Journal of Nonlinear and Convex Analysis
Volume 25, Number 4, 2024, 715–724

FINITE-TIME ROBUST CONTROL FOR MARKOV JUMP DELAYED SYSTEMS

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ABSTRACT. The main goal of this article is to deals with the problem for the stochastic delayed Markov jump system, a novel design method of dynamic finite-time control is proposed. By designing the robust controller and constructing Lyapunov Krasovskii function, the stability criterion and robust performance of the system are given by using LMIs. The control strategy can switch stochastic time delay to Markov process stochastic time delay, so the big range of value of delay can be dealt with. Finally, the simulation results had been illustrated the effectiveness.

1. INTRODUCTION

In recent years, more and more researches focus on finite-time problem subject to input saturation. Many results have been reported due to its practical and theoretical. For example, He and Liu et al. [7,10] developed a state feedback controller based on observe. In [8], H_∞ control approach and dynamic observer are designed for the nonlinear jump system. On the other hand, the nonlinearity caused by saturation which may lead to great instability to system performance, due to the increasing complexity of controlled objects and the increasing requirements of engineering for control, the theory and application of robust optimal control has become a common concern of many scholars and experts all over the world. Lots of results have been reported in [1,3–5,9,11–14], because of the existence of time-delay, it is easy to lead to system reliability performance deterioration, so the Markov jump delayed system has not been fully studied and remain open and challenging. In addition, since to achieve infinite energy of the signal is usually impossible, it is more and more common for input saturation in the real world. In recent years, many researchers have addressed the issue, as noted in the references herein [2,6]. This problem will become more complex, thus, the method of the Markov jump model can be used in dealing with the problems discussed. In this paper, the main contributions are listed:

(1) Analyze the stability of the Markov jump delayed systems with LMI.

(2) A novel control scheme of compensator is robust against the effects of the perturbations.

2020 *Mathematics Subject Classification.* 93C10, 93D40.
Key words and phrases. Input saturation, dynamic anti-windup, finite-time control.
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2. System description

Considering the discrete system (Σ):

(2.1)
$$\dot{x}\left(t\right)=A\left(\delta(t),\ t\right)x\left(t\right)+A_{d}\left(\delta(t),\ t\right)x\left(t-h(t)\right)+B\left(\delta(t),\ t\right)V\left(t\right)+D\left(\delta(t),\ t\right)d(t)$$

(2.2)
$$z\left(t\right)=C_{1}\left(\delta(t),\ t\right)x\left(t\right)+D_{1}\left(\delta(t),\ t\right)V\left(t\right)+D_{d}\left(\delta(t),\ t\right)d(t)$$

(2.3)
$$y\left(t\right)=C(\delta(t),\ t)x\left(t\right)$$

(2.4)
$$x(t+\theta)=\phi(\theta),\ \forall\theta\in[-\tau,\ 0]$$

where $V\left(t\right)\in\mathbb{R}^{m}$ is the input, $y\left(t\right)\in\mathbb{R}^{q}$ is the output, $x\left(t\right)\in\mathbb{R}^{n}$ is the state vector. The delay $\dot{h}(t)\leq h<1$ and $h(t)\leq\tau$, τ>0.

$$\Pr\left\{\delta\left(t+\Delta\right)=j\left|\delta\left(t\right)=i\right.\right\}=\left\{\begin{array}{cc}\pi_{ij}\Delta+o\left(\Delta\right) & i\neq j,\\ 1+\pi_{ii}\Delta+o\left(\Delta\right) & i=j,\end{array}\right.$$

where $\pi_{ij}\geq 0$ and $\Delta>0$, $\lim_{\Delta\to 0}\left(o\left(\Delta\right)/\Delta\right)=0$, for $j\neq i$, and

$$\pi_{ii}=-\sum_{j\in S,j\neq i}\pi_{ij}.$$

The matrix is expressed as follows:

$$\left[\begin{array}{ccccc}\pi_{11} & ? & \pi_{13} & \cdots & ?\\ ? & ? & ? & \cdots & \pi_{2n}\\ \cdots & ? & \cdots & \cdots & \cdots\\ \pi_{n1} & ? & \pi_{n3} & \cdots & ?\end{array}\right],$$

We define

$$S^{i}=S_{k}^{i}\bigcup S_{uk}^{i},$$

$$S_{k}^{i}\doteq\{j:\pi_{ij}\ is\ known\ for\ j\in S\},$$
$$S_{uk}^{i}\doteq\{j:\pi_{ij}\ is\ unknown\ for\ j\in S\}.$$

In addition, if $S_{k}^{i}\neq\varnothing$,we have:

(2.5)
$$S_{k}^{i}=\{k_{1}^{i},\ k_{2}^{i},\cdots,\ k_{m}^{i}\},$$

The input u_i is:

(2.6)
$$-u_{0(i)}\ \leq\ u_{(i)}\leq u_{0(i)},\ u_{0(i)}>0,\ i=1,\cdots,\ m.$$

Let

(2.7)
$$[\Delta A_{i}(t)\ \Delta A_{di}(t)\ \Delta B_{i}(t)]\ =\ M_{i}F_{i}(t)[N_{ai}\ N_{adi}\ N_{bi}].$$

(2.8)
$$\dot{x}_{c}(t)\ =\ A_{ci}x_{c}(t)+B_{ci}u_{c}(t),$$

(2.9)
$$y_{c}(t)\ =\ C_{ci}x_{c}(t)+D_{ci}u_{c}(t),$$

where $x_c(t)\in\mathbb{R}^{n_c}$, $u_c(t)\in\mathbb{R}^{n_p}$, $y_c(t)\in\mathbb{R}^{m}$ and we have:

$$u_{c}(t)=y(t),\quad V(t)=sat(y_{c}(t)).$$

Base on the above, it can be obtained that:

$$\dot{x}_a(t) = A_{ai}x_a(t) + B_{ai}\psi(y_c(t)), \tag{2.10}$$

$$y_a(t) = C_{ai}x_a(t) + D_{ai}\psi(y_c(t)), \tag{2.11}$$

Then the new controller is:

$$\dot{x}_c(t) = A_{ci}x_c(t) + B_{ci}y(t) + y_a(t),$$
$$y_c(t) = C_{ci}x_c(t) + D_{ci}u_c(t).$$

By defining $\xi(t) = [x(t)^T \; x_c(t)^T \; x_a(t)^T]^T$, we have:

$$\dot{\xi}(t) = \mathbf{A}\xi(t) + \mathbf{A}_d\xi(t - h(t)) + \mathbf{B}\psi(y_c(t)) + \overline{D}_i d(t), \tag{2.12}$$

$$z\,(t) = \overline{C}_1\,(\delta(t),\ t)\,x\,(t) + D_1\,(\delta(t),\ t)\,\psi(y_c(t)) + D_d\,(\delta(t),\ t)\,d(t), \tag{2.13}$$

$$u(t) = \mathbf{K}\xi(t), \tag{2.14}$$

$$\xi(t+\theta) = \phi(\theta),\ \forall\theta \in [-\tau,\ 0], \tag{2.15}$$

where $\mathbf{A} = \hat{A}_i + \Delta\hat{A}_i(t)$, $\mathbf{A}_d = \hat{A}_{di} + \Delta\hat{A}_{di}(t)$, $\mathbf{B} = \hat{B}_i + \Delta\hat{B}_i(t)$, $\overline{C}_{1i} = C_{1i} + D_{1i}\mathbf{K}$, $\mathbf{K} = [D_{ci}C_i \;\; C_{ci} \;\; 0] = [K_1 \; 0]$, and $[\Delta\hat{A}_i \; \Delta\hat{A}_{di} \; \Delta\hat{B}_i] = \bar{M}_iF_i(t)\mathbf{N}$, with the following matrices:

$$\bar{M}_i^T = [M_i^T \; 0 \; 0],\ \mathbf{N} = [\bar{N}_i \; N_{di} \; N_{bi}],$$
$$\bar{N}_i = [N_{ai} + N_{bi}D_{ci}C_i \;\; N_{bi}D_{ci}C_i \;\; 0] = [N_{i1} \;\; 0],$$
$$N_{di} = [N_{adi} \; 0 \; 0] = [N_{di1} \;\; 0],$$
$$\hat{A}_i = \begin{bmatrix} \bar{A}_i & \check{I}C_{ai} \\ 0 & A_{ai} \end{bmatrix},\ \hat{A}_{di} = \begin{bmatrix} \bar{A}_{di} & 0 \\ 0 & 0 \end{bmatrix},\ \hat{B}_i = \begin{bmatrix} \bar{B}_i + \check{I}D_{ai} \\ B_{ai} \end{bmatrix},$$
$$\bar{A}_i = \begin{bmatrix} A_i + B_iD_{ci}C_i & B_iC_{ci} \\ B_{ci}C_i & A_{ci} \end{bmatrix},\ \bar{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix},$$
$$\check{I}^T = [0 \;\; I].$$

3. Design process

The controller is designed which guarantees the stabilization, some processes are given as follows:

Lemma 3.1. *For the matrices $A,\ D,\ S,\ W > 0$, the matrix $F(t)^TF(t) \le I$ with appropriate dimensions, the following inequalities hold:*

1) $\forall\epsilon > 0$ *and* $x,\ y \in R^n$

$$2x^TDFSy \le \epsilon^{-1}x^TDD^Tx + \epsilon y^TS^TSy.$$

2) $\forall\epsilon > 0$, *if* $W - \epsilon DD^T > 0$,

$$(A + DFS)^TW^{-1}(A + DFS) \le A^T(W - DD^T)^{-1}A + \epsilon^{-1}S^TS.$$

Definition 3.2. Under the assumed zero initial condition, for the system(Σ), the closed-loop system (2.12) – (2.15) is SFTB, there exists the following cost function inequality for $T > 0$.

$$J = E\{\int_0^T z^Tzdt\} - \gamma^2E\{\int_0^T d^Tddt\} \le 0.$$

Theorem 3.3. *For the system* (Σ)*, let* $T>0$*,* $\delta(t)=i\in S$*,* $h<1$ *and* $\epsilon_i>0$*, if there exist* $P_i=P^T,\ R=R^T,\ W_i=W^T$*, satisfies the following LMIs:*

$$H_0+\Sigma_{j\in(S_k^i)}\pi_{ij}(H_w+H_p)<0,\ i\in S_k^i, \tag{3.1}$$

$$P_i-W_i<0,\ j\neq i\in S_{uk}^i, \tag{3.2}$$

$$P_i-W_i\geq 0,\ j=i\in S_{uk}^i, \tag{3.3}$$

$$C_1(\sigma_P+\tau\sigma_Q)+\frac{\gamma^2 d}{\alpha}(1-e^{-aT})<e^{-aT}C_2\sigma_p \tag{3.4}$$

where

$$H_0=\begin{bmatrix} He(P_iA_i)+R+ \\ \varepsilon_i N_{ai}^T N_{ai} & P_iA_{di} & P_i\overline{D}_i+\bar{C}_{1i}\bar{D}_{di} & P_iM_i \\ * & \varepsilon_i N_{adi}^T N_{adi}-(1-h)R & 0 & 0 \\ * & * & -\gamma^2 I+\bar{D}_{di}^T\bar{D}_{di} & 0 \\ * & * & * & -\frac{1}{2}\varepsilon_i I \end{bmatrix},$$

$$H_p=\begin{bmatrix} P_j & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix},\ H_w=\begin{bmatrix} -W_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix},$$

$$\sigma_P=max_{i\in S}\sigma_{max}(\tilde{P}_i),\sigma_p=min_{i\in S}\sigma_{min}(\tilde{P}_i),$$
$$\sigma_R=max_{i\in S}\sigma_{max}(\tilde{R}_i),\sigma_r=min_{i\in S}\sigma_{min}(\tilde{R}_i),$$
$$\tilde{R}_i=\hat{R}_i^{-1/2}R\hat{R}_i^{-1/2},\tilde{P}_i=\hat{R}_i^{-1/2}P_i\hat{R}_i^{-1/2}.$$

then system (Σ) *is finite-time stable.*

Proof. We define

$$V(x(t),i,t)=x(t)^TP_ix(t)+\textstyle\int_{t-h(t)}^{t}x(\theta)^TRx(\theta)d\theta, \tag{3.5}$$

Then

$$\begin{aligned}\dot{V}(x(t),i,t) &= x(t)^T(P_iA_i+A_i^TP_i)x(t)+2x(t)^TP_iA_{di}x(t-h(t)) \\ &\quad +\Sigma_{j\in(s)}\pi_{ij}x(t)^TP_jx(t)+2x^T(t)P_i\bar{D}_{di}d(t) \\ &\quad +x(t)^TR_ix(t)-(1-\dot{h}(t))x(t-h(t))^TRx(t-h(t)) \\ &\quad +2x(t)^TP_i\Delta A_{it}x(t)+2x(t)^TP_i\Delta A_{dit}x(t-h(t)).\end{aligned}$$

$$2x(t)^TP_i\Delta A_{it}x(t)\leq\varepsilon_i^{-1}x(t)^TP_iM_iM_i^TP_ix(t)+\varepsilon_ix(t)^TN_{ai}^TN_{ai}x(t), \tag{3.6}$$

Denote:

$$H_0=\begin{bmatrix}\mathcal{H} & P_iA_{di} & P_i\overline{D}_i+\bar{C}_{1i}\bar{D}_{di} \\ * & \varepsilon_iN_{adi}^TN_{adi}-(1-h)R & 0 \\ * & * & -\gamma^2I+\bar{D}_{di}^T\bar{D}_{di}\end{bmatrix}$$

$$\xi(t)=[x(t)^T\ \ x(t-h(t))^T\ \ d(t)^T].$$

By using of the Schur complements,

$$\dot{V}(\hat{x},\ i)<\alpha V(\hat{x},\ i)+\gamma^2d^Td-z^Tz.$$

We can get the follows:

$$[e^{-\alpha t}V(\hat{x},\ i)] < e^{-\alpha t}[\gamma^2 d^T d - z^T z].$$

$$e^{-\alpha t}V(\hat{x},\ i) < \int_0^T e^{-\alpha t}[\gamma^2 d^T d - z^T z]dt.$$

Consider that $t \in [0\ T]$, we have

$$\int_0^T z^T z dt < \gamma^2 e^{\alpha T}\int_0^T d^T d dt.$$

The proof is completed. □

Theorem 3.4. *For the system* (2.12) − (2.15)*, let* $T > 0$*,* $\delta(t) = i \in S$*,* $h < 1$ *and scalars* $\epsilon_i > 0$*, if there exist* $X_i,\ Y_i,\ R_{i1},\ R_{i3},\ W_{i1}, W_{i3},\ G_1,\ G_2,\ N_i,\ \hat{A}_{ai},\ \hat{B}_{ai},\ \hat{C}_{ai},\ \hat{D}_{ai}, U_i,\ V_i,\ R_{i2},\ W_{i2}$*, and diagonal positive definite matrices* S_i *satisfies the following LMIs:*

$$\begin{bmatrix} H_0 + \Sigma_{j\in(S_k^i)}\pi_{ij}H_w + \pi_{ii}H_i & * \\ \Lambda_{1i}^T & -\Xi_{1i} \end{bmatrix} < 0,\ i \in S_k^i \tag{3.7}$$

$$\begin{bmatrix} H_0 + \Sigma_{j\in(S_k^i)}\pi_{ij}H_w & * \\ \Lambda_{2i}^T & -\Xi_{2i} \end{bmatrix} < 0,\ i \in S_{uk}^i, \tag{3.8}$$

$$\begin{bmatrix} -W_{i1} & * & * & * \\ -W_{i2} & -W_{i3} & * & * \\ X_i & Y_i & -G_1 & 0 \\ 0 & N_i & 0 & -G_2 \end{bmatrix} < 0,\ j \neq i \in S_{uk}^i, \tag{3.9}$$

$$\begin{bmatrix} X_i - W_{i1} & * \\ X_i - W_{i2} & Y_i - W_{i3} \end{bmatrix} \geq 0,\ j = i \in S_{uk}^i, \tag{3.10}$$

$$\begin{bmatrix} X_i & * & * \\ X_i & Y_i & * \\ K_1X_i + U_i & K_1Y_i + V_i & \mu_{o(k)}^2 \end{bmatrix} > 0,\ k = 1, \cdots,\ m, \tag{3.11}$$

$$G_1 - Y_i \leq 0, \tag{3.12}$$

$$\sigma_1\hat{R}_i^{-1} < X_i < \hat{R}_i^{-1}, \tag{3.13}$$

$$0 < R_{1i} < \sigma_2\hat{R}_i, \tag{3.14}$$

$$\begin{bmatrix} -e^{-\alpha T}C_2 + C_1\tau\sigma_2 + \frac{\gamma^2 d}{\alpha}(1 - e^{-\alpha T}) & \sqrt{C_1} \\ * & -\sigma_1 \end{bmatrix} < 0, \tag{3.15}$$

where

$$H_i = \begin{bmatrix} X_i & X_i & \mathbf{0} \\ X_i & Y_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},\ H_w = \begin{bmatrix} -W_{i1} & * & \mathbf{0} \\ -W_{i2} & -W_{i3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Xi_{1i} = diag\ \{G_1,\ G_2, \cdots,\ G_1,\ G_2, ...,\ G_1,\ G_2\},$$

$$\Xi_{2i} = diag\ \{G_1,\ G_2, \cdots,\ G_1,\ G_2\},$$

$$\Lambda_{1i} = \begin{bmatrix} \sqrt{\pi_{ik_1^i}}\bar{Q}_i & \sqrt{\pi_{ik_2^i}}\bar{Q}_i & ... & \sqrt{\pi_{ik_{r-1}^i}}\bar{Q}_i & \sqrt{\pi_{ik_r^i}}\bar{Q}_i & ... & \sqrt{\pi_{ik_m^i}}\bar{Q}_i \\ \mathbf{0} & \mathbf{0} & ... & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Lambda_{2i} = \begin{bmatrix} \sqrt{\pi_{ik_1^i}}\bar{Q}_i & \sqrt{\pi_{ik_2^i}}\bar{Q}_i & \cdots & \sqrt{\pi_{ik_m^i}}\bar{Q}_i \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix},$$

with

$$\begin{aligned} &H_{11} = He(\bar{A}_iX_i) + R_{i1} + \epsilon_i M_0, \\ &H_{21} = \bar{A}_iX_i + Y_i\bar{A}_i^T + \hat{C}_{ai}^T\check{I}^T + \hat{A}_{ai}^T + R_{i2} + \epsilon_i M_0, \\ &H_{22} = He(\bar{A}_iY_i + \check{I}\hat{C}_{ai}) + R_{i3} + \epsilon_i M_0, \\ &H_{51} = S_i\bar{B}_i^T + \hat{D}_{ai}^T\check{I}^T + \hat{B}_{ai}^T, H_{52} = S_i\bar{B}_i^T + \hat{D}_{ai}^T\check{I}^T, \\ &\bar{Q}_i = \begin{bmatrix} X_i & 0 \\ Y_i & N_i^T \end{bmatrix}, M_0 = \begin{bmatrix} M_iM_i^T & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

the system $(2.12)-(2.15)$ *is locally asymptotically finite-time stable with parameters as follows:*

$$N_i^T\tilde{M}_i = I - Y_iX_i^{-1},\ A_{ai} = \tilde{M}_i^{-T}X_i^{-1}\hat{A}_{ai}N_i^{-1}, \tag{3.16}$$

$$B_{ai} = \tilde{M}_i^{-T}X_i^{-1}\hat{B}_{ai}S_i^{-1}, C_{ai} = \hat{C}_{ai}N_i^{-1}, D_{ai} = \hat{D}_{ai}S_i^{-1}. \tag{3.17}$$

Proof. For any $\delta(t) = i \in S$,

$$V(\xi(t), i, t) = \xi(t)^T P_i\xi(t) + \int_{t-h(t)}^{t} \xi(\theta)^T R\xi(\theta)d\theta, \tag{3.18}$$

According to the Lemma 3.1, then

$$\begin{aligned} &2\xi(t)^T P_i(\Delta\hat{A}_i(t)\xi(t) + \Delta\hat{A}_{di}(t)\xi(t-h(t)) + \Delta\hat{B}_i(t)\psi(t)) \\ \leq\ &\epsilon_i\xi(t)^T P_i\bar{M}_i\bar{M}_i^T P_i\xi(t) + \epsilon_i^{-1}\beta(t)^T\mathbf{N}^T\mathbf{N}\beta(t), \end{aligned} \tag{3.19}$$

We have

$$\dot{V}(\xi(t), i, t) \leq \beta(t)^T\Phi_i\beta(t) + \epsilon_i^{-1}\beta(t)^T\mathbf{N}^T\mathbf{N}\beta(t) + 2x^T(t)P_i\bar{D}_{di}d(t), \tag{3.20}$$

where

$$\Phi_i = \begin{bmatrix} \Omega_i & * & * & * \\ \hat{A}_{di}^T P_i & -(1-h)R & * & * \\ \hat{B}_i^T P_i + T_iL_i + D_{1i}^T\bar{C}_{1i} & 0 & -2T_i + D_{1i}^T D_{1i} & * \\ D_i^T P_i & 0 & D_{di}^T D_{1i} & -\gamma^2 I + D_{di}^T D_{di} \end{bmatrix},$$

with

$$\Omega_i = P_i\hat{A}_i + \hat{A}_i^T P_i + \Sigma_{j\in(s)}\pi_{ij}P_j + R + \epsilon_i P_i\bar{M}_i\bar{M}_i^T P_i. \tag{3.21}$$

We have

$$\Omega_i = P_i\hat{A}_i + \hat{A}_i^T P_i + \Sigma_{j\in(s)}\pi_{ij}P_j - \Sigma_{j\in(s)}\pi_{ij}O_i + R + \epsilon_i P_i\bar{M}_i\bar{M}_i^T P_i. \tag{3.22}$$

By employing the Schur complements, and (3.18) with $diag(P_i,\ P_i,\ T_i,\ I, \ldots,\ I)$ respectively, we derive

$$\dot{V}(x,\ i) < \alpha V(x,\ i) + \gamma^2 d^T d - z^T z.$$

where

$$\begin{aligned} &\bar{\Omega}_i = \hat{A}_iQ_i + Q_i\hat{A}_i^T + \Sigma_{j\in(s)}\pi_{ij}Q_iP_jQ_i - \Sigma_{j\in(s)}\pi_{ij}W_i + \mathfrak{R}_i + \epsilon_i\bar{M}_i\bar{M}_i^T, \\ &Z_i = L_iQ_i = [V_i\ \ \tilde{U}_i],\ \mathfrak{R}_i = Q_iRQ_i,\ W_i = Q_iO_iQ_i, Q_i = P_i^{-1},\ S_i = T_i^{-1}. \end{aligned}$$

In addition, we define the follows

$\tilde{X}_i = \hat{R}_i^{1/2}\bar{X}_i\hat{R}_i^{1/2},\ \tilde{R}_i = \hat{R}_i^{-1/2}\bar{R}\hat{R}_i^{-1/2},\ \hat{X}_i = R_i^{1/2}X_iR_i^{1/2},\ \hat{R}_i = R_i^{-1/2}RR_i^{-1/2},$

where $\bar{X}_i = diag\{X_i,\ X_i\}$, $\hat{R}_i = diag\{R_i,\ R_i\}$.

$$\begin{aligned}\sigma_X &= max_{i\in S}\sigma_{max}(\hat{X}_i) = max_{i\in S}\sigma_{max}(\tilde{X}_i),\\ \sigma_x &= min_{i\in S}\sigma_{min}(\hat{X}_i) = min_{i\in S}\sigma_{min}(\tilde{X}_i),\\ \sigma_R &= max_{i\in S}\sigma_{max}(\hat{R}_i) = max_{i\in S}\sigma_{max}(\tilde{R}_i).\end{aligned}$$

and

$$max_{i\in S}\sigma_{max}(X_i) = \frac{1}{min_{i\in S}\sigma_{min}P_i}.$$

It follows that

$$\frac{C_1}{\sigma_x} + C_1\tau\sigma_Q + \frac{\gamma^2 d}{\alpha}(1-e^{-\alpha T}) < \frac{e^{-\alpha T}C_2}{\sigma_X}. \tag{3.23}$$

Defining $H = \Phi_i$, it follows that

$$H \ = \ H_0 + \Sigma_{j\in(S)}\pi_{ij}H_j + \Sigma_{j\in(S)}\pi_{ij}H_w$$

(3.24)

where

$$H_j = \begin{bmatrix} Q_iP_jQ_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix},\ H_w = \begin{bmatrix} -W_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix},$$

with $\bar{\Omega}_{i0} = \hat{A}_iQ_i + Q_i\hat{A}_i^T + \mathfrak{R}_i + \epsilon_i\bar{M}_i\bar{M}_i^T$.
From (3.23), then we have $H < 0$:

$$H_0 + \Sigma_{j\in(S_k^i,\ j\neq i)}\pi_{ij}H_j + \Sigma_{j\in(S_k^i)}\pi_{ij}H_w + \pi_{ii}H_i < 0,\ j = i\in S_k^i, \tag{3.25}$$

$$H_0 + \Sigma_{j\in(S_k^i,\ j\neq i)}\pi_{ij}H_j + \Sigma_{j\in(S_k^i)}\pi_{ij}H_w < 0,\ j = i\in S_{uk}^i, \tag{3.26}$$

$$H_j + H_w < 0,\ j\in S_{uk}^i,\ j\neq i, \tag{3.27}$$

$$H_j + H_w \geq 0,\ j\in S_{uk}^i,\ j = i. \tag{3.28}$$

Consider the conditions (3.24) and (3.25), we derive

$$\begin{bmatrix} H_0 + \Sigma_{j\in(S_k^i)}\pi_{ij}H_w + \pi_{ii}H_i & \Lambda_{1i} \\ \Lambda_{1i}^T & -\Xi_{1i}\end{bmatrix} < 0,\ i\in S_k^i, \tag{3.29}$$

$$\begin{bmatrix} H_0 + \Sigma_{j\in(S_k^i)}\pi_{ij}H_w & \Lambda_{2i} \\ \Lambda_{2i}^T & -\Xi_{2i}\end{bmatrix} < 0,\ i\in S_{uk}^i, \tag{3.30}$$

where

$$\begin{aligned}\Xi_{1i} &= diag\ \{Q_{k_1^i},\ Q_{k_2^i},\cdots,\ Q_{k_{r-1}^i},\ Q_{k_r^i},...,\ Q_{k_m^i}\},\\ \Xi_{2i} &= diag\ \{Q_{k_1^i},\ Q_{k_2^i},...,\ Q_{k_m^i}\},\\ \Lambda_{1i} &= \begin{bmatrix} \sqrt{\pi_{ik_1^i}}Q_i & \sqrt{\pi_{ik_2^i}}Q_i & ... & \sqrt{\pi_{ik_{r-1}^i}}Q_i & \sqrt{\pi_{ik_r^i}}Q_i & ... & \sqrt{\pi_{ik_m^i}}Q_i \\ \mathbf{0} & \mathbf{0} & ... & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},\end{aligned}$$

$$\Lambda_{2i} = \begin{bmatrix} \sqrt{\pi_{ik_1^i}}Q_i & \sqrt{\pi_{ik_2^i}}Q_i & \cdots & \sqrt{\pi_{ik_m^i}}Q_i \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}.$$

Here, we define that

$$P_i = \begin{bmatrix} X_i^{-1} & * \\ \tilde{M}_i & E_i \end{bmatrix}, \; Q_i = P_i^{-1} = \begin{bmatrix} Y_i & * \\ N_i & F_i \end{bmatrix}, \gamma = \begin{bmatrix} I & I \\ \tilde{M}_i X_i & 0 \end{bmatrix},$$

and $\hat{A}_{ai} = X_i\tilde{M}_i^T A_{ai} N_i$, $\hat{B}_{ai} = X_i\tilde{M}_i^T B_{ai} S_i$, $\hat{C}_{ai} = C_{ai}N_i$, $\hat{D}_{ai} = D_{ai}S_i$, $U_i = V_i + \tilde{U}_i\tilde{M}_i X_i$, $G = G^T \leq Q_i$, $\forall i \in S$ can be find.

$$G = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix},$$

then replace the $Q_{k_m^i}$ of (3.28) and (3.29) by G_1 and G_2, considering the condition (3.26), by use of the Schur complements, we have

$$\begin{bmatrix} -W_i & Q_i \\ Q_i & -Q_j \end{bmatrix} < 0, \tag{3.31}$$

$$\begin{bmatrix} P_i & * \\ \mathbf{K}_{i(k)} + L_{i(k)} & u_{0(k)}^2 \end{bmatrix} > 0, \; k = 1, \cdots, \; m, \tag{3.32}$$

Based on the above, the system (2.12)-(2.15) is locally asymptotically finite-time stable. The proof is completed. □

4. Simulation study

Consider the system(Σ) with the following parameters:

$$A_1 = \begin{bmatrix} 0.70 & -0.75 \\ 1.50 & -1.50 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0.15 & 4.5 \\ 2.10 & -0.4 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.20 & 2.50 \\ 1.20 & -2.1 \end{bmatrix}, \; A_4 = \begin{bmatrix} 0.95 & -0.35 \\ 1.75 & -1.50 \end{bmatrix},$$
$$B_1 = B_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \; B_3 = \begin{bmatrix} 1 \\ 0.6 \end{bmatrix},$$
$$A_{di} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.2 \end{bmatrix}, i = 1, \; 2, \; 3, \; 4,$$
$$C_i = \begin{bmatrix} -0.1 & -0.2 \end{bmatrix}, i = 1, \; 2, \; 3, \; 4,$$

with

$$A_{c1} = -5.25 \;, \; A_{c2} = -5.5 \;, \; A_{c3} = -4.5 \;, \; A_{c4} = -7.5 \;,$$
$$B_{c1} = -1.5 \;, B_{c2} = -0.8 \;, B_{c3} = -0.8 \;, B_{c4} = -1.0 \;,$$
$$C_{c1} = -1.0 \;, \; C_{c2} = -1.00 \;, \; C_{c3} = -1.50 \;, \; C_{c4} = -1.50 \;,$$
$$D_{c1} = 0 \;, \; D_{c2} = 5 \;, \; D_{c3} = 6 \;, \; D_{c4} = -2 \;.$$

In this case, we chose the initial values for $c_1 = 0.25,\ \alpha = 1.0,\ T = 2,\ R_i = I_2,\ d = 4,\ \tau = 0.5$, the input $u_0 = 0.05$ and the following transition rate matrix

$$\begin{bmatrix} -0.6 & 0.5 & ? & ? \\ ? & -0.1 & 0.05 & ? \\ ? & ? & ? & 0.05 \\ 0.05 & ? & ? & -0.2 \end{bmatrix}.$$

By using the Theorem 3.4, we derive that

$$A_{a1} = \begin{bmatrix} -3.6724 & -370.0899 & 0.7034 \\ 400.0015 & -2.1444 & 4.1037 \\ -0.8996 & -5.3571 & -3.1314 \end{bmatrix} \tag{4.1}$$

$$A_{a2} = \begin{bmatrix} -2.8648 & -501.3760 & -0.2201 \\ 405.6848 & -2.5844 & 13.0015 \\ 0.3211 & -13,9987 & -2.0035 \end{bmatrix} \tag{4.2}$$

$$B_{a1} = \begin{bmatrix} -0.0173 \\ -0.0085 \\ -0.0001 \end{bmatrix} \ B_{a2} = \begin{bmatrix} -0.0071 \\ -0.0149 \\ -0.0001 \end{bmatrix} \ B_{a3} = \begin{bmatrix} -0.0062 \\ -0.0161 \\ -0.0001 \end{bmatrix} \ B_{a4} = \begin{bmatrix} -0.0164 \\ -0.0075 \\ -0.0001 \end{bmatrix}$$

$$C_{a1} = \begin{bmatrix} 17.2335 & 0.6817 & -19.3071 \end{bmatrix} \ C_{a2} = \begin{bmatrix} 68.4433 & -48.3066 & -2.1287 \end{bmatrix}$$

$$C_{a3} = \begin{bmatrix} 68.7359 & -49.2900 & -2.1654 \end{bmatrix} \ C_{a4} = \begin{bmatrix} 17.2752 & 0.6631 & -18.7779 \end{bmatrix}$$

$$D_{a1} = \ 0.2040 \ \ D_{a2} = \ 0.1881 \ \ D_{a3} = \ 0.1894 \ \ D_{a4} = \ 0.1966.$$

Numerical simulation examples show that the state of $x(t)$ in open-loop systems and closed-loop systems within a transition jump rates are asymptotically finite-time stable, respectively.

5. Conclusion

We consider the robust stability of discrete generalized Markov systems at the general transfer rate, and obtain the stochastic admissibility conditions based on the linear matrix inequalities, the simulation results had been illustrated the effectiveness.

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Manuscript received June 5, 2023
revised December 29, 2023

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