Journal of Nonlinear and Convex Analysis
Volume 26, Number 6, 2025, 1645–1652

Yokohama Publishers
ISSN 1880-5221 ONLINE JOURNAL


# DISTRIBUTED SOURCE CODING WITH LDPC FOR QUANTUM COMMUNICATION IN METROPOLITAN AREAS

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Abstract. This study aimed to propose an improved scheme of DSC considering the shortcomings of distributed source coding (DSC) in quantum communication in metropolitan areas, such as high bit error rates and low compression ratios. First, the redundancy technology was adopted in the source encoder design to provide error protection, and then the joint design of low-density parity-check code with redundancy was used to conduct DSC. The experimental results demonstrated that the proposed method achieved a high compression rate and could be easily implemented when a strong internal correlation of the quantum communication sources existed in metropolitan areas.

## 1. Introduction

Quantum science is a novel frontier discipline formed by the cross integration of physical science, information science, and other disciplines, mainly including "quantum computing" [9], "quantum communication" [6], and "quantum precision measurement" [10]. It holds significant scientific value and offers broad application prospects. Quantum communication is complex. No ideal breakthroughs or solutions have been achieved yet for a series of key technical problems such as the single-photon quantum communication model. Hence, quantum communication needs to be improved to achieve better compression performance. In 2017, a satellite-based satellite-to-ground quantum communication experiment was realized. Pan's research group successively completed the satellite-to-ground quantum key distribution experiment, earth-to-satellite quantum teleportation experiment, and satellite-based thousand-kilometer entanglement distribution experiment. Further, in 2018, Pan's research group completed an encrypted video call between Beijing and Vienna, initially showing the value of satellite-to-ground quantum communication. International attention has been paid to the quantum communication satellite-to-ground network, with many countries conducting research on quantum communication satellites in metropolitan areas. Research teams from the United Kingdom, Singapore, Italy, Germany, the Netherlands, Switzerland, Austria, and other countries are planning to develop a 6U quantum communication cube satellite payload jointly using the International Space Station to achieve satellite-to-ground quantum communication [5]. In other words, although the transmission efficiency of satellite-to-ground quantum communication is high, it does not apply to quantum

2020 *Mathematics Subject Classification.* 94A29, 94A40.

*Key words and phrases.* Distributed source coding, low-density parity check, metropolitan area, quantum communication.

The author was partially supported by the Yunnan Fundamental Research Projects under Grant 202401AT070450 and the Science Foundation of Yunnan Education Department under Grant 2024J1141.

communication in metropolitan areas. Therefore, further exploring the distributed source coding (DSC) technology for quantum communication in metropolitan areas may hold great practical significance for improving transmission efficiency.

DSC is usually used in quantum communication in metropolitan areas, image coding, audio coding, video coding, and other relevant fields. Quantum communication sources in metropolitan areas can be employed for robust and secure information communication in a noisy and eavesdropping channel. In Joyner's wiretap channel theory, the effect of eavesdropping and noise is treated as a wiretap channel. According to Joyner's theory, the secrecy capacity is the capacity difference between the main and wiretap channels [5]. If the secrecy capacity is nonzero, a classical coding scheme exists that enables the reliable and secure transmission of information at a rate lower than the secrecy capacity. This study aimed to present a DSC method based on side information, designed to address the problem of data compression in quantum communication within metropolitan areas.

Recently, Griod and Aaron from Stanford University proposed a pixel-based DSC scheme, a DCT-based coding scheme, and a decoder that uses hash codes to assist with decoding in the DCT domain [1, 2]. However, the bit error rate of their algorithm was high. Xiong [7] proposed a layered Wyner-Ziv coding to extract basic information from the basic layer of traditional coding and higher-resolution information and details from the enhancement layer for distributed coding and decoding low-density parity-check (LDPC) encoder. However, the error rate of this algorithm was also high. Aljohani [3] proposed a DSC algorithm based on wavelet transform, which could improve the robustness but was highly complex.

## 2. Quantum communication sourecs in metropolitan areas

Quantum communication is the first practical application in the field of quantum information. The quantum random access code is a communication scheme that can represent the superiority of quantum. The three-party quantum random access codes in cascaded mode involve only one sender and two receivers. When the measured intensity of the first receiver is weighed, the probability of success for both receivers in a sample is greater than three fourths of the classical bound. This study extended the original protocol and introduced a theoretical model of $3 \to 1$ cascading quantum random access codes involving a preparation-transform-measurement scenario, which included one sender and three receivers.

Quantum communication is a new way of communication that combines quantum mechanics with information theory. It is one of the main applications of quantum information science. According to the uncertainty principle in quantum mechanics and quantum non-cloning theorem, quantum communication can guarantee absolute security. Quantum communication is a communication mode that realizes information transmission between two places by transmitting quantum states. It includes quantum key distribution, quantum teleportation, and other aspects. Quantum key distribution refers to the transmission of quantum states by the information-sending end as well as the sharing of a key string between the information-sending end and the information-receiving end through the quantum channel. Currently, many domestic teams have successfully implemented the practical application of quantum key distribution. Quantum teleportation is usually interpreted as transmitting the information carried by an unknown quantum state from ground A to ground B by consuming quantum channel resources and combining them with joint measurement. During quantum teleportation, the original particles do not move physically, but the state changes. Quantum teleportation not only ensures complete security in information transmission but also promotes the development of quantum technology. Quantum communication can address information security challenges across various industries. Nowadays, quantum communication has received immense attention due to a prominent increase in information security challenges. Quantum communication, as a new method of information transmission based on the quantum entanglement effect, is characterized by high efficiency and absolute security. It has evolved through the research of international quantum physics and information science.

The quantum communication scheme based on the free space channel was analyzed. The security key rate and transmission distance in metropolitan areas were calculated through the performance simulation of the protocol. A microwave modulator was used to modulate the key information into the quantum state, and the signal was transmitted through the antenna. The signal reached the receiver after being transmitted through the loss thermal noise channel. The input($x$)−−output($y$) relationship of the channel is shown in equation (2.1).

$$y = \sqrt{\eta_c}x + z \tag{2.1}$$

where $\eta_c$ is the channel transitivity and $z$ is the channel noise correlation term.

## 3. DSC with LDPC

DSC is a relatively new data compression technology developed in recent years. It is used to encode each source separately at the encoding end and to decode jointly by using the correlation between the source sequences at the decoding end. This is equivalent to transferring part of the task at the coding end to the understanding of the code end, greatly reducing the computation of the source coding algorithm. In addition, DSC has a certain degree of error resilience, indicating that DSC has potential for application.

The direction classification was carried out for the spatial correlation of each pixel, and the corresponding row-based linear filter was designed based on the classification to explore the spatial correlation of images in the same perspective. In the DSC with LDPC code and binary symmetric channel, each line of the multi-view image was encoded independently and the coded and compressed code stream was transmitted to the same decoder for joint decoding.

At the encoding end, each row of data used a DSC scheme. First, the maximum and minimum grayscale values were calculated for each pixel, with the minimum value directly transmitted as a meaningless bit plane to the decoding end. The meaningful bit plane, required for the row data, was formed by subtracting the minimum value from the grayscale values of all pixels. The pixel grayscale value data were generated from the meaningful bit plane as the processed data. DSC was not complex for prediction, including transformation and quantization at the encoding end. The encoding process only involved sampling each row of data to achieve a meaningful bit plane. This approach reduced complexity, especially given the two independent transmission channels, where the channel noise was modeled as Gaussian white noise with a mean of 0 and a variance of 1.

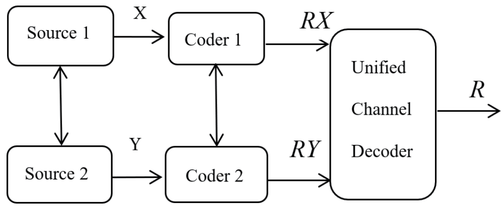



FIGURE 1. Distributed source coding and decoding.

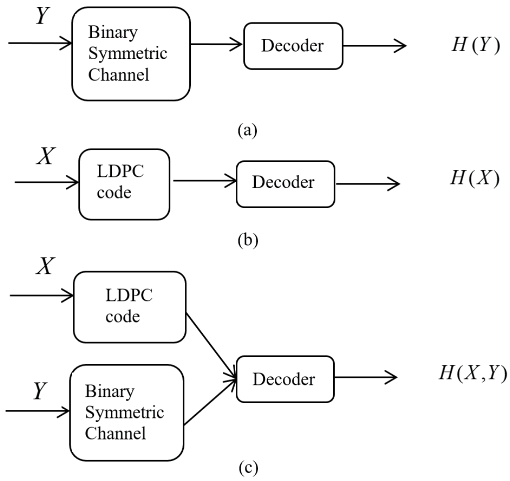



FIGURE 2. Three lossless encoding and decoding schemes for $X$ and $Y$.

Suppose $X$ and $Y$ both are discrete memory-less sources, and a correlation exists between $X$ and $Y$. The $H(X)$ and $H(Y)$ represent the information entropy of $X$ and $Y$, respectively; $H(X, Y)$ represents the joint entropy of $X$ and $Y$; and $RX$ and $RY$ represent the coding rate of $X$ and $Y$, respectively. Usually, three types of schemes exist for encoding and decoding $X$ and $Y$, as shown in Figs. 1 and 2. The first solution was encoded and decoded using $X$ and $Y$, respectively, as shown in Figure 2a. At this point, the total encoding rate was $R = RX + RY$, and $R \geq H(X) + H(Y)$ was satisfied. The second solution was to jointly encode and decode $X$ and $Y$, as shown in Figure 2b, with the total bit rate $R \geq H(X, Y)$. The last solution was the DSC scheme. The coder was encoded using $X$ and $Y$ separately, and the decoder was decoded using $X$ and $Y$ jointly, as shown in Figure 2c.

The theorem of Slepian and Wolf [8] proved that, under the conditions of equations (3.1)-(3.3), $X$ and $Y$ were encoded in $RX$ and $RY$ at the coding end, and the joint decoding was performed at the decoding end. The coding performance was equivalent to the joint coding and decoding of $X$ and $Y$. Compared with the findings in Figure 2b, the DSC did not make any losses in the encoding performance.

$$RX \geq H(X|Y) \tag{3.1}$$

$$RY \geq H(Y|X) \tag{3.2}$$

$$RX + RY \geq H(X|Y) \tag{3.3}$$

where $H(X|Y)$ and $H(Y|X)$ are conditional entropies, and $H(X, Y)$ is the joint entropy.

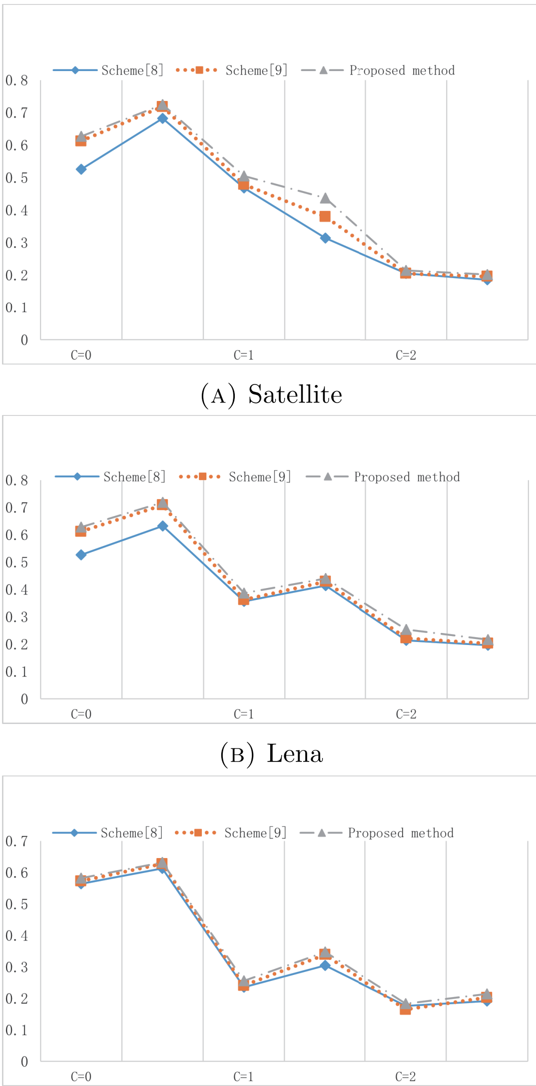
## 4. Simulations

In our simulations, the LDPC code and the check matrices were described as $R = 0.5$, $R = 0.75$, and $R = 0.875$. The experimental image was gray ($512 \times 512$), and each plane embedded 262,144 bits, which was regarded as the source sequence $X$. $Y$ was obtained from the binary symmetric channel to simulate the correlation of the source. The experiment tested the bit error rate and the compression ratio of previous studies [3, 7] (referred to as scheme 7 and scheme 3, respectively). The information stream of the DSC included the code stream, which was obtained after the arithmetic coding of $X$ and the adjoins corresponding to $Y$. For example, when the LDPC code rate was $R = 0.75$, the 262,144 bits were sampled from $X$, and the remaining 235,930 bits were obtained after arithmetic coding (code length was $S$). At the same time, the adjoining formula of 262,144 bits (total length was $L$) was employed with the code rate, and the total code rate was $(L + S)/262144$.

Table 1. Comparison of bit error rate.

| | R | Scheme 7 | Scheme 3 | Proposed method |
|---|---|---|---|---|
| Satellite | 1/2 | 0.0012 | 0.0015 | 0.0011 |
| | 3/4 | 0.0631 | 0.0721 | 0.0536 |
| | 7/8 | 0.1301 | 0.1406 | 0.1287 |
| | R | Scheme 7 | Scheme 3 | Proposed method |
| Lena | 1/2 | 0.0017 | 0.0021 | 0.0014 |
| | 3/4 | 0.0735 | 0.0814 | 0.0728 |
| | 7/8 | 0.1271 | 0.1324 | 0.1203 |
| | R | Scheme 7 | Scheme 3 | Proposed method |
| Men | 1/2 | 0.0019 | 0.0022 | 0.0016 |
| | 3/4 | 0.0735 | 0.0814 | 0.0728 |
| | 7/8 | 0.1271 | 0.1324 | 0.1227 |

The highest bit plane of the standard image Lena was taken as the source. Table 1 shows the bit error rate of schemes 7 and 3 and that of this study. The error codes in this study were obtained from the reconstruction of $X$; therefore, the bit error rate was calculated using the number of error bits between $X$ and $Y$. When the source correlation was strong, the performance of schemes 7 and 3 was better. When the bit rate of LDPC was adjusted to 0.75 and 0.875, the performance of this study was reduced with $R = 0.5$. When the bit error rate was good, schemes 7 and 3 were high requirements for the correlation between sources. When the correlation between sources was not strong, the bit error rate was relatively high in schemes 7 and 3. When the correlation was weak, the bit error rate significantly increased. When the correlation of the source was weak in this study, the bit error rate was big, and the errors came into being only when the correlation decreased to close to 0.1. However, even if the correlation was weak, the bit error rate was lower than those of schemes 8 and 9.




(A) Satellite

(B) Lena

(C) Men

FIGURE 3. Compression ratio on images.

The eighth bit plane of the standard images of satellite, Lena, and men was taken as the source, and the compression ratio of the different sources was examined. The DSC with LDPC had $BER = 0.5$, and the bit error rate under the experimental conditions was 0.

Figure 3 shows that if the internal correlation of the sequence $X$ could not be used sufficiently, the bit rate in this study was approximately 0.6, which was inferior compared with that of schemes 8 and 9. However, when high-order entropy encoding was used, the bit error rate of this study was lower than that of schemes 8 and 9, and the compression ratio was not turned with the alteration of the correlations. Then, taking advantage of the internal correlation of $X$ was important. In fact, $H(X|Y)$ represented the bounds to which source $X$ could be compressed when only $Y$ was used. In this experiment, the bit error rate of scheme 8 was 0.5, and that of scheme 9 was also approximately 0.5. Only when $H(X|Y)$ was significantly less than 0.5 and the correlation between $X$ and $Y$ was strong, remarkable results were achieved. When the correlation between $X$ and $Y$ was weak and $H(X|Y)$ was close to $R = 0.5$, the bit error rate of schemes 8 and 9 was higher than that of this study.

## 5. Conclustion

The traditional DSC algorithm works efficiently based on correlations between sources, with less use of correlations within sources. The algorithm proposed in this study made use of the correlations within the source sequence of quantum communication in metropolitan areas, resulting in a lower overall bit error rate. Compared with the existing DSC scheme, the proposed DSC scheme was effective with extremely low BER and high CR when the internal correlation of the sources was stronger.

The results demonstrated the advantages of improving distributed source code. In the future, two-dimensional sampling will be considered to maximize the correlation with the source and improve the accuracy of the symbol estimation algorithm.

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*Manuscript received February 5, 2024*
*revised September 9, 2024*

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