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AIR QUALITY ASSESSMENT METHOD BASED ON NORMAL CLOUD MODEL AND ITS APPLICATION

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Abstract. In this paper, a normal cloud-based air quality assessment method is constructed. The method uses the characteristics that normal cloud can process fuzzy and random information to represent and characterize the various uncertainty indexes in qualitaitve evaluation. Firstly, six kinds of air pollutants include sulfur dioxide (SO_2), nitrogen dioxide (NO_2), carbon monoxide(CO), ozone(O_3), $PM_{2.5}$, and PM_{10} are selected as evaluation indexes, and the evaluation grades of these six kinds of evaluation indexes are obtained according to the Ambient Air Quality Standards (GB 3095-2012), which are converted into the standard cloud concept; Secondly, the air pollutant data of Shijiazhuang in the past five years published by the China environmental monitoring station is converted into the normal cloud concept through the dynamic incremental backward cloud transformation. Finally, for each evaluation index, the standard cloud concept level with the smallest drift of its evaluation level is selected as the final evaluation level. The air quality assessment method mainly based on the normal cloud exporession model comprehensively considers the average situation and dispersion of data, which produces a brand-new perspective for the air quality assessment of Shijiazhuang city, China.

1. Introduction

In recent years, the environmental protection concept has been deeply rooted in people's hearts, and people are paying more and more attention to adhering to the harmonious coexistence between humans and nature, constantly practicing the concept that green mountains and clear waters are invaluable assets, and strengthening environmental governance. Air governance is an important component of environmental governance and is also highly concerned by people. Air quality evaluation, as a means of evaluating the effectiveness of air governance, has become an important segment in air governance. Meanwhile, with the advancement of global industrialization, air quality has become one of the major issues of global concern. The issue of air quality assessment has become one of the hot research areas in recent years, and been conducted in-depth research from different perspectives using different methods [7, 9, 14]. Air quality assessment is an important and complex task that not only affects the natural environment in which people live, but also affects their daily production and life. The air quality evaluation system is to some extent complex, and during the evaluation process, there may be problems such as variable influencing factors, randomness of detection data, fuzziness of evaluation grading

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standards, and the time domain uncertainty, logical inconsistency, data incompleteness, mixed uncertainty data with grey and rough uncertainty features, and so on. The evaluation results will also be affected by features such as randomness, fuzziness, grayness and rough uncertainty of the data. It is inevitable to be influenced by subjectivity. Thus, the main objective of this article is to use the advantages of using cloud model to represent uncertain information, especially randomness and fuzziness, as well as the advantages of backward cloud transformation to convert quantitative data into qualitative cloud concepts. Meanwhile, based on the good performance of cloud model in qualitative evaluation, the method of a normal cloud model-based air quality evaluation is constructed. Furthermore, we apply the constructed evaluation method to the air quality evaluation of Shijiazhuang City, Hebei Province, China, in order to provide more scientific guidance for air governance. Thus, research on the normal cloud method for air quality evaluation is of great significance and practical application value.

The main structure of this article is as follows. Firstly, the current status and progress of research on air quality assessment are introduced, as detailed in Section 2. Secondly, the basic concept of normal cloud model and the implementation process of dynamic incremental backward cloud transformation method are mainly introduced, as shown in Section 3. Thirdly, it mainly introduces the construction of an air quality evaluation procedure based on normal cloud, including: air quality evaluation index and reference value of evaluation grade; determination of standard cloud concept in air quality assessment; evaluation grade determination based on dynamic incremental backward cloud transformation and cloud concept drift measurement, as shown in Section 4. Fourthly, the air pollutant quality concentration data of Shijiazhuang City from January 2019 to September 2023 are used to evaluate and analyze according to the evaluation indicators include sulfur dioxide (SO_2), nitrogen dioxide (NO_2), carbon monoxide(CO), ozone(O_3), $PM_{2.5}$ and PM_{10}, as shown in Section 5. Finally, summarize the full text in Section 6.

2. State of the Art

Due to the uncertainty of evaluation indicators, many authors have conducted specific research on air quality assessment from different perspectives, such as using deep graph learning [10], set pair analysis model [20], fuzzy comprehensive evaluation model [4, 15], monitoring network [13], Entropy weight method [19] and other methods to evaluate and analyze air quality from different angles. Raheja et al. proposed a two-stage air quality assessment model by calculating the scores of each influencing factor, and thus classifying the air quality index is based on different levels of pollution by introducing a fuzzy inference system [16]. This model considers the fuzziness of the evaluation index in the evaluation, so the evaluation results have a certain reliability. The applicability of the Raheja's model was demonstrated by comparing the obtained air quality index with the air quality indicators proposed by Indian government agencies, which provides a direction for constructing air quality evaluation methods. Thereafter, Luo et al. conducted a related study on the influence of different quantities of air quality monitoring stations on the final air quality [13]. The Luo's research results showed that one of the major uncertainties in air quality assessment is the constantly changing number of monitoring stations. Meanwhile, this also indicates that in air quality assessment, the number of detection stations is also an important influencing factor on the final evaluation results. In addition, authors such as Coelho et al. conducted an exploratory analysis of air quality impacts based on three high-priority scenarios using coupled model Intercomparison project phase 6 (CMIP6): SSP2-4.5, SSP3-7.0, and SSP5-8.5 in the IPCC's sixth evaluation report [3]. They found that in similar schemes with similarities (and/or differences), it is impossible to build a obvious trend of bigger/smaller severity between representative concentration pathway (RCP) and the Shared Socio-Economic Pathway (SSP), depending on climate change and its spatial and temporal location. There are also scholars such as Rabadi et al. selected the concentration of sulfur dioxide (SO_2), nitrogen dioxide (NO_2), carbon monoxide(CO), ozone(O_3), $PM_{2.5}$ and PM_{10} as the evaluation index to evaluate the air quality of the industrial area and the surrounding environment [1]. Kaya et al. evaluated the air quality of Sivas province with $PM_{2.5}$ as the main research index, and analyzed the pollution period using the sand dust regional atmospheric model [8]. Wang et al. conducted a comprehensive analysis on the key themes, trends, and driving factors of air quality detection in mobile environments [17], and proposed that in the future, air quality information can be explored from unconventional data such as images and natural language processing to better grasp air quality data and improve air quality detection.

Because the air environment is influenced by various factors, the system is complex and the influencing factors restrict each other, and there exists a lot of uncertainties in air quality evaluation. These methods have achieved certain results in air quality evaluation, but the comprehensive impact of randomness and fuzziness of evaluation indicators is not fully considered in the evaluation process. The normal cloud model can effectively uncertain information and reflect the randomness and fuzziness contained therein [2, 18, 24], thereby enabling the uncertainty conversion between qualitative and quantitative. It has been widely used in various important fields, such as qualitative evaluation and uncertainty decision-making by scholars [5, 6, 11, 12]. The backward cloud transformation algorithm can realize the conceptual leap, and can convert a large amount of data into cloud concepts representing these data, which has something in common with the air assessment process. Therefore, this article utilizes the advantage of using the normal cloud model to represent uncertain information in assessment information, constructs an evaluation model for air quality assessment, and applies the proposed model to evaluate the air quality problems of Shijiazhuang City, Hebei Province in recent years. The study considers both fuzziness and randomness in traditional air quality assessment, which helps to find loopholes in air quality management and strive to seek a more scientific and reasonable air quality assessment.

3. Normal cloud model and backward cloud transformation

In cloud model, the uncertain information is characterized by the parameters (Ex, En, He) [21, 23, 26], in which Ex reflects the overall expected value of the uncertain object, and it is therefore called the expectation,while En and He are used to measure the uncertainty indicators of the research object, and He further reflects the uncertainty of En, hence En is called entropy and He is called hyper-entropy. Normally, the three parameters (Ex, En, He) are used to represent an uncertainty concept as a whole from conceptual cognitive perspective.

Definition 3.1 ([23]). Assuming C represents a qualitative concept on the quantitative universe U. If $x \in U$ is a randomized representation of the uncertainty concept C, and the certainty $\mu(x)$ $(0 \le \mu(x) \le 1)$ of each object x for the concept C is a random number with stable tendency, in that way each x is named a cloud drop and the distribution of x on the universe U is named cloud.

As definition 3.1 indicates, when the given probability distribution is different, different cloud models are formed. Because of the universality of the normal distribution in uncertain phenomena, the normal cloud based on the normal distribution has been widely used. The normal cloud represented by three parameters (Ex, En, He) is defined as follows.

Definition 3.2 ([21,23]). Assuming C be a qualitative concept (Ex, En, He). If the x in U is a random realization of the (Ex, En, He), namely, x satisfies: x is a normal random number with Ex as the expectation and $|y|$ as the standard deviation, wherein y is also a normal random number with En as the expectation and He as the standard deviation. Furthermore, the uncertainty degree $\mu(x)$ $(0 \le \mu(x) \le 1)$ of x which respect to C is a random number with stable tendency, *i.e.*, $\mu(x) = \mathrm{e}^{-\frac{(x-Ex)^2}{2y^2}}$, then the distribution of the cloud drop x is called normal cloud.

Backward cloud transformation (BCT) can implement the conversion from quantitative data to qualitative concept, and can transform the index data in air quality assessment into the corresponding normal cloud concept, which is the key to the application of cloud model in various fields. At present, many scholars have conducted relevant research on backward cloud transformation from different angles, among which the dynamic incremental backward cloud transformation method algorithm (denoted as DIBCT(S^2)) proposed by Xu et al. has better effect [21], see algorithm 1 for details.

Algorithm 1. DIBCT(S^2) [21]

Input: Quantitative values X_i, where, $i = 1, \ldots, N$;
Output: Estimated value $(\hat{E}x, \hat{E}n, \hat{H}e)$ of $X_1, X_2, \ldots, X_N$;

Step 1: Calculate $\bar{X} = \frac{1}{N}\sum\limits_{i=1}^{N} X_i$ and $S^2 = \frac{1}{N-1}\sum\limits_{i=1}^{N}(X_i - \bar{X})^2$, then $\hat{E}x = \bar{X}$, $\hat{E}n = \sqrt{\frac{\pi}{2}} \times \frac{1}{N}\sum\limits_{i=1}^{N}\left|X_i - \hat{E}x\right|$

Step 2: Let $G^2 = \frac{1}{N-1}\sum\limits_{i=1}^{N}(X_i - \hat{E}x)^2$.

Step3: If $G^2 - \hat{E}n^2 \ge 0$, then execute the Step5, otherwise execute the Step4.

Step4: Generate $e \times M$ $(0 \le e \le 1)$ new normal random samples with the expectation $\hat{E}x$ and the variance $\lambda \times \hat{E}n(\lambda \ge 1)$, and fuse them with the original samples as the final sample data. At this time, the sample size N is revised to $N + e \times M$, and then turn to Step2.

Step5: $S^2 = G^2$, $\hat{H}e = \sqrt{S^2 - \hat{E}n^2}$.

4. Air quality assessment method based on normal cloud

Air quality assessment can reflect the air quality of a certain area at a certain time. It is an important and complex work, which is not only related to the natural environment where people live, but also affects people's normal daily life and production. According to the Technical Regulation for Ambient Air Quality Assessment (on Trial) (HJ663-2013) (denoted as TR-AAQA-oT(HJ663-2013)), the air quality rating system mainly uses the air pollutant index to assess the ambient air quality. The evaluation period is divided into hourly evaluation, daily evaluation and annual evaluation. The hourly evaluation examines the average hourly concentrations of sulfur dioxide (SO_2), nitrogen dioxide (NO_2), carbon monoxide(CO), and ozone(O_3). The daily evaluation investigated the average concentration of SO_2, NO_2, CO, O_3, $PM_{2.5}$ and PM_{10} in 24 hours and the sliding average of the maximum concentration of O_3 in 8 hours. There are two kinds of annual evaluation criteria: annual average evaluation and quantile evaluation. In the paper, the annual average value of SO_2, NO_2, $PM_{2.5}$ and PM_{10} pollutants and the quantile value of CO and O_3 are used to evaluate.

4.1. **Air quality assessment method based on normal cloud.**

The normal cloud, which studies the uncertainty of randomness and fuzziness [25], is used in air quality evaluation. First, the air quality evaluation standard cloud is constructed, that is, the evaluation level generates the concept of standard cloud; Then, the normal cloud concept of each index is generated from the air quality data by using the backward cloud transformation, and the similarity between the normal cloud concept of the index and the standard cloud concept of each grade is calculated [25]. Finally, the evaluation grade is determined according to the similarity. The following is the steps of the evaluation method constructed in this paper:

Step1: determine the air quality assessment indicators and methods based on the regional characteristics and the specifications of the TR-AAQA-oT(HJ663-2013).

Step2: determine the standard set of air quality evaluation indexes and the index value range of each evaluation level, and generate the standard cloud concept of each evaluation index level.

Step3: use dynamic incremental backward cloud transformation to generate the corresponding normal cloud concept from the collected air quality data of each indicator.

Step4: calculate the drift degree between the normal cloud concept of the indicator data obtained in Step3 and the corresponding standard cloud concept of each level of the indicator, find the standard cloud with the maximum similarity corresponding to it, and take the evaluation level it belongs to as the air quality level of the indicator.

The evaluation system is mainly to convert each index data and index evaluation criteria into cloud concepts, and determine the evaluation level by calculating the similarity between cloud concepts. It should be noted that this step should assume that the air quality data used therein belongs to the index evaluation level, and the distribution of the certainty is normal.

4.2. **Air quality evaluation index and reference value of evaluation grade.** In the paper, the assessment is carried out on a yearly basis. According to the assessment index specification formulated in TR-AAQA-oT(HJ663-2013), the mass concentrations of six air pollutants(sulfur dioxide, nitrogen dioxide , carbon monoxide, ozone, $PM_{2.5}$ and PM_{10}) are selected. Moreover, we take the 90th percentile of the maximum 8-hour daily moving average and the 95th percentile of the 24-hour average.. According to the basic concentration limits of ambient air pollutants specified in Ambient Air Quality Standards (GB 3095-2012), the above six pollutants can be divided into two levels. In this paper, in order to further accurately assess the air quality and improve the readability of the assessment results, the air quality is classified into five evaluation standards: excellent, good, mild-pollution, moderate-pollution and severe-pollution. Combined with literature [27], the reference values of the six air quality evaluation indexes $\{\mathrm{SO_2}, \mathrm{NO_2}, \mathrm{CO}, \mathrm{O_3}, \mathrm{PM_{2.5}}, \mathrm{PM_{10}}\}$ under the rating grade $\mathrm{D} = \{\mathrm{excellentI}, \mathrm{goodII}, \mathrm{lightpollutionIII}, \mathrm{moderatepollutionIV}, \mathrm{heavypollutionV}\}$ are shown in Table 1.

TABLE 1. Classification standards for ambient air quality assessment

grade d_j / index v_i	excellent(I)	good(II)	lightpollution(III)	moderatepollution(IV)	heavypollution(V)
SO_2	20	60	100	140	180
NO_2	20	40	60	80	100
CO	2	4	6	8	10
O_3	100	160	220	280	340
$PM_{2.5}$	15	35	75	120	165
PM_{10}	40	70	140	210	320

Notes: except that the mass concentration of CO is mg/m^3, the unit of other evaluation indexes is ug/m^3.

4.3. **Determination of standard cloud concept in air quality assessment.** According to the criterion and the reference values of each level given above, the standard cloud concept of the evaluation level can be determined. If the evaluation index v_i is $(a_{ij}, b_{ij}]$ in the range of evaluation grade d_j, then the digital feature of the evaluation index v_i in the standard cloud concept $C_{ij} = (Ex_{ij}, En_{ij}, He_{ij})$ of evaluation grade d_j is

$$Ex_{ij} = \frac{(a_{ij} + b_{ij})}{2}, En_{ij} = \frac{(a_{ij} - b_{ij})}{6}, (i = 1, 2, \dots, 6, j = 1, 2, \dots, 5). \tag{4.1}$$

Among them, $v_i(i = 1, 2, \dots, 6)$ represents six evaluation indicators (i.e. v_1 to v_6 represent $\mathrm{SO_2}, \mathrm{NO_2}, \mathrm{CO}, \mathrm{O_3}, \mathrm{PM_{2.5}}$ and $\mathrm{PM_{10}}$, respectively) and d_j $(j = 1, 2, \dots, 5)$ represents five evaluation levels (i.e. d_1 to d_5 represent excellent, good, mild pollution, moderate pollution and heavy pollution, respectively). Combining with literature [27], $He_{3j} = 0.01$ for CO in the standard cloud concept C_{3j}, while $He_{ij} = 0.1$ for other evaluation indexes in the standard cloud concept, so the standard cloud concept of air quality evaluation is obtained as shown in Table 2.

4.4. **Evaluation grade determination based on dynamic incremental backward cloud transformation and cloud concept drift measurement.**

TABLE 2. The standard cloud concepts of ambient air quality assessment level

grade d_j / index v_i	excellent(I)	good(II)	lightpollution(III)	moderatepollution(IV)	heavypollution(V)
SO_2	(10,3.33,0.1)	(40,6.67,0.1)	(80,6.67,0.1)	(120,6.67,0.1)	(160,6.67,0.1)
NO_2	(10,3.33,0.1)	(30,3.33,0.1)	(50,3.33,0.1)	(70,3.33,0.1)	(90,3.33,0.1)
CO	(1,0.33,0.01)	(3,0.33,0.01)	(5,0.33,0.01)	(7,0.33,0.01)	(9,0.33,0.01)
O_3	(50,16.67,0.1)	(130,10,0.1)	(190,10,0.1)	(250,10,0.1)	(310,10,0.1)
$PM_{2.5}$	(7.5,2.5,0.1)	(25,3.33,0.1)	(55,6.67,0.1)	(97.5,7.5,0.1)	(142.5,7.5,0.1)
PM_{10}	(20,6.67,0.1)	(55,5,0.1)	(105,11.67,0.1)	(180,11.67,0.1)	(265,18.33,0.1)

Firstly, the air evaluation index data is converted into cloud concept by the backward cloud transformation algorithm DIBCT(S^2), and the evaluation index cloud concept C_{ik} of each evaluation area k is obtained.

Secondly, using the drift measure of KL divergence given in literature [22], the drift degrees of the evaluation index cloud concept C_{ik} and the standard cloud concept C_{ij} of each evaluation level are calculated respectively, i.e.,

$$D(C_{ij} \parallel C_{ik}) = \frac{1}{2}\left[(Ex_{ij} - Ex_{ik})^2 + \left(\sigma_{ij}^2 + \sigma_{ik}^2\right)\right]\left(\frac{1}{\sigma_{ij}^2} + \frac{1}{\sigma_{ik}^2}\right) - 2. \tag{4.2}$$

Among them, $\sigma_{ij} = En_{ij} + 3He_{ij}, \sigma_{ik} = En_{ik} + 3He_{ik}$, the smaller the drift degree $D(C_{ij} \parallel C_{ik})$, the more similar the cloud concept.

Finally, take the minimum value according to the drift degree

$$\min_j D(C_{ij} \parallel C_{ik}). \tag{4.3}$$

The evaluation grade d_j is determined as the evaluation grade of the evaluation index v_i in k region.

5. Application of air quality assessment method–Taking Shijiazhuang city as an example

5.1. An explanation of data sources and assessment index.

Data for this study came from Ministry of Ecology and Environment of the People's Republic of China [1]. We sort out the daily air pollutant mass concentration data of Shijiazhuang city from January 2019 to September 2023, with a total of 1732 relevant data points.

The specific index system used for air quality assessment here include SO_2, NO_2, CO, O_3, $PM_{2.5}$ and PM_{10}. Among them, the daily collected data are used for SO_2, NO_2, CO and O_3. Due to the significant impact of seasons on $PM_{2.5}$ and PM_{10}, especially in winter, the mass concentrations of $PM_{2.5}$ and PM_{10} are the highest. Therefore, the monthly collected data are used for $PM_{2.5}$ and PM_{10}.

5.2. Assessment and analysis of air quality in Shijiazhuang city in recent five years.

According to the proposed assessment method in Section 4, the following is an

[1] https://www.mee.gov.cn/

assessment and analysis of the air quality situation in Shijiazhuang City in recent five years (from January 2019 to September 2023).

(1) The specific data of assessment index is transformed into the corresponding cloud concepts. The data of each index from January 2019 to September 2023 will be converted to the corresponding assessment index cloud concepts through the backward cloud transformation algorithm DIBCT(S^2), respectively, as shown in Table 3.

TABLE 3. The assessment index cloud concepts in Shijiazhuang city

year \ index	$PM_{2.5}$	PM_{10}	SO_2
2019	(63.13,48.97,26.92)	(118.21,74.38,29.88)	(16.33,8.95,4.32)
2020	(58.23,41.65,18.62)	(101.43,62.43,17.46)	(12.4,5.7,1.59)
2021	(45.67,33.68,13.15)	(84.46,56.55,3.05)	(9.13,5.1,1.73)
2022	(45.58,31.37,7.95)	(82.32,45.86,1.57)	(8.16,3.72,0.08)
2023	(43.23,31.2,14.86)	(94.46,66.7,75.14)	(6.8,4,0.76)
	NO_2	CO	O_3
2019	(46.07,21.34,2.71)	(1.02,0.6,0.38)	(106.65,72.14,0.6)
2020	(41.02,19.79,2.15)	(0.93,0.55,0.29)	(99.94,59.41,1.51)
2021	(31.6,15.14,0.46)	(0.73,0.31,0.12)	(101.35,52.99,0.52)
2022	(32.71,15.16,0.31)	(0.7,0.28,0.09)	(105.44,59.53,5.96)
2023	(28.43,14.65,3.12)	(0.66,0.27,0.11)	(123.53,56.97,0.6)

(2) According to the metric $D(C_{ij} \parallel C_{ik})$, calculate the drift degree between the assessment index cloud concept and the standard cloud concept for each index at each level, separately. And then select the grade of the standard cloud concept with the lowest drift degree as the final assessment grade of each index based on the $\min_j D(C_{ij} \parallel C_{ik})$, as shown in Table 4.

TABLE 4. The final air quality assessment results of Shijiazhuang city

year \ index	$PM_{2.5}$	PM_{10}	SO_2	NO_2	CO	O_3
2019	moderatepollution(IV)	lightpollution(III)	good(II)	lightpollution(III)	excellent(I)	excellent(I)
2020	lightpollution(III)	lightpollution(III)	excellent(I)	lightpollution(III)	excellent(I)	excellent(I)
2021	lightpollution(III)	lightpollution(III)	excellent(I)	good(II)	excellent(I)	excellent(I)
2022	lightpollution(III)	lightpollution(III)	excellent(I)	good(II)	excellent(I)	excellent(I)
2023	lightpollution(III)	lightpollution(III)	excellent(I)	good(II)	excellent(I)	excellent(I)

The results of the Table 4 show that the overall form of air quality in Shijiazhuang in recent five years is still good, especially NO_2, CO, O_3, which is excellent. Combined with Table 3, the changes of the expected values of $PM_{2.5}$ and PM_{10} show that the performance has gradually improved in recent five years. According to the ecological environment bulletin of Hebei Province in the past five years, it was found that the air quality in Shijiazhuang City, Hebei Province has been stable and improving in the past five years. The actual performance of various pollutants is roughly consistent with the evaluation in this article, indicating that the evaluation of air quality in this article is reliable and effective.

6. Summary

In this paper, the air quality of Shijiazhuang in Hebei Province was evaluated and analyzed based on normal cloud and dynamic incremental backward cloud transformation method. The concept of evaluation index and rating level standard cloud is established, and the evaluation cloud concept of each evaluation index is generated by dynamic incremental backward cloud transformation method. Finally, the drift degree between each evaluation cloud concept and the corresponding standard cloud concept of each level is calculated to determine its evaluation level. The annual average concentration of pollutants SO_2, NO_2, CO in Shijiazhuang have reached the national secondary standard, and the treatment of $PM_{2.5}$ and PM_{10} need to be strengthened. However, there are still shortcomings and areas for further optimization in this article: the research and superiority of the inverse cloud transformation method are mainly reflected in the case of a large ratio of entropy (En) and hyper-entropy (He). Therefore, further exploration is needed to improve the evaluation of air quality in Shijiazhuang when the ratio of entropy and hyper entropy in pollutant data is large. In addition, for one thing, the cloud model is a model that expresses uncertain knowledge at a coarse-grained level. For the decision and the evaluation of large-scale data, rapid decision and evaluation at the appropriate coarse-grained level can not only deliver the efficiency of decision making, but also simplify the decision making process. Therefore, the study of large-scale data uncertainty decision and evaluation model based on normal cloud is a topic worth studying in the future. For another thing, the researches on high-dimensional normal cloud, including high-dimensional normal cloud transformation, uncertainty evaluation and decision based on high-dimensional normal cloud and intelligent classifier based on high-dimensional normal cloud, are worth studying. The above areas will be the main directions worth exploring in the future.

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