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DOMINANT CONDITIONAL ENTROPY-BASED INCREMENTAL ALGORITHM FOR DOMINANT DATASETS

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Abstract. Since dynamic dominant datasets change with time, their efficient reduction in artificial intelligence (AI) applications becomes problematic. This paper analyzes the incremental mechanisms of dominant conditional entropy when adding some objects to dominant datasets and proposes a matrix-based incremental reduction approach. To this end, fusion techniques for the dominant matrix and incremental reduction approach are developed. Finally, the execution time and classification accuracy of incremental and non-incremental reduction approaches applied to four information systems from UCI are estimated and compared. While the classification accuracies of both approaches are close, the incremental reduction approach significantly outperforms the non-incremental one by execution time, providing a more effective reduction of dynamic dominant datasets.

1. Introduction

The rough set theory (RST), a powerful mathematical tool for multi-criterial description and analysis, has been widely used in the last few years to deal with uncertain data mining in many fields, including artificial intelligence (AI) [22], feature selection and recognition [21], image processing technology [6], and machine learning [1].

In RST, the equivalence relationship in the relation domain plays a vital role, and upper and lower approximation sets describe the decision-making objectives. However, it is not effective to process the problem of the dominant dataset. Hence, an extension of RST called dominant rough set theory (DRST) has been developed to reduce and rule-mine dominant information systems, whose major concept is the approximation of class unions [4, 5, 19].

In RST, the attribute reduction method is an important research subject and a crucial process in data mining. In the past few years, some attribute reduction approaches have been presented. However, they are suitable only for computing reductions in the static information system, which are referred to as non-incremental methods [24, 26]. They can not effectively handle the reduction of dynamic data. Several incremental learning approaches have been designed to acquire knowledge of

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dynamic information systems and their reduct update in the following three aspects to fill this gap [2, 3, 11–13, 15, 17].

Firstly, the object variation aspect was explored by Liang et al. [14], who developed an incremental approach to updating the reduct of a dataset under the change of some objects. Alternatively, Jing et al. [9] designed an incremental tool for calculating the reduction of an information system under object variation conditions. Besides, Xu et al. [25] developed an incremental method for updating the reduct of a decision table under the change of some objects.

Secondly, the feature variation aspect was studied by Shu et al. [18], who presented an incremental method to compute the reduct of the dataset under feature variation conditions. In addition, Wang et al. [20] designed an incremental means to calculate the reduct of information systems under a variation of features.

Thirdly, the feature value variation aspect was investigated by Wang et al. [23] and Jing et al. [10], who independently developed incremental means to calculate the reduct of the information system under a variation of feature values.

While the above research efforts promoted the development of a reduction of dynamic information systems, they did not cover dynamic dominant datasets. This paper aims to fill this gap.

The rest of this paper is structured as follows. Some notations of the dominance rough set approach and a reduction approach of DRST are reviewed in Section 2. The incremental mechanisms of dominant conditional entropy and dominant matrix are introduced, and an incremental reduction method based on dominant conditional entropy is proposed in Section 3. The experimental analysis results are provided in Section 4. The research results and future tasks are summarized in Section 5.

2. Preliminaries

This section reviews some notations and the correlation theory of the dominance RST approach [5, 16].

2.1. The major notations of dominance RST.

Definition 2.1. Suppose $IS = (U, A, V, f)$ be an information system. It must comply with:

(1) $U = \{u_1, u_2, \ldots, u_n\}$;
(2) $A = C \cup D$;
(3) $V = \cup_{a \in A} V_a$;
(4) $f : U \times (C \cup D) \to V$.

where U is described as samples, A is described as features,C is described as condition features, D is described as decision features, V is described as domain of feature A, f called information function.

Definition 2.2. Suppose $DIS = (U, A, V, f)$ be a dominant data set. $\forall b \in (C \cup D)$, $u_i, u_j \in U, 1 \leq i, j \leq n$. If $f(u_j, b) \geq f(u_i, b)$, then u_j dominates u_i, represented by $u_j \succeq_a u_i$. In addition, if $f(u_j, b) \leq f(u_i, b)$, then u_j is inferior to u_i, denoted by $u_j \preceq_a u_i$.

To keep it simple, $DIS = (U, A, V, f)$ can be written as DIS below.

Definition 2.3. Suppose DIS be a dominant data set and feature set E is a subset of condition attribute set C, $\forall a \in A$, $u_s, u_t \in U, 1 \leq s, t \leq n$. Then u_s-dominating set $[u_s]_E^{\geq}$ and u_s-dominated set $[u_s]_E^{\leq}$ are described as:

$$[u_s]_E^{\geq} = \{u_s \mid u_t \succeq_a u_s\}, [u_s]_E^{\leq} = \{u_s \mid u_t \preceq_a u_s\} \tag{2.1}$$

Definition 2.4. Suppose DIS be a dominant data set. The decision feature D has a preference order, and $U/D = \{dl_1, dl_2, \ldots, dl_n\}$ is decision classes. $[dl]_s^{\geq}$ is upward unions of $[dl]_s$, $[dl]_s^{\leq}$ is downward unions of $[dl]_s$. Then, $[dl]_s^{\geq}$ and $[dl]_s^{\leq}$ are described as:

$$[dl]_s^{\geq} = \cup_{t \geq s} dl_t, 1 \leq t, s \leq n, [dl]_s^{\leq} = \cup_{t \leq s} dl_t, 1 \leq t, s \leq n \tag{2.2}$$

Definition 2.5. Suppose DIS be a dominant data set and feature set E is a subset of C. The upper approximation set $\underline{P}([dl]_s^{\geq})$ of $[dl]_s^{\geq}$ and lower approximation set $\overline{P}([dl]_s^{\geq})$ of $[dl]_s^{\geq}$ are described as:

$$\underline{P}([dl]_s^{\geq}) = \left\{x \in U \mid [u_i]_E^{\geq} \subseteq [dl]_s^{\geq}\right\}, \overline{P}([dl]_s^{\geq}) = \left\{x \in U \mid [u_i]_E^{\leq} \cap [dl]_s^{\geq} \neq \varnothing\right\} \tag{2.3}$$

Definition 2.6. Let DIS be a dominant data set and feature set E is a subset of C. The upper approximation set $\underline{P}([dl]_s^{\leq})$ of $[dl]_s^{\leq}$ and lower approximation set $\overline{P}([dl]_s^{\leq})$ of $[dl]_s^{\leq}$ are described as:

$$\begin{aligned} &\underline{P}([dl]_s^{\leq}) = \left\{x \in U \mid [u_i]_E \leq\subseteq [dl]_s^{\leq}\right\}, \\ &\overline{P}([dl]_s^{\leq}) = \left\{x \in U \mid [u_i]_E \geq \cap [dl]_s^{\leq} \neq \varnothing\right\} \end{aligned} \tag{2.4}$$

Definition 2.7. Suppose DIS be a dominant data set and feature set E is a subset of C, $U/E = \{[u_1], [u_2], \ldots, [u_m]\}$. Then, dominant entropy $[DH]_E^{\geq}$ is described as [7,8] :

$$[DH]_E^{\geq}(U) = -\frac{1}{|U|}\sum_{i=1}^{m} \log \frac{|[u_i]_E^{\geq}|}{|U|} \tag{2.5}$$

where $|\cdot|$ denotes the cardinality.

Definition 2.8. Suppose DIS be a dominant data set and feature set E is a subset of attribute set A, $E \subseteq A$, $D \subseteq A$,$|n| = |U|$. Then, dominant conditional entropy $[DH]_{D|E}^{\geq}(U)$ is described as [7,8] :

$$[DH]_{D|E}^{\geq}(U) = -\frac{1}{n}\sum_{i=1}^{m} \log \frac{|[u_i]_E^{\geq} \cap [u_i]_D^{\geq}|}{|[u_i]_E^{\geq}|} \tag{2.6}$$

where $[u_i]_E^{\geq} \cap [u_i]_D^{\geq} = [u_i]_{E \cup D}^{\geq}$.

Definition 2.9. Suppose DIS be a dominant data set. E is features, $u_i, u_j \in U, 1 \leq i, j \leq n$, $M_{n\times n}$ is dominant matrix of U in feature E, m_{ij} is element of $M_{n\times n}$. Then, m_{ij} is described as:

$$m_{ij} = \begin{cases} 1, & f(u_j, E) \geq f(u_i, E) \\ 0, & f(u_j, E) < f(u_i, E) \end{cases} \quad 1 \leq i, j \leq n$$

Definition 2.10. Suppose DIS be a dominant data set. A are features, $u_i, u_j \in U, 1 \le i, j \le n$. $M'_{n\times n}$ is dominant matrix of U in feature A, (m'_{ij}) is element of $M'_{n\times n}$. Then, m'_{ij} is described as:

$$m'_{ij} = \begin{cases} 1, & f(u_j, A) \ge f(u_i, A) \\ 0, & f(u_j, A) < f(u_i, A) \end{cases} \quad 1 \le i, j \le n$$

Definition 2.11. Suppose DIS be a dominant data set and feature set E is a subset of attribute set A, $B \subseteq A$, $D \subseteq A$,$n = |U|$. Then, dominant conditional entropy $[DH]^{\ge}_{D|E}$ is described as:

$$[DH]^{\ge}_{D|E}(U) = -\frac{1}{n}\sum_{i=1}^{m} \log \frac{\sum_{j=1}^{n} |sum(m'_{ji})|}{\sum_{j=1}^{n} |sum(m_{ji})|} \tag{2.7}$$

Definition 2.12. Suppose DIS be a dominant data set and feature set E is a subset of condition attribute set C, $\forall a' \in E$. Then, inner significance of a' is described as:

$$Sig_U^{inner}(a', E, D) = [DH]^{\ge}_{D|(E-\{a'\})}(U) - [DH]^{\ge}_{D|E}(U) \tag{2.8}$$

Definition 2.13. Suppose DIS be a dominant data set and feature set E is a subset of condition attribute set C, $\forall a' \in E$. Then, outer significance of a' is described as:

$$Sig_U^{outer}(a', E, D) = [DH]^{\ge}_{D|E}(U) - [DH]^{\ge}_{D|(E\cup\{a'\})}(U) \tag{2.9}$$

Definition 2.14. Suppose DIS be a dominant data set and feature set E is a subset of condition attribute set C. Then, E is reduct of DIS. It must comply with:

(1) $[DH]^{\ge}_{D|E}(U) = [DH]^{\ge}_{D|C}(U)$

(2) $\forall a' \in E, [DH]^{\ge}_{D|(E-\{a'\})}(U) \ne [DH]^{\ge}_{D|E}(U)$.

2.2. **A reduction approach of DRST.** In the subsection, a reduction approach of DRST is introduced.

3. Incremental reduction method

The incremental mechanisms of dominant conditional entropy and the dominant matrix are analyzed, and an incremental reduction method is proposed.

3.1. **Incremental mechanisms of dominant conditional entropy.** In the subsection, some incremental mechanisms of dominant conditional entropy are introduced.

Definition 3.1. Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Then, dominant matrix $Q_{t\times n} = (q_{ij})_{t\times n}$ of U' and U in attribute set C is described as:

$$q_{ij} = \begin{cases} 1, f(u_{n+i}, C) \ge f(u_j, C) \\ 0, f(u_{n+i}, C) < f(u_j, C) \end{cases} \quad 1 \le i, j \le t$$

Algorithm 1: A reduction approach of DRST

Input: A dominance information system DIS.
Output: A reduct P_{RED_U}.

begin
  $P_{RED_U} \leftarrow \varnothing$
  for i=1 *to* $|U|$ **do**
    compute $[DH]^{\geq}_{D|C}(U)$
  end
  for *each* $a_k \in C$ **do**
    calculate $[DH]^{\geq}_{D|(C-\{a_k\})}(U)$,$Sig^{inner}_U(a_k, C, D)$
    if $Sig^{inner}_U(a_k, C, D) > 0$ **then**
      $P_{RED_U} \leftarrow (P_{RED_U} \cup \{a_k\})$
    end
    Let $E \leftarrow P_{RED_U}$;
  end
  while $[DH]^{\geq}_{D|E}(U) \neq [DH]^{\geq}_{D|C}(U)$ **do**
    for *each* $a_k \in (C - E)$ **do**
      calculate $Sig^{outer}_U(a_k, E, D)$
      $a' = max\left\{Sig^{outer}_U(a_k, E, D)\right\}$
      $E \leftarrow (E \cup \{a'\})$
    end
  end
  $P_{RED_U} \leftarrow E$;
  return P_{RED_U}.
end

Definition 3.2. Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Then, dominant matrix $Q'_{t\times n} = (q'_{ij})_{t\times n}$ of U' and U in attribute set A is described as:

$$q'_{ij} = \begin{cases} 1, f(u_{n+i}, A) \geq f(u_j, A) \\ 0, f(u_{n+i}, A) < f(u_j, A) \end{cases} \quad 1 \leq i, j \leq t$$

Definition 3.3. Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Then, dominant matrix $Z_{t\times t} = (z_{ij})_{t\times t}$ of U' in attribute set C is described as:

$$z_{ij} = \begin{cases} 1, f(j, C) \geq f(u_i, C) \\ 0, f(j, C) < f(u_i, C) \end{cases} \quad 1 \leq i, j \leq t$$

Definition 3.4. Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Then, dominant matrix $Z'_{t\times t} = (z_{ij})_{t\times t}$ of U' in attribute set A is described as:

$$z'_{ij} = \begin{cases} 1, f(u_j, A) \geq f(u_i, A) \\ 0, f(u_j, A) < f(u_i, A) \end{cases} \quad 1 \leq i, j \leq t$$

Definition 3.5. Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Then, dominant matrix $W_{n\times t} = (w_{ij})_{n\times t}$ of U and U' in attribute set C is described as:

$$w_{ij} = \begin{cases} 1, f(u_j, C) \geq f(u_{n+i}, C) \\ 0, f(u_j, C) < f(u_{n+i}, C) \end{cases} \quad 1 \leq i, j \leq t$$

Definition 3.6. Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Then, dominant matrix $W'_{n\times t} = (w'_{ij})_{n\times t}$ of U and U' in attribute set A is described as:

$$w'_{ij} = \begin{cases} 1, f(u_j, A) \geq f(u_{n+i}, A) \\ 0, f(u_j, A) < f(u_{n+i}, A) \end{cases} \quad 1 \leq i, j \leq t$$

Theorem 3.7. *Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Let $F = U \cup U'$, we can get the dominant matrix $W_{n\times t}$, $M_{n\times n}$, $Q_{t\times n}$ and $Z_{t\times t}$. Then, dominant matrix $U_{n'\times n'}$ of F in attribute set C is described as:*

$$U_{n'\times n'} = \begin{bmatrix} M_{n\times n} & W_{n\times t} \\ Q_{t\times n} & Z_{t\times t} \end{bmatrix}$$

Theorem 3.8. *Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added to DIS. Let $n' = |U \cup U'|$, $n = |U|$, we can get the dominant matrix $W'_{n\times t}$, $M'_{n\times n}$, $Q'_{t\times n}$ and $Z'_{t\times t}$. Then, dominant matrix $H'_{n'\times n'}$ of F in attribute set A is described as:*

$$H'_{n'\times n'} = \begin{bmatrix} M'_{n\times n} & W'_{n\times t} \\ Q'_{t\times n} & Z'_{t\times t} \end{bmatrix}$$

Theorem 3.9. *Suppose DIS be a dominant data set. $U' = \{u_{n+1}, u_{n+2}, \ldots, u_{n+t}\}$ is added into a dominant information system DIS. Let $F = U \cup U'$, $n' = |U \cup U'|$, $n = |U|$, the dominant conditional of U entropy is $[DH]^{\geq}_{D|B}(U)$. We can get the dominant matrix $W_{n\times t}$, $M_{n\times n}$, $Q_{t\times n}$, $Z_{t\times t}$, $W'_{n\times t}$, $M'_{n\times N}$, $Q'_{t\times n}$ and $Z'_{t\times t}$. Then, the dominant conditional entropy $[DH]^{\geq}_{D|C}(F)$ is described as:*

$$\begin{aligned}
[DH]^{\geq}_{D|C}(F) &= -\frac{n}{n'}[DH]^{\geq}_{D|C}(U) \\
&\quad - \frac{1}{n'}\Big(\sum_{i=1}^{n} \log \frac{\sum_{j=1}^{n} |sum(m'_{ji}) + sum(w'_{ji})|}{\sum_{j=1}^{n} |sum(m'_{ji})|} \\
&\quad + \sum_{i=1}^{n} \log \frac{\sum_{j=1}^{n} |sum(m_{ji})|}{\sum_{j=1}^{n} |sum(m_{ji}) + sum(w_{ji})|} \\
&\quad + \sum_{i=1}^{n} \log \frac{\sum_{j=1}^{n} |sum(q'_{ji}) + sum(z'_{ji})|}{\sum_{j=1}^{n} |sum(q_{ji}) + sum(z_{ji})|}\Big)
\end{aligned} \tag{3.1}$$

3.2. **Incremental reduction method when many objects are added to the dominant dataset.** This subsection proposes an incremental reduction method of DRST according to the incremental mechanisms of dominant conditional entropy and the dominant matrix.

4. Experimental evaluations

To verify the efficiency and reliability of the incremental algorithm (further referred to as Algorithm 2) and its superiority of non-incremental one (called Algorithm 1), we perform some experiments using four dominant datasets from the UCI. In testing, the dominant data are illustrated in Table 1. The Car dataset of Table 1 is a heterogeneous dominant dataset, while the remaining three ones are numerical dominant datasets. Furthermore, there are values in some dominant datasets

Algorithm 2: An incremental reduction method of DRST

Input: A dominance data set DIS, P_{RED_U}, new object set U', let $F = U \cup U'$.
Output: A new reduct P_{RED_F}.

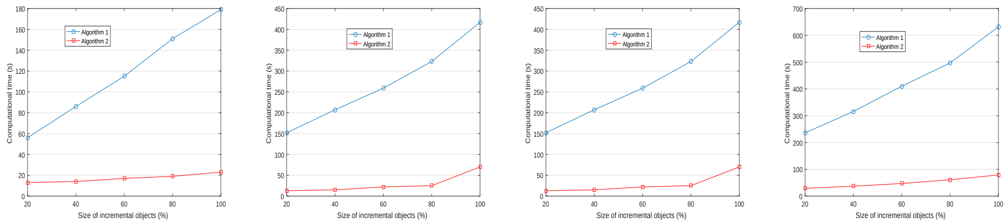
1 **begin**
2   $E \leftarrow P_{RED_U}$
3   **calculate** $[DH]^{\geq}_{D|E}(F)$, $[DH]^{\geq}_{D|C}(F)$
4   **if** $[DH]^{\geq}_{D|E}(F) = [DH]^{\geq}_{D|C}(F)$ **then**
5   **go to** 16
6   **else**
7   **go to** 9
8   **end**
9   **while** $[DH]^{\geq}_{D|E}(F) \neq [DH]^{\geq}_{D|C}(F)$ **do**
10   **for** *each* $a_k \in (C - E)$ **do**
11   **compute** $Sig^{outer}_{(F)}(a_k, B, D)$
12       $a' = max\left\{Sig^{outer}_{(F)}(a_k, B, D)\right\}$
13       $E \leftarrow (E \cup \{a'\})$
14   **end**
15   **end**
16   $P_{RED_F} \leftarrow E$;
17   **return** P_{RED_F}.
18 **end**

and some characters in others that need to be added. Hence, both algorithms cannot directly process these datasets. To ensure the experiment's validity, we must deal with them before the experiment. For some data with some missing values, we directly delete these objects. For some data with some characters, we replace characters with numbers. The codes for computing reduction are written in Java, and the codes for computing classification accuracy are written in Python. The programs are executed with a 64-bit Win10 operation system, 3.2GHz CPU AMD Ryzen 7 5800H with Radeon Graphics, and random-access memory of 16.0 GB.

TABLE 1. A overview of dominant data set

	Dominant data	samples	Condition features	Decision attributes
1	BCW	699	9	2
2	Dermatology	366	34	6
3	Spectf	267	45	2
4	Car	1728	6	4

4.1. **Comparison of execution times for Algorithms 2 and 1 when adding some objects.** The main objective is to investigate the relevance of the incremental reduction algorithm's function to the objects' sizes. Each dominant dataset in Table 1 is divided into two identical partitions. One partition is the original dominant dataset, while the second is subdivided into one, two, three, four, and five dominant data. That with 1 dominant data is described as the 1st incremental dataset, the combination of 1 dominant data and 2 dominant data is described as the 2nd incremental dataset, . . . , the combination of all five dominant data is described as the 5th incremental dataset. We compute the execution time of reduct obtained by Algorithms 1 and 2 when the 1st, 2nd, 3rd, 4th, or 5th incremental dataset is appended into the original dominant dataset, respectively. The operation times of Algorithms 1 and 2 with growing sizes of dominant data are shown in Fig. 1, where



(a) BCW (b) Dermatology (c) Spectf (d) Car

Figure 1. A comparison execution time between Algorithm 2 and Algorithm 1 when adding some objects.

the abscissa and ordinate indicate sizes of dominant datasets and execution time, respectively.

Figure 1 shows that the execution times of both algorithms grow with the increasing size of each dominant dataset, but that of Algorithm 2 increases slower than that of Algorithm 1, reaching much lower values as a result. Hence, Algorithm 2 is a more efficient method for updating the reduct of dynamic dominant datasets.

4.2. **Comparison of classification accuracies.** Numerous tests were performed in this study to compute the classification accuracy of reducts obtained by Algorithms 1 and 2, respectively. In the process of testing, each dominant dataset in Table 1 is divided into two identical partitions: basic and new dominant datasets. Then, we compute the reduction of each dominant dataset in Table 1 by Algorithms 1 and 2 when the new dominant dataset is added to the basic ones. Finally, the classification accuracy of each dominant dataset in Table 1 is calculated by 10-fold cross-validation and the Bayes criterion. The 90% and 10% of data of each dominant dataset in Table 1 are used as training and testing samples, respectively. Computed results are represented in a percentage in TableTable 2, indicating that the classification accuracy of reducts obtained by both algorithms is similar. However, the execution time of the incremental reduction approach based on dominant conditional entropy is much lower than that of the non-incremental one, making it more effective in reducing dynamic dominant datasets' calculation.

Table 2. A Comparison of Classification Accuracy.

Dominant data sets	Raw data sets	Algorithm 1	Algorithm 2
BCW	97.35	97.35	97.35
Dermatology	95.45	98.12	98.12
Spectf	92.98	96.98	96.98
Car	92.34	92.67	93.56

5. Conclusions

The evolution of various object sets and datasets requires policy-makers to modify their strategies frequently. DRST is a useful mathematical tool for describing and analyzing multi-criterial problems and has been extensively applied to feature selection and recognition. In the paper, some incremental mechanisms of dominant conditional entropy are analyzed when some objects are added to dominant datasets. Then, the fusion techniques for the dominant matrix and incremental reduction approach are developed. Finally, execution times and classification accuracies of non-incremental and incremental approaches (defined as Algorithm 1 and Algorithm 2) are computed for the four information systems from UCI, proving the latter's supremacy. The main limitation of the latter proposed approach is that it cannot directly process attribute reduction of big dominant data. For this purpose, a parallel incremental reduction method will be developed in the follow-up study.

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