.5]	[2,2.5]	[0,0]
Intellectual education literacy	[0,1]	[4,6.5]	[0,2]	[0,0.5]
Physical education literacy	[0,0]	[0,2]	[5,7.5]	[0,0.5]
Aesthetic education literacy	[0,0.5]	[2,3.5]	[3,4.5]	[1,1.5]
Labor education literacy	[1,2]	[5,6.5]	[1,1.5]	[0,0]

the multiple-choice one in the following, respectively:

$$D^1 = \begin{pmatrix} 0.4 & 0.3 & 0.3 & 0 \\ 0.3 & 0.4 & 0.3 & 0 \\ 0.1 & 0.3 & 0.5 & 0.1 \\ 0.2 & 0.4 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{pmatrix}, D^2 = \begin{pmatrix} 0.1 & 0.5 & 0.4 & 0 \\ 0.3 & 0.5 & 0.2 & 0 \\ 0.2 & 0.7 & 0.1 & 0 \\ 0 & 0.3 & 0.7 & 0 \\ 0.1 & 0.5 & 0.3 & 0.1 \end{pmatrix},$$

$$D^3 = \begin{pmatrix} 0.3 & 0.4 & 0.3 & 0 \\ 0.1 & 0.6 & 0.2 & 0.1 \\ 0 & 0.2 & 0.7 & 0.1 \\ 0.1 & 0.3 & 0.4 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 \end{pmatrix}, G^1 = \begin{pmatrix} [2,3.5] & [1, 3.5] & [2, 3] & [0, 0] \\ [1,1.5] & [3,5] & [2,3.5] & [0,0] \\ [0,0.5] & [1,2.5] & [4,6] & [0,1] \\ [1,2] & [3,5] & [0,1.5] & [1,1.5] \\ [2,3.5] & [3,5] & [1,1.5] & [0,0] \end{pmatrix},$$

$$G^2 = \begin{pmatrix} [1,1.5] & [3,4.5] & [3,4] & [0,0] \\ [3,4.5] & [2,4.5] & [0,1] & [0,0] \\ [2,2.5] & [6,7] & [0,0.5] & [0,0] \\ [0,0] & [2,3] & [6,7] & [0,0] \\ [0,0.5] & [4,5.5] & [2,3.5] & [0,0.5] \end{pmatrix},$$

$$G^3 = \begin{pmatrix} [2,3] & [3,4.5] & [2,2.5] & [0,0] \\ [0,1] & [4,6.5] & [0,2] & [0,0.5] \\ [0,0] & [0,2] & [5,7.5] & [0,0.5] \\ [0,0.5] & [2,3.5] & [3,4.5] & [1,1.5] \\ [1,2] & [5,6.5] & [1,1.5] & [0,0] \end{pmatrix}.$$

Step 4: For Class I, II, and III, based on the weight vector of influencing factors and the fuzzy evaluation matrices of the experts' voting results, considering the case of single-choice evaluation set, use Eq. (2.5) to calculate the weighted evaluation vectors as β_S^1, β_S^2, β_S^3, and for the case of multiple-choice one, use Eq. (3.3) and (3.4) to calculate the weighted evaluation vectors as β_M^1, β_M^2, β_M^3, respectively. Compared with other mathematical softwares, we employ the MATLAB software to analyze the data due to its excellent computing performance and ability to handle complex mathematical operations. It can efficiently calculate the weighted evaluation vectors and the total scores of the comprehensive evaluation, ensuring the accuracy and reliability of the results. We obtain the following results through MATLAB programming as follows, where

$$\beta_S^1 = \alpha \oplus D^1 = (0.15, 0.55, 0.1, 0.1, 0.1) \oplus \begin{pmatrix} 0.4 & 0.3 & 0.3 & 0 \\ 0.3 & 0.4 & 0.3 & 0 \\ 0.1 & 0.3 & 0.5 & 0.1 \\ 0.2 & 0.4 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{pmatrix}$$
$$= (0.295, 0.385, 0.300, 0.020);$$

$$\beta_M^1 = \alpha \odot G^1 = (0.15, 0.55, 0.1, 0.1, 0.1) \odot \begin{pmatrix} [2,3.5] & [1, 3.5] & [2, 3] & [0, 0] \\ [1,1.5] & [3,5] & [2,3.5] & [0,0] \\ [0,0.5] & [1,2.5] & [4,6] & [0,1] \\ [1,2] & [3,5] & [0,1.5] & [1,1.5] \\ [2,3.5] & [3,5] & [1,1.5] & [0,0] \end{pmatrix}$$
$$= ([0.55, 0.825], [1.65, 2.75], [1.1, 1.925], [0.1, 0.15]).$$

Here, "$\oplus$" represents the synthesis operation "$(\cdot, +)$". By the same calculation, it can be concluded that

$$\beta_S^2 = \alpha \oplus D^2 = (0.21, 0.50, 0.28, 0.01);\ \beta_S^3 = \alpha \oplus D^3 = (0.130, 0.500, 0.285, 0.085);$$
$$\beta_M^2 = \alpha \odot G^2 = ([1.65, 2.475], [1.1, 2.475], [0.6, 0.7], [0, 0.05]);$$

$$\beta_M^3 = \alpha \odot G^3 = ([0.3, 0.55], [2.2, 3.575], [0.5, 1.1], [0.1, 0.275]).$$

Step 5: According to the weighted evaluation vector β_S^1, β_S^2, β_S^3, β_M^1, β_M^2, β_M^3 and the quantization vector γ of the evaluation set, use Eq. (2.6) to calculate the comprehensive total scores for Classes I, II, and III as follows. For the case of single-choice evaluation set, they are represented as $Score_S^1, Score_S^2, Score_S^3$, where

$$Score_S^1 = \beta_S^1 \cdot \gamma = (0.295, 0.385, 0.300, 0.020) \cdot (1, 0.75, 0.5, 0.25)^T = 0.7388;$$
$$Score_S^2 = \beta_S^2 \cdot \gamma = (0.21, 0.50, 0.28, 0.01) \cdot (1, 0.75, 0.5, 0.25)^T = 0.7275;$$
$$Score_S^3 = \beta_S^3 \cdot \gamma = (0.130, 0.500, 0.285, 0.085) \cdot (1, 0.75, 0.5, 0.25)^T = 0.6687.$$

For the case of multiple-choice evaluation set, they are represented as $Score_M^1$, $Score_M^2$, $Score_M^3$, where

$$\begin{aligned} Score_M^1 &= \beta_M^1 \cdot \gamma \\ &= ([0.55, 0.825], [1.65, 2.75], [1.1, 1.925], [0.1, 0.15]) \cdot (1, 0.75, 0.5, 0.25)^T \\ &= [2.3625, 3.8882]; \\ Score_M^2 &= \beta_M^2 \cdot \gamma \\ &= ([1.65, 2.475], [1.1, 2.475], [0.6, 0.7], [0, 0.05]) \cdot (1, 0.75, 0.5, 0.25)^T \\ &= [2.775, 4.69375]; \\ Score_M^3 &= \beta_M^3 \cdot \gamma \\ &= ([0.3, 0.55], [2.2, 3.575], [0.5, 1.1], [0.1, 0.275]) \cdot (1, 0.75, 0.5, 0.25)^T \\ &= [2.225, 3.85]. \end{aligned}$$

Step 6: According to Step 5, for the case of single-choice evaluation set, it is evident to figure out that $Score_S^1 \geq Score_S^2 \geq Score_S^3$. Therefore, in this case, the result of comprehensive evaluation of the core competencies of college students in the three classes shows that Class I is better than Class II, and Class II is better than Class III.

Meanwhile, according to Step 5 and the order relationship defined in Section 3.2, for the case of the multiple-choice evaluation set, it is not difficult to verify that $Score_M^2 \geq Score_M^1 \geq Score_M^3$. Therefore, the result of comprehensive evaluation of the core competencies of college students in the three classes shows that Class II is better than Class I, and Class I is better than Class III. □

Remark 4.2. Obviously, it is different on the comprehensive evaluation conclusions of the single-choice evaluation set and the multiple-choice evaluation set. The main difference focuses on the evaluation results on the Class I and Class II. As matter of fact, compared with the case of the single-choice evaluation set, the results show that the multiple-choice evaluation set can provide more detailed and accurate information about the students' core competencies. Essentially, the single-choice evaluation set may overlook the students' potential in certain areas, while our method can better capture the uncertainty and complexity of the evaluation process, and the interval value and its operation rules can handle the hesitant voting results more effectively.

Remark 4.3. The findings of this example have important implications for educational practice. Educators can use the results to identify areas where students need further development and design targeted interventions to improve their core competencies. Additionally, policymakers can use the information to make informed decisions about educational policies and resource allocation.

5. Discussion

The aim of this study was to develop a fuzzy comprehensive evaluation method based on the multiple-choice evaluation set to evaluate the core competencies of Chinese college students in the 21st century. We consider the multiple-choice evaluation set, which better reflects the uncertainty and complexity of the real-world evaluation situations. Previous studies often used the single-choice evaluation set, which may oversimplify the evaluation process and lead to incomplete or inaccurate results. The main findings are that the proposed method is effective and valid, and can provide more accurate and comprehensive evaluation results than the traditional single-choice evaluation set. The multiple-choice evaluation set can better handle the uncertainty and subjectivity in the evaluation process, and the interval value and its operation rules can calculate the comprehensive evaluation total score more precisely.

Based on the findings, we recommend that educators and policymakers pay more attention to the development of students' core competencies and use the multiple-choice evaluation set in the evaluation process to obtain more accurate and comprehensive information. They can also design targeted education and training programs to improve the students' core competencies based on the evaluation results.

One limitation of this study is the relatively small sample size, which may limit the generalizability of the results. Future studies could expand the sample size to include more classes and universities. Furthermore, future research could explore the use of other evaluation methods or combinations of methods to further enhance the accuracy and reliability of the evaluation.

6. Concluding remarks

Evaluating the core competencies of Chinese college students is of great importance for their growth and the implementation of the fundamental task of "cultivating virtue and nurturing people". In this study, we extend the previous theory of the single-choice evaluation set by proposing a fuzzy comprehensive evaluation method based on the multiple-choice evaluation set, providing a more flexible and realistic approach to evaluate college students' core competencies. The method takes into account the subjectivity and uncertainty in the evaluation process, resulting in more accurate and comprehensive evaluation results.

The classical fuzzy comprehensive evaluation method is only applicable to models with single-choice evaluation sets. In order to evaluate the situation with multiple-choice evaluation sets, we propose a fuzzy comprehensive evaluation method based on interval value and its operation rules and evaluates the core competencies of three classes of college students in a certain university. The experts' voting results with the multiple-choice evaluation sets can be characterized by interval value with fuzzy numbers. Then the corresponding operation rules can be used to calculate the comprehensive evaluation total score of each evaluated object and sort or evaluate it. Overall, accurately evaluating the core competencies of college students is crucial for their personal development and the future of our society. Our research provides a valuable approach to address this important issue and lays the foundation for further research and practice in this area.

Future research could focus on further validating and refining the proposed method, exploring its application in different educational contexts, and investigating the relationship between core competencies and other factors such as students' academic performance and future career success. Moreover, the method proposed in this paper is also applicable to other comprehensive evaluation models with multiple-choice evaluation sets, such as environmental assessment, engineering project evaluation, personnel assessment and so on.

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