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A STUDY OF QUINTIC B-SPLINE DIFFERENTIAL QUADRATURE METHOD FOR SOLVING GRAY-SCOTT MODEL

ANISHA AND RAJNI ROHILA

Abstract. In this article, we have applied the quintic B-spline differential quadrature approach to study numerical solutions of the Gray-Scott model. The Gray-Scott system is a reaction-diffusion process that generates self-reproducing patterns such as spots and stripes. These structural combinations are useful for analyzing the relationship between diffusion and reaction. The corresponding partial differential system has been solved using the quintic B-spline approach to improve accuracy while preserving system stability. The calculated error norm is reported in the table, and the results are illustrated graphically.

1. Introduction

Reaction-diffusion systems are used in chemistry to simulate how chemical reactions occur and substances spread through diffusion and are also used across various fields such as physics, geology, and biology to model processes such as neutron diffusion theory, electrical phenomena, and the cell cycle [29]. Reaction-diffusion systems describe the evolution of patterns in biological systems through mechanisms such as self-organization, turing instability, multi-component systems, reactions deriving from biological processes, and experimental validation [11, 26, 30, 34, 37]. These models have been used in many different biological situations and provide valuable insights into the fundamental mechanics of pattern formation.

We consider a reaction-diffusion system with Neumann and Dirichlet boundary conditions in the domain $[f, g]$.

$$\begin{cases} \frac{\partial X}{\partial t} = b_1 \frac{\partial^2 X}{\partial z^2} + g_1(X, Y), \\ \frac{\partial Y}{\partial t} = b_2 \frac{\partial^2 Y}{\partial z^2} + g_2(X, Y), \end{cases} \tag{1.1}$$

where b_1 and b_2 represent diffusion constants and X, Y are real valued functions. The Gray-Scott model is one example of a semi-linear parabolic partial differential equation that represents the reaction-diffusion system mathematically. Many researchers have worked on reaction-diffusion systems. Gray and Scott [16] introduced the Gray-Scott (GS) model in 1983. Matsuya [27] has investigated the spatial patterns of the GS model. Gizaw [15] used the finite method to solve the GS model. Korkmaz *et al*. [24] solved the GS model using sine cardinal functions and the exponential cubic B-spline collocation technique. Joshi [21] applied the Lagrange polynomial to obtain numerical solutions to the Gray-Scott model. Ali and Saleem [2] used the Chebyshev spectral method to obtain numerical solutions

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to the GS model. The discontinuous Galerkin finite element method was used to obtain numerical solutions to the GS model by Satyvir [31].

The differential quadrature method (DQM) is a numerical approach used to approximate solutions to differential equations. It discretizes differential equations by converting them into algebraic equations. The technique has been used in many engineering challenges, including structural issues, reservoir modification, and free vibration analysis [8, 10, 13]. DQM was first introduced by Bellman *et al.* [9] in 1972. Many researchers have used various types of DQMs to find weighting coefficients and solve linear and non-linear differential equations. Tamsir and Huntul [32] introduced the cubic uniform algebraic trigonometric tension B-spline and a unified extended spline to determine the weighting coefficients of Fisher's reaction-diffusion equation. Umar *et al.* [36] introduced the iterative approach to solve nonlinear equations, while Garodia and Uddin [14] utilised it to solve delay differential equations. Bashan [4, 5] presented a modified cubic B-spline technique to find numerical solutions to the coupled Korteweg-de Vries (KdV) equation and used the quartic B-spline approach to solve the Schrodinger equation. Ahmad *et al.* [1] used a local meshless method to find solutions for regularized long wave, coupled Burger's, and Klein-Gordon equations numerically. Aliyi *et al.* [3] introduced the radial basis function approach to solve the heat equation. The modified cubic B-spline technique was used by Yusuf *et al.* [35] to determine the weighting coefficients of the modified Burger's equation, and this method was also used by Bashan [6] to solve the Kawahara equation. Hashmi *et al.* [18] used the cubic B-spline technique to determine the numerical solutions to the Burger ' s equation. The hyperbolic B-Spline DQM was presented by Tamsir *et al.* [33] for solving 3D wave equations. Kaur and Joshi [22] introduced a quintic hyperbolic B-spline method to evaluate the numerical solutions of the coupled KdV equation. The cubic B-spline least squares technique was used by Dag *et al.* [12] to solve the advection-diffusion equation. The modified cubic B-spline and Lagrange interpolation methods were used by Jiwari [19] to numerically solve the hyperbolic partial differential equations. Korkmaz and Dag [25] solved the advection-diffusion equation using quartic and quintic B-spline techniques. Khatoon *et al.* [23] utilized the iterative method for generalized α-Reich–Suzuki non-expansive mappings.

In this work, we studied a reaction-diffusion system in which we solved the Gray-Scott model using the quintic B-spline differential quadrature method. The five-band Thomas algorithm has been used to determine the weighting coefficients of the differential quadrature approach. After applying this method, the given problem was converted into a system of ordinary differential equations and solved using the strong stability preserving fourth-order Runge-Kutta technique (SSP-RK43). DQM can be applied to both linear and nonlinear problems, making it versatile for various engineering and scientific applications. The rest of the paper is structured as follows. First, we describe the differential quadrature method used in this study in Section 2. In Section 3, we apply the method to the reaction-diffusion equation with boundary conditions. In Section 4, we solved the Gray-Scott model and presented the obtained solution both graphically and in tabular form. A conclusion is given in the last Section.

2. Quintic B-spline function

We consider a domain $[f,g]$ partitioned by nodes z_n such that $f = z_1 < z_2 < \cdots < z_{N-1} < z_N = g$ with the distance $d = z_n - z_{n-1}$ between consecutive nodes. Let $W(z_n,t)$ be a given smooth function over the domain at each grid. The approximation of the derivatives of $W(z_n,t)$ with respect to z at the nodes z_n is

$$\frac{\partial^{(j)} W(z_n,t)}{\partial z^{(j)}} = \sum_{m=1}^{N} C_{nm}^{(j)} W(z_m,t), \quad n = 1,2,\dots,N. \tag{2.1}$$

here, j represents the order of the derivatives, and $C_{nm}^{(j)}$ denotes the weighting coefficients of the approximation. Let $\delta_i(z)$ be the quintic B-spline function with nodes at points z_n. The quintic B-spline basis function is defined as [7, 25, 28]

$$\delta_i(z) = \frac{1}{d^5}\begin{cases} (z-z_{i-3})^5, & z \in [z_{i-3}, z_{i-2}),\\ (z-z_{i-3})^5 - 6(z-z_{i-2})^5, & z \in [z_{i-2}, z_{i-1}),\\ (z-z_{i-3})^5 - 6(z-z_{i-2})^5 + 15(z-z_{i-1})^5, & z \in [z_{i-1}, z_i),\\ (z_{i+3}-z)^5 - 6(z_{i+2}-z)^5 + 15(z_{i+1}-z)^5, & z \in [z_i, z_{i+1}),\\ (z_{i+3}-z)^5 - 6(z_{i+2}-z)^5, & z \in [z_{i+1}, z_{i+2}),\\ (z_{i+3}-z)^5, & z \in [z_{i+2}, z_{i+3}),\\ 0, & \textit{otherwise}. \end{cases} \tag{2.2}$$

here $\delta_{-1}, \delta_0, \dots, \delta_{N+2}$ form the basis for functions defined over $[f,g]$. Each quintic B-spline encloses six nodes, so a total of six quintic B-splines enclose one node. Non-zero values of $\delta_i(z)$ and the first four derivatives at the given nodes are summarized in Table 1. Substituting each quintic B-spline function into the differential quadrature method equation (2.1) for a fixed z_n gives

$$\frac{\partial^{(j)} \delta_i(z_n)}{\partial z^{(j)}} = \sum_{m=i-2}^{i+2} C_{n,m}^{(j)} \delta_i(z_m), \tag{2.3}$$

here $i = -1, 0, \dots, N+2$ and $n = 1,2,\dots,N$. Now, we can write equation (2.3) in matrix notation for any z_n in the domain $[f,g]$ as given below:

(2.4)

$$\begin{bmatrix} \delta_{-1,-3} & \delta_{-1,-2} & \delta_{-1,-1} & \delta_{-1,0} & \delta_{-1,1} & & & & & \\ & \delta_{0,-2} & \delta_{0,-1} & \delta_{0,0} & \delta_{0,1} & \delta_{0,2} & & & & \\ & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \\ & & & & \delta_{N+1,N-1} & \delta_{N+1,N} & \delta_{N+1,N+1} & \delta_{N+1,N+2} & \delta_{N+1,N+3} & \\ & & & & & \delta_{N+2,N} & \delta_{N+2,N+1} & \delta_{N+2,N+2} & \delta_{N+2,N+3} & \delta_{N+2,N+4} \end{bmatrix} \times A_1 = \psi_1,$$

here $\delta_{n,m}$ denotes $\delta_n(z_m)$, $A_1 = [C_{n,-3}^{(j)}, C_{n,-2}^{(j)}, \dots, C_{n,N+4}^{(j)}]^T$ and $\psi_1 = [\frac{\partial^{(j)}\delta_{-1}(z_n)}{\partial z^{(j)}}, \frac{\partial^{(j)}\delta_0(z_n)}{\partial z^{(j)}}, \dots, \frac{\partial^{(j)}\delta_{N+2}(z_n)}{\partial z^{(j)}}]^T$.

System (2.4) contains N+8 unknowns and N+4 equations. By adding four more equations to the system, we obtain a unique solution.

$$\frac{\partial^{(j+1)}\delta_{-1}(z_n)}{\partial z^{(j+1)}} = \sum_{m=-3}^{1} C^{(j)}_{n,m}\delta_{-1}(z_m), \tag{2.5}$$

$$\frac{\partial^{(j+1)}\delta_{0}(z_n)}{\partial z^{(j+1)}} = \sum_{m=-2}^{2} C^{(j)}_{n,m}\delta_{0}(z_m), \tag{2.6}$$

$$\frac{\partial^{(j+1)}\delta_{N+1}(z_n)}{\partial z^{(j+1)}} = \sum_{m=N-1}^{N+3} C^{(j)}_{n,m}\delta_{N+1}(z_m), \tag{2.7}$$

$$\frac{\partial^{(j+1)}\delta_{N+2}(z_n)}{\partial z^{(j+1)}} = \sum_{m=N}^{N+4} C^{(j)}_{n,m}\delta_{N+2}(z_m). \tag{2.8}$$

After adding the above four equations, system (2.4) becomes $R_1 \times A_1 = \psi_2$, here $R_1 =$

$$\begin{bmatrix}
\delta_{-1,-3} & \delta_{-1,-2} & \delta_{-1,-1} & \delta_{-1,0} & \delta_{-1,1} & & & & & \\
\delta'_{-1,-3} & \delta'_{-1,-2} & \delta'_{-1,-1} & \delta'_{-1,0} & \delta'_{-1,1} & & & & & \\
 & \delta_{0,-2} & \delta_{0,-1} & \delta_{0,0} & \delta_{0,1} & \delta_{0,2} & & & & \\
 & \delta'_{0,-2} & \delta'_{0,-1} & \delta'_{0,0} & \delta'_{0,1} & \delta'_{0,2} & & & & \\
 & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \\
 & & & & \delta_{N+1,N-1} & \delta_{N+1,N} & \delta_{N+1,N+1} & \delta_{N+1,N+2} & \delta_{N+1,N+3} & \\
 & & & & \delta'_{N+1,N-1} & \delta'_{N+1,N} & \delta'_{N+1,N+1} & \delta'_{N+1,N+2} & \delta'_{N+1,N+3} & \\
 & & & & & \delta_{N+2,N} & \delta_{N+2,N+1} & \delta_{N+2,N+2} & \delta_{N+2,N+3} & \delta_{N+2,N+4} \\
 & & & & & \delta'_{N+2,N} & \delta'_{N+2,N+1} & \delta'_{N+2,N+2} & \delta'_{N+2,N+3} & \delta'_{N+2,N+4}
\end{bmatrix}$$

$A_1 = [C^{(j)}_{n,-3}, C^{(j)}_{n,-2}, \dots, C^{(j)}_{n,N+3}, C^{(j)}_{n,N+4}]^T$ and $\psi_2 = \Big[\frac{\partial^{(j)}\delta_{-1}(z_n)}{\partial z^{(j)}}, \frac{\partial^{(j+1)}\delta_{-1}(z_n)}{\partial z^{(j+1)}}, \dots, \frac{\partial^{(j)}\delta_{N+2}(z_n)}{\partial z^{(j)}}, \frac{\partial^{(j+1)}\delta_{N+2}(z_n)}{\partial z^{(j+1)}}\Big]^T$.

We used the derivatives and values of the quintic B-spline at the nodes and eliminated the terms $C^{(j)}_{n,-3}$, $C^{(j)}_{n,-2}$, $C^{(j)}_{n,N+3}$, and $C^{(j)}_{n,N+4}$ to obtain a five-banded matrix of the form $R_2 \times A_2 = \psi_3$. here

$$R_2 = \begin{bmatrix}
37 & 82 & 21 & & & & & \\
8 & 33 & 18 & 1 & & & & \\
1 & 26 & 66 & 26 & 1 & & & \\
 & \dots & \dots & \dots & \dots & \dots & & \\
 & & & 1 & 26 & 66 & 26 & 1 \\
 & & & & 1 & 18 & 33 & 8 \\
 & & & & & 21 & 82 & 37
\end{bmatrix}$$

$A_2 = [C^{(j)}_{n,-1}, C^{(j)}_{n,0}, \dots, C^{(j)}_{n,N+1}, C^{(j)}_{n,N+2}]^T$ and $\psi_3 = [\lambda_{-1}, \lambda_0, \dots, \lambda_{N+1}, \lambda_{N+2}]$

$$\lambda_{-1} = \frac{1}{30}\Big[-5\delta^{(j)}_{-1}(z_n) + \delta^{(j+1)}_{-1}(z_n)d + 40\delta^{(j)}_{0}(z_n) + 8\delta^{(j+1)}_{0}(z_n)d\Big], \tag{2.9}$$

$$\lambda_0 = \frac{1}{10}\Big[5\delta^{(j)}_{0}(z_n) - \delta^{(j+1)}_{0}(z_n)d\Big], \tag{2.10}$$

$$\lambda_m = \delta^{(j)}_m(z_n), \quad \text{for} \quad m = 1, 2, 3, \dots, N-1, N, \tag{2.11}$$

TABLE 1. $\delta_i(z)$ and its derivatives on nodes.

z	z_{i-3}	z_{i-2}	z_{i-1}	z_i	z_{i+1}	z_{i+2}	z_{i+3}
$\delta_i(z)$	0	1	26	66	26	1	0
$\delta_i^{'}(z)$	0	$5/d$	$50/d$	0	$-50/d$	$-5/d$	0
$\delta_i^{''}(z)$	0	$20/d^2$	$40/d^2$	$-120/d^2$	$40/d^2$	$20/d^2$	0

$$\lambda_{N+1} = \frac{1}{10}\Big[5\delta_{N+1}^{(j)}(z_n) + \delta_{N+1}^{(j+1)}(z_n)d\Big], \tag{2.12}$$

$$\lambda_{N+1} = \frac{1}{10}\Big[5\delta_{N+1}^{(j)}(z_n) + \delta_{N+1}^{(j+1)}(z_n)d\Big], \tag{2.13}$$

We now choose $j = 1$ and $n = 1$ in equations (2.9)-(2.13) to approximate the first-order derivatives at the grid point z_1

$$\lambda_{-1} = \frac{1}{30}\Big[-5\delta_{-1}^{(1)}(z_1) + \delta_{-1}^{(2)}(z_1)d + 40\delta_0^{(1)}(z_1) + 8\delta_0^{(2)}(z_1)d\Big] = \frac{-109}{2d},$$

$$\lambda_0 = \frac{1}{10}\Big[5\delta_0^{(1)}(z_1) - \delta_0^{(2)}(z_1)d\Big] = \frac{-29}{d},$$

$$\lambda_1 = \delta_1^{(1)}(z_1) = 0,$$

$$\lambda_2 = \delta_2^{(1)}(z_1) = \frac{50}{d},$$

$$\lambda_3 = \delta_3^{(1)}(z_1) = \frac{5}{d},$$

$$\lambda_{N+1} = \frac{1}{10}\Big[5\delta_{N+1}^{(1)}(z_1) + \delta_{N+1}^{(2)}(z_1)d\Big] = 0,$$

$$\lambda_{N+2} = \frac{1}{30}\Big[40\delta_{N+1}^{(1)}(z_1) - 8\delta_{N+1}^{(2)}(z_1)d - 5\delta_{N+2}^{(1)}(z_1) - \delta_{N+2}^{(2)}(z_1)d\Big] = 0.$$

We can write the above approximation in matrix form as:

$$\begin{bmatrix} 37 & 82 & 21 & & & & & & \\ 8 & 33 & 18 & 1 & & & & & \\ 1 & 26 & 66 & 26 & 1 & & & & \\ & \vdots & \vdots & \vdots & \vdots & \vdots & & & \\ & & & 1 & 26 & 66 & 26 & 1 \\ & & & & 1 & 18 & 33 & 8 \\ & & & & & 21 & 82 & 37 \end{bmatrix} \times \begin{bmatrix} C_{1,-1}^{(1)} \\ C_{1,0}^{(1)} \\ C_{1,1}^{(1)} \\ \vdots \\ C_{1,N}^{(1)} \\ C_{1,N+1}^{(1)} \\ C_{1,N+2}^{(1)} \end{bmatrix} = \begin{bmatrix} -109/2d \\ -29/d \\ 0 \\ 50/d \\ 5/d \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Similarly, at grid points z_p, $2 \leq p \leq N-1$, for determining the weighting coefficients $C_{p,m}^{(1)}$, $m = -1, 0, \ldots, N+2$, we obtain the following algebraic equation:

$$\begin{bmatrix} 37 & 82 & 21 & & & & & & \\ 8 & 33 & 18 & 1 & & & & & \\ 1 & 26 & 66 & 26 & 1 & & & & \\ & \vdots & \vdots & \vdots & \vdots & \vdots & & & \\ & & & 1 & 26 & 66 & 26 & 1 \\ & & & & 1 & 18 & 33 & 8 \\ & & & & & 21 & 82 & 37 \end{bmatrix} \times \begin{bmatrix} C^{(1)}_{p,-1} \\ \vdots \\ C^{(1)}_{p,p-1} \\ C^{(1)}_{p,p} \\ C^{(1)}_{p,p+1} \\ \vdots \\ C^{(1)}_{p,N+2} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ -5/d \\ -50/d \\ 0 \\ 50/d \\ 5/d \\ \vdots \\ 0 \end{bmatrix}$$

To determine weighting coefficients $C^{(1)}_{N,m}$ at the grid point z_N, we obtain an algebraic system as follows:

$$\begin{bmatrix} 37 & 82 & 21 & & & & & & \\ 8 & 33 & 18 & 1 & & & & & \\ 1 & 26 & 66 & 26 & 1 & & & & \\ & \vdots & \vdots & \vdots & \vdots & \vdots & & & \\ & & & 1 & 26 & 66 & 26 & 1 \\ & & & & 1 & 18 & 33 & 8 \\ & & & & & 21 & 82 & 37 \end{bmatrix} \times \begin{bmatrix} C^{(1)}_{N,-1} \\ C^{(1)}_{N,0} \\ \vdots \\ C^{(1)}_{N,N-1} \\ C^{(1)}_{N,N} \\ C^{(1)}_{N,N+1} \\ C^{(1)}_{N,N+2} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ -5/d \\ -50/d \\ 0 \\ 29/d \\ 109/2d \end{bmatrix}$$

The pentadiagonal Thomas algorithm is used to solve the linear system $R_2 \times A_2 = \psi_3$, so that we can obtain second-order derivative approximations using the same method.

3. Discretization of the Gray-Scott model

By applying the differential quadrature method to equation (1.1), we obtain the following system:

$$\begin{cases} \frac{dX(z_n,t)}{dt} = b_1 \sum\limits_{m=1}^{N} C^{(2)}_{nm} X(z_m,t) + g_1(X_n,Y_n), \quad n = 1,2,...,N. \\ \frac{dY(z_n,t)}{dt} = b_2 \sum\limits_{m=1}^{N} C^{(2)}_{nm} Y(z_m,t) + g_2(X_n,Y_n), \quad n = 1,2,...,N. \end{cases} \tag{3.1}$$

with initial conditions

$$X(z_n,0) = X_0(z_n) \quad \text{and} \quad Y(z_n,0) = Y_0(z_n), n = 1,2,..N \tag{3.2}$$

This system of ordinary differential equations, which provides numerical solutions at different time levels, is solved by using the SSP-RK43 [17] scheme.

When the boundary conditions are given as the Neumann boundary condition, we denote their values at the boundary as follows:

$$X_z(f,t) = f_1, X_z(g,t) = f_2, Y_z(f,t) = f_3 \quad \text{and} \quad Y_z(g,t) = f_4, \tag{3.3}$$

Similarly, when the boundary conditions are in Dirichlet form, the conditions are

$$X(f,t) = g_1, X(g,t) = g_2, Y(f,t) = g_3 \quad \text{and} \quad Y(g,t) = g_4, \tag{3.4}$$

Using the differential quadrature technique, we discretized the conditions and determined the values of X and Y at the end points as follows:

$$\sum_{m=1}^{N} C_{1m}^{(1)} X(z_m, t) = f_1, \tag{3.5}$$

$$\sum_{m=1}^{N} C_{Nm}^{(1)} X(z_m, t) = f_2, \tag{3.6}$$

The equations (3.5) and (3.4) can be rewritten as

$$\sum_{m=2}^{N-1} C_{1m}^{(1)} X(z_m, t) + C_{11}^{(1)} X(z_1, t) + C_{1N}^{(1)} X(z_N, t) = f_1, \tag{3.7}$$

$$\sum_{m=2}^{N-1} C_{Nm}^{(1)} X(z_m, t) + C_{N1}^{(1)} X(z_1, t) + C_{NN}^{(1)} X(z_N, t) = f_2, \tag{3.8}$$

The values of $X(z_1, t)$ and $X(z_N, t)$ are determined from here. The Neumann conditions corresponding to Y can be treated in a similar manner.

4. Results of Gray-Scott Model

The proposed technique has been used in this section to derive numerical solutions to the Gray-Scott model. To check the accuracy, the error norm has been calculated using the given formula

$$L_\infty = \|W_N - W_{2N}\|_\infty.$$

Here, W_N and W_{2N} represent the numerical solution with N and $2N$ grid points, respectively.

We consider the Gray-Scott model [20, 21] as follows:

$$\begin{cases} \frac{\partial X}{\partial t} = \alpha_1 \frac{\partial^2 X}{\partial z^2} - XY^2 + a(1 - X), \\ \frac{\partial Y}{\partial t} = \alpha_2 \frac{\partial^2 Y}{\partial z^2} + XY^2 - (a + b)Y, \end{cases} \tag{4.1}$$

with the following initial and boundary conditions in (-50,50)

$$X(z, 0) = 1 - \frac{1}{2} sin^{100}\Big(\frac{\pi(z - 50)}{100}\Big), \tag{4.2}$$

$$Y(z, 0) = \frac{1}{4} sin^{100}\Big(\frac{\pi(z - 50)}{100}\Big), \tag{4.3}$$

$$X(-50, t) = X(50, t) = 1, \tag{4.4}$$

$$Y(-50, t) = Y(50, t) = 0. \tag{4.5}$$

here, X denotes the activator concentration and Y denotes the inhibitor concentration. α_1 and α_2 are the diffusion rates for X and Y, respectively. a is the feed rate of X, and b is the decay rate of Y. The calculated results are shown in the figures at different values of a, b, α_1, and α_2.

(i) For our first experiment, we choose $a = 0.064$, $b = 0.062$, $\alpha_1 = 1$, and $\alpha_2 = 0.01$. The Turing patterns are shown in Figure 1 at different time levels

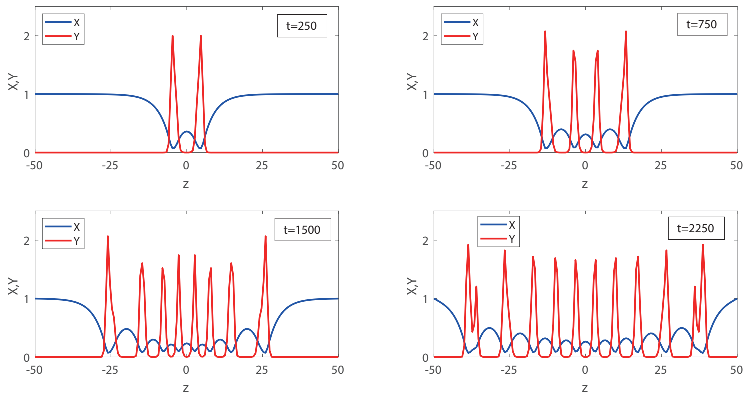

FIGURE 1. Turing patterns of Gray-Scott model for $a = 0.064$, $b = 0.062$, $\alpha_1 = 1$, and $\alpha_2 = 0.01$

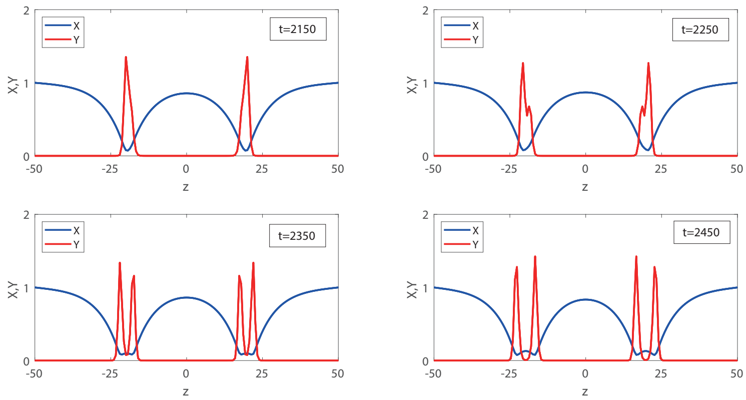

FIGURE 2. Turing patterns of Gray-Scott model for $a = 0.02$, $b = 0.066$, $\alpha_1 = 1$, and $\alpha_2 = 0.01$

t=250, 750, 1500, and 2250. In each figure, self-replicating patterns are observed. The error norm has been computed and compared with those obtained by [21] in Table 2. It is observed from the table that the proposed method giving good results.

(ii) For our second experiment, we choose $a = 0.02$, $b = 0.066$, $\alpha_1 = 1$, and $\alpha_2 = 0.01$. At different time levels t =2150, 2250, 2350, and 2450, we observe the progression of bifurcation in Figure 2. It can be seen from the figure that the two solitary waves generated from the beginning split into four more waves and separated with increasing time until an equilibrium position was achieved.

5. CONCLUSION

In this article, we have proposed a quintic B-spline differential quadrature approach to find numerical solutions to the Gray-Scott model. The main purpose of the differential quadrature method is to convert a partial differential equation

TABLE 2. Comparison of error norm with $a = 0.064$, $b = 0.062$, $\alpha_1 = 1$, and $\alpha_2 = 0.01$.

Method	t	X	Y
Present	5	0.0015	0.0010
	10	0.0037	0.0012
[21]	5	0.0026	0.0019
	10	0.0049	0.0023

into an ordinary differential equation. To obtain the weighting coefficients of the derivative approximations, linear algebraic systems have been solved. Using quintic B-spline functions, the reaction-diffusion system is discretized in space, thus we obtain an ordinary differential equation system. To solve the system of ordinary differential equations, the SSP-RK43 technique has been applied. The error norm has been compared with earlier work and reported in Table 2. It is concluded that the proposed method is very effective for solving the Gray-Scott model.

REFERENCES

[1] I. Ahmad, M. Ahsan, I. Hussain, P. Kumam and W. Kumam, *Numerical simulation of PDEs by local meshless differential quadrature collocation method*, Symmetry **11** (2019): 394.

[2] I. Ali and M. T. Saleem, *Spatiotemporal dynamics of reaction–diffusion system and Its application to turing pattern formation in a Gray–Scott Model*, Mathematics **11** (2023): 1459.

[3] K. Aliyi, A. Shiferaw and H. Muleta, *Radial basis functions based differential quadrature method for one dimensional heat equation*, American Journal of Mathematical and Computer Modelling **6** (2021), 35–42.

[4] A. Bashan, *A mixed algorithm for numerical computation of soliton solutions of the coupled KdV equation: Finite difference method and differential quadrature method*, Applied Mathematics and Computation **360** (2019), 42–57.

[5] A. Bashan, *A mixed methods approach to Schrodinger equation: Finite difference method and quartic B-spline based differential quadrature method*, An International Journal of Optimization and Control: Theories add Applications (IJOCTA) **9** (2019), 223–235.

[6] A. Bashan, *An efficient approximation to numerical solutions for the Kawahara equation via modified cubic B-spline differential quadrature method*, Mediterranean Journal of Mathematics **16** (2019): 14.

[7] A. Bashan, S. B. G. Karakoc and T. Geyikli, *Approximation of the KdVB equation by the quintic B-spline differential quadrature method* **42** (2015), 67–92.

[8] L. Behera and S. Chakraverty *Application of Differential Quadrature method in free vibration analysis of nanobeams based on various nonlocal theories*, Computers and Mathematics with Applications **69** (2015), 1444–1462.

[9] R. Bellman, B. G. Kashef and J. Casti, *Differential quadrature: a technique for the rapid solution of nonlinear partial differential equations*, Journal of computational physics **10** (1972), 40–52.

[10] F. Civan, *Differential Quadrature for Reservoir Simulation*, in: SPE Reservoir Simulation Conference, SPE, 1989.

[11] E. Crampin and P. Maini, *Reaction-diffusion models for biological pattern formation*, 2001.

[12] I. Dag, B. Ay and B. Saka, *Cubic b-spline least squares method for the numerical solution of advection-diffusion equation*, Journal of Applied and Computational Sciences **1** (2022), 53–58.

[13] T. C. Fung, *Stability and accuracy of differential quadrature method in solving dynamic problems*, Computer Methods in Applied Mechanics and Engineering **191** (2002), 1311–1331.

[14] C. Garodia and I. Uddin, *A new fixed point algorithm for finding the solution of a delay differential equation*, Aims Mathematics **5** (2020), 3182–3200.

[15] D. Gizaw, *Finite element method applied to Gray-Scott reaction-diffusion problem. Using FEniCs software*, Advances in Physics Theories and Applications **58** (2016), 5–8.

[16] P. Gray and S. K. Scott, *Autocatalytic reactions in the isothermal, continuous stirred tank reactor: isolas and other forms of multistability*, Chemical Engineering Science **38** (1983), 29–43.

[17] G. Gurarslan and M. Sari, *Numerical solutions of linear and nonlinear diffusion equations by a differential quadrature method (DQM)*, International Journal for Numerical Methods in Biomedical Engineering **27** (2011), 69–77.

[18] M. S. Hashmi, M. Wajiha, S. W. Yao, A. Ghaffar and M. Inc, *Cubic spline based differential quadrature method: A numerical approach for fractional Burger equation*, Results in Physics **26** (2021): 104415.

[19] R. Jiwari, *Lagrange interpolation and modified cubic B-spline differential quadrature methods for solving hyperbolic partial differential equations with Dirichlet and Neumann boundary conditions*, Computer Physics Communications **193** (2015), 55–65.

[20] R. Jiwari, S. Singh, and A. Kumar, *Numerical simulation to capture the pattern formation of coupled reaction-diffusion models*, Chaos, Solitons and Fractals **103** (2017), 422–439.

[21] V. Joshi, *Numerical solution of Gray Scott reaction-diffusion equation using Lagrange polynomial*, Journal of Physics: Conference Series **1531** (2020): 012058.

[22] N. Kaur and V. Joshi, *Numerical solution of coupled Korteweg-de Vries equation employing quintic hyperbolic B-spline based differential quadrature method*, International Journal of Applied and Computational Mathematics **9** (2023): 8.

[23] S. Khatoon, I. Uddin and D. Baleanu, *Approximation of fixed point and its application to fractional differential equation*, Journal of Applied Mathematics and Computing **66** (2021), 507–525.

[24] A. Korkmaz, O. Ersoy and I. Dag, *Motion of patterns modeled by the Gray-Scott autocatalysis system in one dimension*, arXiv preprint arXiv:1605.09712, 2016.

[25] A. Korkmaz and I. Dag, *Quartic and quintic B-spline methods for advection–diffusion equation*, Applied Mathematics and Computation **274** (2016), 208–219.

[26] A. N. Landge, B. M. Jordan, X. Diego and P. Muller, *Pattern formation mechanisms of self-organizing reaction-diffusion systems*, Developmental biology **460** (2020), 2–11.

[27] K. Matsuya and M. Murata, *Spatial pattern of discrete and ultradiscrete Gray-Scott model*, Discrete and continuous Dynamical Systems-Series B20 (2015),173–187.

[28] R. C. Mittal and S. Dahiya, *A study of quintic B-spline based differential quadrature method for a class of semi-linear Fisher-Kolmogorov equations*, Alexandria Engineering Journal **55** (2016), 2893–2899.

[29] G. Nicolis and A. De Wit, *Reaction-diffusion systems* , Scholarpedia **2** (2007): 1475.

[30] H. Rahim, N. Iqbal, C. Cong and Z. Ding, *Pattern selection of three components Gray-Scott model*, In Journal of Physics: Conference Series **1324** (2019): 012012.

[31] S. Singh, *Numerical investigation of wave pattern evolution in Gray–Scott model using discontinuous Galerkin finite element method*, in: Advances in Mathematical and Computational Modeling of Engineering Systems, CRC Press, 2023, pp. 47–58.

[32] M. Tamsir and M. J. Huntul, *A numerical approach for solving Fisher ' s reaction–diffusion equation via a new kind of spline functions*, Ain Shams Engineering Journal **12** (2021), 3157–3165.

[33] M. Tamsir, M. Z. Meetei and A. H. Msmali, *Hyperbolic B-spline function-based differential quadrature method for the approximation of 3D wave equations*, Axioms **11** (2022): 597.

[34] B. Tsang, *Patterns in Reaction Diffusion System*, Department of Physics, University of Illinois Urbana-Champaign, 2011.

[35] Y. Ucar, M. Yagmurlu and A. Bashan, *Numerical solutions and stability analysis of modified Burgers equation via modified cubic B-spline differential quadrature methods*, Sigma Journal of Engineering and Natural Sciences **37** (2019), 129–142.

[36] X. Umar, M. M. S. Sehrish, and A. W. Shaikh, *A new quadrature-based iterative method for scalar nonlinear equations*, J. Mech. Cont. and Math. Sci **15** (2020), 79–93.

[37] V. K. Vanag and I. R. Epstein, *Pattern formation mechanisms in reaction-diffusion systems*, International Journal of Developmental Biology **53** (2009), 673–681.

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Anisha
Department of Applied Sciences, The Northcap University, Gurugram
E-mail address: `anisha21asd003@ncuindia.edu`

R. Rohila
Department of Applied Sciences, The Northcap University, Gurugram
E-mail address: `rajnirohila@ncuindia.edu`