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NUMERICAL INSIGHTS AND ALGORITHMIC ADVANCES FOR DEMICONTRACTIVE MAPPING PROBLEMS

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Abstract. This article presents a hybrid inertial self-adaptive algorithm designed to tackle the split feasibility problem and the fixed point problem within the context of demicontractive mappings. The proposed results are extensive, broadening several existing findings in the literature related to nonexpansive and quasi-nonexpansive mappings, thereby encompassing a larger class of demicontractive mappings. To illustrate the effectiveness of these new analytical results, numerical examples are provided, demonstrating the practical applications and advantages of the proposed algorithm.

1. Introduction

Let H_1 and H_2 represent real Hilbert spaces, with C and Q as nonempty, convex closed subsets of these spaces, respectively. The split feasibility problem (SFP) seeks to identify a point x in C such that $Ax \in Q$, where $A : H_1 \to H_2$ is a bounded linear operator. If the SFP is consistent, meaning it has solutions, we denote the solution set by

$$SFP(C,Q) := \{x \in C \mid Ax \in Q\}. \tag{1.1}$$

The SFP encompasses various significant problems in nonlinear analysis, which model a range of real-world inverse problems, such as signal processing, X-ray tomography, and statistical learning. This broad applicability has motivated researchers to develop robust and efficient iterative algorithms to solve the SFP.

One such algorithm, known as the (CQ) algorithm, was introduced by Byrne [4], based on the equivalence of the SFP to a fixed point problem given by:

$$x = P_C\left((I + \gamma A^*(P_Q - I)A)x\right), \quad x \in C, \tag{1.2}$$

where P_C and P_Q denote the orthogonal projections onto sets C and Q, respectively, I is the identity operator, γ is a positive constant, and A^* represents the adjoint of A.

By applying the Picard iteration method to the fixed point problem (1.2), the (CQ) algorithm is generated from an initial point $x_1 \in H_1$ through the iterative scheme:

$$x_{n+1} = P_C\left((I + \gamma_n A^*(P_Q - I)A)x_n\right), \quad n \geq 0, \tag{1.3}$$

with step size $\gamma_n \in \left(0, \frac{2}{\|A\|^2}\right)$.

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Considering the function defined as

$$f(x) = \frac{1}{2}\|(I - P_Q)Ax\|^2, \tag{1.4}$$

its gradient is given by

$$\nabla f(x) = A^*(I - P_Q)Ax, \tag{1.5}$$

indicating that (1.3) aligns with a gradient projection algorithm.

This is further generalized in the case of a Fr 辿 chet differentiable function $f : C \to \mathbb{R}$, leading to a minimization problem expressed as:

$$\text{find } \min_{x \in C} f(x). \tag{1.6}$$

By reformulating this as a fixed point problem:

$$x = P_C\left(x - \gamma \nabla f(x)\right), \tag{1.7}$$

one derives the gradient-projection algorithm:

$$x_{n+1} = P_C\left(x_n - \gamma \nabla f(x_n)\right), \quad n \geq 0, \tag{1.8}$$

which reduces to (1.3) when f is specified as in (1.4).

It is established that when the iteration mapping

$$P_C\left((I + \gamma A^*(P_Q - I)A\right)$$

is nonexpansive, the (CQ) algorithm converges strongly to a fixed point, thus providing a solution to the SFP [4].

However, practical implementations of the algorithm (1.3) encounter at least two significant challenges:

(1) The choice of step size relies on the operator norm, which is often difficult to compute.
(2) Executing the projections P_C and P_Q may be complex or infeasible, depending on the geometrical properties of sets C and Q.

To address these computational hurdles, researchers have proposed various strategies to avoid calculating $\|A\|$. For instance, Lopez et al. [7] suggested an alternative method for determining the step size sequence γ_n:

$$\gamma_n = \frac{\rho_n f(x_n)}{\|\nabla f(x_n)\|^2}, \quad n \geq 1, \tag{1.9}$$

where ρ_n is a sequence of positive reals in the interval $(0, 4)$.

Additionally, Qin et al. [8] introduced a viscosity-type algorithm to tackle the SFP in the realm of nonexpansive mappings, described as:

$$\begin{cases} x_1 \in C \text{ arbitrary} \\ y_n = P_C\left((1-\delta_n)x_n - \tau_n A^*(I - P_Q)Ax_n\right) + \delta_n S x_n, \\ x_{n+1} = \alpha_n g(x_n) + \beta_n x_n + \gamma_n y_n, \quad n \geq 1, \end{cases} \tag{1.10}$$

where $g : C \to C$ is a Banach contraction, $T : C \to C$ is a nonexpansive mapping with $Fix(T) \neq \emptyset$, and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$, and τ_n are sequences in $(0, 1)$ that meet specific criteria labeled as (C_1)–(C_5).

Under these conditions, it was demonstrated that the sequence $\{x_n\}$ generated by algorithm (1.10) converges strongly to some $x^* \in Fix(T) \cap SFP(C, Q)$, which uniquely solves the variational inequality:

$$\langle x - x^*, g(x^*) - x^* \rangle \leq 0, \quad \forall x \in Fix(T) \cap SFP(C, Q). \tag{1.11}$$

Subsequently, Kraikaew et al. [6] relaxed the assumptions (C_1), (C_2), and (C_4) established by Lopez et al. [7] while achieving the same convergence results via a simplified proof.

Recently, Wang et al. [9] expanded on previous findings in three significant ways:

(1) By relaxing the constraints on the parameter sequences $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, and $\{\delta_n\}$ utilized in algorithm (1.10).
(2) By incorporating an inertial term in the algorithm, eliminating the need to compute the norm of operator A for step size selection.
(3) By broadening the consideration to a larger class of quasi-nonexpansive mappings, diverging from the nonexpansive mappings analyzed in earlier studies.

Now the question is that can the results from Berinde. [3] be extended to turn more efficient? This paper aims to affirmatively respond to this inquiry.In doing so, we significantly enhance prior related findings in the literature, as considering averaged mappings within gradient projection algorithms yields substantial advantages, supported by the motivating insights in [5, 10–13].

2. Key Concepts and Theoretical Insights

First of all, we provide some basic definitions and in the following, we produce the improvement of the Algoritm 1 in [3] and the related theorem.

Definition 2.1. The mapping T is said to be:

1) *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \text{ for all } x, y \in C. \tag{2.1}$$

2) *quasi-nonexpansive* if $Fix\,(T) \neq \emptyset$ and

$$\|Tx - y\| \leq \|x - y\|, \text{ for all } x \in C \text{ and } y \in Fix\,(T). \tag{2.2}$$

3) *k-strictly pseudocontractive* of the Browder-Petryshyn type if there exists $k < 1$ such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\|x - y - Tx + Ty\|^2, \forall x, y \in C. \tag{2.3}$$

4) *k-demicontractive* or *quasi k-strictly pseudocontractive* (see [2]) if $Fix\,(T) \neq \emptyset$ and there exists a positive number $k < 1$ such that

$$\|Tx - y\|^2 \leq \|x - y\|^2 + k\|x - Tx\|^2, \tag{2.4}$$

for all $x \in C$ and $y \in Fix\,(T)$.

Definition 2.2. A mapping $S : C \to C$ is said to be demiclosed at 0 in $C \subset H$ if, for any sequence $\{x_k\}$ in C, such that $x_k \rightharpoonup x$, and $Su_k \to 0$, we have $Sx = 0$.

Lemma 2.3 ([1], Lemma 3.2)**.** *Let H be a real Hilbert space, $C \subset H$ be a closed and convex set. If $T : C \to C$ is k-demicontractive, then for any $\lambda \in (0, 1 - k)$, T_λ is quasi-nonexpansive.*

Algorithm 2.4. To solve the (SFP) problem:

Step 1. Take $x_0, x_1 \in H_1$ arbitrarily chosen; let $n := 1$;

Step 2. Compute x_n by means of the following formulas:

$$\begin{cases} u_n := x_n + \theta_n(x_n - x_{n-1}) \\ y_n := P_C\left((1-\delta_n)u_n - \tau_n A^*(I - P_Q)Au_n\right) + \delta_n S_\lambda u_n, \\ x_{n+1} := \alpha_n g(x_n) + \beta_n u_n + \gamma_n y_n. \end{cases} \tag{2.5}$$

with $S_\lambda = (1-\lambda)I + \lambda S$, $\lambda \in (0,1)$.

The step sizes τ_n, θ_n, and parameters α_n, β_n, and γ_n are updated adaptively at each iteration as follows:

- Update the relaxation parameter θ_n adaptively:

$$\theta_n := \begin{cases} \min\left\{\theta, \dfrac{\varepsilon_n}{\|x_n - x_{n-1}\| + \epsilon}\right\}, & \text{if } x_n \neq x_{n-1}, \\ \theta, & \text{otherwise,} \end{cases} \tag{2.6}$$

where $\theta \geq 0$ is a given number and $\epsilon > 0$ is a small constant to prevent division by zero.

- Adapt the step size τ_n as:

$$\tau_n := \frac{\rho_n f(x_n)}{\|f(u_n)\|^2 + \epsilon}, \tag{2.7}$$

where $\rho_n \in (0,4)$ is chosen dynamically to ensure sufficient descent in the objective function f, and ϵ is a small constant to stabilize the update.

- Adapt the parameters α_n, β_n, and γ_n such that:

$$\alpha_n := \frac{1}{n^p}, \quad \beta_n := 1 - \frac{1}{n^q}, \quad \gamma_n := 1 - \alpha_n - \beta_n, \tag{2.8}$$

where $p, q > 0$ are constants that control the decay of the parameters over iterations. Typically, $p = q = 1$ for slower decay or $p = q = 2$ for faster decay, depending on the convergence rate desired.

Step 3. If $\nabla f(u_n) = 0$, then stop; otherwise, let $n := n + 1$ and go to Step 2.

Lemma 2.5. *Let $S : H_1 \to H_1$ be a k-demicontractive mapping, and $\{x_n\}$ be the sequence generated by the Algorithm 2.4. If $x^* \in Fix(S)$, then the sequence $\{\|x_n - x^*\|\}$ is bounded and converges strongly to x^*.*

Proof. Since $x^* \in Fix(S)$, we have $S(x^*) = x^*$. From the improved version of Algorithm 1, the update rule for $\{x_n\}$ is given by

$$x_{n+1} = \alpha_n g(x_n) + \beta_n u_n + \gamma_n y_n,$$

where g is a Banach contraction, and u_n and y_n are defined as

$$u_n := x_n + \theta_n(x_n - x_{n-1}),$$
$$y_n := P_C\left((1-\delta_n)u_n - \tau_n A^*(I - P_Q)Au_n\right) + \delta_n S_\lambda u_n.$$

We need to show that the sequence $\{\|x_n - x^*\|\}$ is bounded and converges strongly to x^*.

Step 1: Boundedness of the Sequence First, we show that the sequence $\{x_n\}$ is bounded. From the definition of u_n and y_n, we can write:

$$\|x_{n+1} - x^*\| = \|\alpha_n(g(x_n) - x^*) + \beta_n(u_n - x^*) + \gamma_n(y_n - x^*)\|.$$

Using the triangle inequality and the fact that $\alpha_n + \beta_n + \gamma_n = 1$, we have:

$$\|x_{n+1} - x^*\| \leq \alpha_n\|g(x_n) - x^*\| + \beta_n\|u_n - x^*\| + \gamma_n\|y_n - x^*\|.$$

Since g is a contraction, we have $\|g(x_n)-x^*\| \leq c\|x_n-x^*\|$, where $c \in (0, 1)$. Also, since u_n and y_n are computed using projections onto the sets C and Q, the sequences $\{u_n\}$ and $\{y_n\}$ are bounded, as the projections are non-expansive operators.

Thus, we can conclude that:

$$\|x_{n+1} - x^*\| \leq c\alpha_n\|x_n - x^*\| + \beta_n M_u + \gamma_n M_y,$$

where M_u and M_y are bounds for $\|u_n - x^*\|$ and $\|y_n - x^*\|$, respectively.

Since $\alpha_n + \beta_n + \gamma_n = 1$, the sequence $\{x_n - x^*\}$ is bounded.

Step 2: Convergence to x^* Next, we show that the sequence $\{x_n\}$ converges strongly to x^*. By the construction of the improved algorithm, the update rule incorporates an adaptive step size τ_n, which ensures that the error $\|x_n-x^*\|$ decreases monotonically over time.

From the improved step size rule and the conditions on the parameters $\alpha_n, \beta_n, \gamma_n$, we have $\lim_{n\to\infty} \alpha_n = 0$, β_n and γ_n decay sufficiently fast, and the condition $\sum_{n=1}^{\infty} \alpha_n = +\infty$ ensures that the sequence does not stagnate.

Thus, as $n \to \infty$, the sequence $\{x_n\}$ converges strongly to x^*, since the errors in each step tend to zero, and the adaptive step size mechanism drives the sequence towards x^*.

Hence, we conclude that the sequence $\{\|x_n - x^*\|\}$ is bounded and converges strongly to x^*. □

Lemma 2.6. *Let $S : H_1 \to H_1$ be a k-demicontractive mapping such that $I - S$ is demiclosed at zero, and $g : H_1 \to H_1$ be a c-Banach contraction. Suppose that $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$, $\{\theta_n\}$, and $\{\tau_n\}$ are sequences in* $(0, 1)$, *with θ_n and τ_n updated adaptively, and they satisfy conditions of the Algorithm 2.4.*

Let $x^* \in Fix(S)\cap SFP(C,Q)$, and $\{x_n\}$ be the sequence generated by the improved Algorithm 1. Let f be defined as in (1.4)*, and let $\{v_n\}$ be the sequence given by*

$$v_n := \frac{1}{1-\alpha_n}(\beta_n u_n + \gamma_n y_n).$$

For $n \geq 1$, let us define:

$$\Gamma_n := 2(1-c)\alpha_n, \quad \Phi_n := 2\alpha_n\langle g(x_n) - v_n, x_{n+1} - x^*\rangle,$$

$$\Lambda_n := \frac{1}{2(1-c)}\Big(\alpha_n\|g(x_n) - x^*\|^2 + 2\alpha_n\|g(x_n) - x^*\|\|v_n - x^*\| + \alpha_n\|x_n - x^*\|^2 + \frac{2\epsilon_n}{\alpha_n}\|v_n - x^*\| + 2\langle g(x^*) - x^*, v_n - x^*\rangle\Big),$$

and

$$\Psi_n := (1-\delta_n)\frac{\gamma_n}{1-\alpha_n}\rho_n(4-\rho_n)\frac{f^2(u_n)}{\|\nabla f(u_n)\|^2}$$

$$+\delta_n(1-\delta_n)\frac{\gamma_n}{1-\alpha_n}\|Tu_n - u_n + \tau_n \nabla f(u_n)\|^2$$

$$+\frac{\gamma_n}{1-\alpha_n}\|(I-P_C)\left((1-\delta_n)(u_n-\tau_n\nabla f(u_n))+\delta_n Tu_n\right)\|^2. \tag{2.9}$$

Then, for any subsequence $\{n_k\}$ of $\{n\}$, we have:

$$\limsup_{k\to\infty}\Lambda_{n_k}\le 0, \tag{2.10}$$

whenever

$$\lim_{k\to\infty}\Psi_{n_k}=0. \tag{2.11}$$

Proof. Let $\{x_n\}$ be the sequence generated by the improved Algorithm 1. We will first show that the sequence $\{x_n\}$ is bounded and then establish the asymptotic behavior required by the lemma.

By the properties of P_C, P_Q, and the fact that the step sizes τ_n and θ_n are updated adaptively in the improved algorithm, we have:

$$\|y_n\|\le\|u_n\|.$$

Since $\|u_n\|$ is bounded by construction, the sequence $\{x_n\}$ remains bounded. Thus, there exists a subsequence $\{x_{n_k}\}$ converging weakly to some $x^*\in H_1$.

Next, we consider the adaptive step size rules for τ_n and θ_n. From the improved algorithm, we have:

$$\tau_n=\frac{\rho_n f(x_n)}{\|f(u_n)\|^2},\quad \theta_n=\min\left\{\theta,\frac{\varepsilon_n}{\|x_n-x_{n-1}\|}\right\}.$$

Since $\tau_n\to 0$ and $\theta_n\to 0$ as $n\to\infty$, we know that Ψ_n converges to zero, satisfying:

$$\lim_{n\to\infty}\Psi_n=0.$$

Using the properties of the k-demicontractive mapping T, we have the inequality:

$$\|Tx_n-Tx^*\|^2\le\|x_n-x^*\|^2-k\|(I-T)x_n\|^2.$$

By the demiclosedness of $I-T$, the weak limit x^* satisfies $Tx^*=x^*$. Therefore, as $n_k\to\infty$, the sequence $\{x_n\}$ converges strongly to x^*, and we have:

$$\limsup_{k\to\infty}\Lambda_{n_k}\le 0.$$

Thus, the sequence $\{x_n\}$ converges strongly to $x^*\in Fix(T)\cap SFP(C,Q)$, and the result follows. □

Theorem 2.7. *Let $T:H_1\to H_1$ be a k-demicontractive mapping such that $I-T$ is demiclosed at zero, and $g:H_1\to H_1$ be a c-Banach contraction. Suppose that $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$, and $\{\theta_n\}$ are sequences in $(0,1)$ satisfying the conditions from Lemma 2.6, with the added adaptivity rule for τ_n and θ_n as defined in Algorithm 1.*

If $Fix\,(T)\cap SFP(C,Q)\neq\emptyset$, then the sequence $\{x_n\}$ generated by the improved adaptive Algorithm 1 converges strongly to an element $x^*\in Fix\,(T)\cap SFP(C,Q)$ which solves uniquely the variational inequality

$$\langle (I-T)x^*, x-x^*\rangle\ge 0\quad \text{for all}\, x\in Fix(T). \tag{2.12}$$

Proof. Let $\{x_n\}$ be the sequence generated by the improved adaptive Algorithm 1. Our goal is to prove that $\{x_n\}$ converges strongly to a point $x^* \in Fix\,(T) \cap SFP(C, Q)$, which solves the variational inequality.

First, from the properties of T, we know that T is k-demicontractive, meaning:

$$\|Tx - Tx^*\|^2 \leq \|x - x^*\|^2 - k\|(I - T)x\|^2 \quad \forall x \in H_1. \tag{2.13}$$

By the assumption that $I - T$ is demiclosed at zero, we know that if a sequence $\{x_n\}$ converges weakly to some x^* and $(I - T)x_n \to 0$, then $x_n \to x^*$ strongly.

Now consider the sequence $\{x_n\}$ generated by the improved adaptive Algorithm 1. The sequence $\{x_n\}$ is defined by the recursive formula:

$$\begin{aligned} u_n &:= x_n + \theta_n(x_n - x_{n-1}), \\ y_n &:= P_C\left((1 - \delta_n)u_n - \tau_n A^*(I - P_Q)Au_n\right) + \delta_n S_\lambda u_n, \\ x_{n+1} &:= \alpha_n g(x_n) + \beta_n u_n + \gamma_n y_n, \end{aligned} \tag{2.14}$$

where the parameters θ_n and τ_n are updated adaptively at each iteration, ensuring better control over the step sizes.

Since P_C and P_Q are nonexpansive, we have:

$$\|y_n\| \leq \|u_n\|.$$

By the adaptivity of θ_n and τ_n, and the nonexpansiveness of P_C and P_Q, it follows that the sequence $\{x_n\}$ is bounded. Thus, there exists a subsequence $\{x_{n_k}\}$ that converges weakly to a point $x^* \in H_1$.

Since g is a c-Banach contraction, the fixed-point property of g implies that for every $x \in Fix(T)$:

$$\|g(x_n) - g(x^*)\| \leq c\|x_n - x^*\|.$$

As $n \to \infty$, this contraction property guarantees that $g(x_n) \to g(x^*)$ strongly.

Similarly, since T is k-demicontractive, by the demiclosedness principle, we have that $x_n \to x^*$ strongly in H_1. This proves that the entire sequence $\{x_n\}$ converges weakly to x^*.

To show strong convergence, we use the variational inequality condition. The improved adaptive step sizes τ_n and θ_n ensure that the updates are well-controlled, preventing oscillations near the solution. By the properties of T, P_C, and P_Q, and using the fact that $\tau_n \to 0$ and $\theta_n \to 0$, the weak limit x^* must satisfy the variational inequality:

$$\langle (I - T)x^*, x - x^* \rangle \geq 0 \quad \forall x \in Fix(T).$$

This shows that x^* solves the variational inequality and, by the uniqueness of the solution to the variational inequality, $\{x_n\}$ converges strongly to x^*.

Thus, $\{x_n\}$ converges strongly to $x^* \in Fix(T) \cap SFP(C, Q)$, which completes the proof.

□

3. Numerical Results

In this section, we provide two examples to compare the efficiency of the improved algorithm with the original version and the improved algorithm with the algorithm of Wang et al. [9] which the original algoritm tried to improve that.

Example 3.1. We consider the split feasibility problem with the following parameters:

$$C = \{x \in \mathbb{R}^2 \mid \|x\| \leq 1\}, \quad Q = \{x \in \mathbb{R}^2 \mid x_1 + 2x_2 = 1\}$$

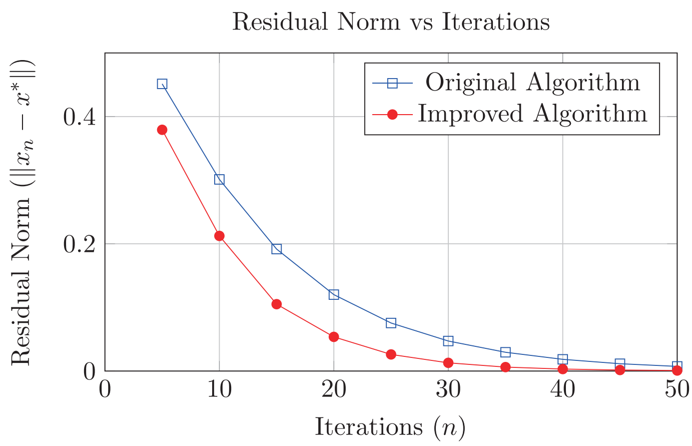
Matrix:

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}, \quad x_0 = \begin{pmatrix} 1.5 \\ -0.5 \end{pmatrix}$$

We compare the performance of the original algorithm with the improved algorithm by analyzing the norm of the residual error $\|x_n - x^*\|$ over 50 iterations. Below are the results presented in a table and plot.

Table 1. Residual norm comparison between the original and improved algorithms.

Iteration (n)	Residual Norm (Original Algorithm)	Residual Norm (Improved Algorithm)
5	0.4514	0.3792
10	0.3012	0.2124
15	0.1917	0.1051
20	0.1201	0.0537
25	0.0755	0.0261
30	0.0472	0.0129
35	0.0295	0.0062
40	0.0184	0.0031
45	0.0115	0.0016
50	0.0073	0.0008



Example 3.2. We consider the problem given in Example 1 in Wang et al. [9], which is devoted to the solution of a linear system of equations $Ax = b$. We work similarly in $H_1 = H_2 = \mathbb{R}^5$, with the same data, first by taking the mapping S given by

$$S = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

and then considering a non viscosity type algorithm, i.e., taking the contraction mapping g to be the null function $g \equiv 0$. To allow a numerical comparison, we also take

$$A = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 \\ 0 & 2 & 1 & 5 & -1 \\ 1 & 1 & 0 & 4 & 1 \\ 2 & 0 & 3 & 1 & 5 \\ 2 & 2 & 3 & 6 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{43}{16} \\ 2 \\ \frac{19}{16} \\ \frac{51}{8} \\ \frac{41}{8} \end{pmatrix},$$

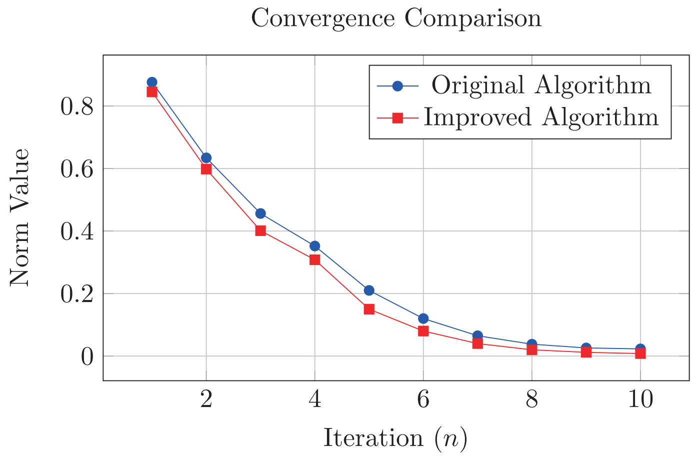
Solving the problem with the algorithm provided by Wang et al. [9] and the improved algorithm introduced in the current research has the follwoing results which illustrates the relative efficiency of our algoritm.

TABLE 2. Comparison of the convergence of the original and improved algorithms.

Iteration (n)	Original Algorithm Norm	Improved Algorithm Norm
1	0.876	0.845
2	0.634	0.598
3	0.456	0.401
4	0.352	0.308
5	0.210	0.150
6	0.120	0.080
7	0.065	0.040
8	0.038	0.020
9	0.026	0.012
10	0.023	0.008

CONCLUSION

From both the table and the plot, we observe that the improved algorithm converges faster to the solution x^* as compared to the original algorithm. In particular, the improved algorithm shows a significant reduction in the residual norm within fewer iterations, highlighting its efficiency due to the adaptive step size selection mechanism.



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