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TWO-STAGE SUPPLY CHAIN DYNAMIC PRICING AND COORDINATION STRATEGY CONSIDERING CONSUMERS' EXPECTED REGRET

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Abstract. Considering consumers expected regret, a two-stage dynamic game model of a two-stage supply chain composed of manufacturers, retailers and strategic consumers is established. The effects of consumers' strategic degree, consumers' high price regret and out of stock regret on the two-stage equilibrium results are discussed. Furthermore, a two-stage revenue sharing contract is designed to achieve the supply chain coordination. Finally, the following results are obtained: as the consumers' strategic degree increase, consumers often tend to wait until the discount stage to buy products; for high regret consumers, manufacturers can take a quality commitment strategy, and retailers adopt a price commitment strategy; for out of stock regret consumers, manufacturers and retailers should strengthen the relationship between each other, and pay more attention to the inventory level of products or make up for consumers in a certain way.

1. Introduction

In response to an incentive competitive market, companies often implement discount products in the promotion festivals, such as "Double 11", "Double 12" and "618". Many consumers tend to hoard goods during these periods and try to delay buying until the discount period. The purpose is to buy the most cost-effective products at the lowest price, and then expand their input-output ratio. However, in practice, they may lose buying opportunities because of limited inventory, which lead to regret. On the other hand, consumers buy products in the original sales stage, but the price of products in this stage is generally greater than that in discount sales stage. Then, consumers may regret the expensive purchase. The above two distinct types of regret in the market are called out of stock regret and high price regret respectively.

In this paper, there are primary concerns with expected regrets, strategic consumers and two stage supply chain.

(i) Expected regrets. Choosing different purchase stages implies that consumers may regret about their purchase decision. Consumers can make trade-offs to minimize their expected regrets. Many studies show that consumers' expected regret can significantly affect the decision-making of enterprises. For example, by considering the uncertainty of consumers' purchase behavior, Jiang, Narasimhan and

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Turut [5] analyze whether and how expected regret affects the profits and product innovation of competitive enterprises. Yang, Wand and Ang [15] present the impact of expected regret on decision-making in closed-loop supply chain via considering the uncertainty of consumers' valuation of re-manufactured products. Under the pre-sale environment with uncertain consumer valuation, Nasiry and Popescu [10] propose the impact of expected regret on consumer decisions, corporate profits and policies. Ratan [13] finds that expected regret will prompt consumers to choose safe options. Li, Reb and Bagger [6] discuss the role of expected regret in time-based work family conflict decision-making. Comparing the price reduction rate and the product percentage, Zhou and Gu [17] study how expected regret affects consumers' impulse purchase intention. Under price discount situation, Jian et al. [4] propose the impact of anticipated regret in the supply chain decision-making and design a revenue-sharing contract to coordinate the profits of supply chain.

(ii) Strategic consumers. As is well-known, more and more information available to consumers, they can make more rational decisions. It means that strategic consumers account for a larger proportion of consumers who often do not choose to buy at the moment, but wait and see to postpone the purchase. The research on the strategic consumers have attracted many authors. For example, Li, Granados and Netessine [7] provide empirical evidence for strategic consumers and their purchase behavior. Zhang and Zhang [16] establish a two-stage model to study the mitigation effect of pricing commitment policy and most favored customer protection policy on disappointment, aversion and value reduction of strategic consumers' purchase behavior. Prasad, Venkatesh and Mahajan [12] present a comparative analysis on the mixed bundling sales and retention product pricing sales adopted by enterprises respectively. When a new product is introduced, Liang, Cakanyildirim and Sethi [8] discuss the interaction between the two strategies and the purchase behavior of strategic consumers whether the old product remained in the market. Considering the strategic waiting behavior of consumers, Liu and Huang [9] analyze the inventory and dynamic pricing strategies of enterprises. Wang, Fu and Yu [14] divide consumers into short-sighted and strategic types, and then discuss the pricing and inventory strategies of enterprises.

(iii) Two-stage supply chain. Due to the different price of the original sales and discount sales stages in the supply chain, consumers will compare them and decide in which stages to buy. Özer and Zheng [11] study the two-stage optimal pricing and inventory strategy of the seller when the behavior motivation affects the purchase decision of consumers. Considering that consumers' valuation of products is strategic and heterogeneous, Adida and Özer [1] analyze the two-stage pricing problem of retailers based on expected regret. In the two stage pre-sale model, Diecidue, Rudi and Tang [3] consider the regret caused by forward purchase and spot purchase, and propose that how the regret affects the purchase decision.

Therefore, considering the degree of strategy and expected regret of consumers, a two-stage dynamic game model of the supply chain is proposed in this paper. Then, the pricing decisions of manufacturers and retailers are explored. Furthermore, by using revenue-sharing contract [2], a coordination mechanism to achieve Pareto optimality of the supply chain is designed. These would promote the supply chain system to achieve mutual benefit and win-win. In addition, different from the existing results, this paper presents a two-stage dynamic game model which combines expected regret theory. The effects of consumers' strategic degree, consumers' high price regret and out of stock regret on profit difference, price difference and strategic behavior threshold difference between the original and discount sales stages are simultaneously taken into consideration.

2. Problem description

In this paper, p_1 and p_2 are used to denote the selling price of the original product and the discount product per unit respectively. Note that, $p_1 \geq p_2$. Furthermore, consumers have valuation heterogeneity, whose valuation for the original product is V and obeys the uniform distribution on $[0, V_1]$. Its probability density function and cumulative distribution function are $f(v)$ and $F(v)$, respectively. Now, assume that N represents the total number of consumers in the market.

The strategic consumers compare the utility of original and discount sales stages, and then make a decision-making. No matter what choice they choose, consumers may regret high prices or lack of products. High price regret means that consumers buy products in the original sales stage, and the price of products in the discount sales stage is lower than the original sales stage which lead to consumers regret the high price they paid. On the other hand, out of stock regret implies that consumers wait for the discount sales stage to chase products, but may regret for missing the purchase opportunity. In general, assume that the high price regret α is less than the out of stock regret β, i.e. $\alpha < \beta$.

Let δ be the consumers strategic degree. The demand functions of the expected regret of consumers who purchase the original products and the discount products are proposed as follows:

$$\begin{aligned} U_1 &= (V - p_1)^+ - \delta\alpha\,(p_1 - p_2) \\ U_2 &= \delta(V - p_2)^+ - (1 - \delta)\,\beta(V - p_1)^+ \end{aligned}$$

The consumers will purchase the original price products when the utility of original sales stage is greater than that of the discount sales stage, i.e. $U_1 \geq U_2$ and $\mathrm{v} \geq p_1$ are satisfied. Otherwise, they will buy the discount products. Let V_0 be the utility of a consumer who is indifferent between in original sales and discount sales stages. Hereinafter, it is called the strategic behavior threshold of consumers. Then, we have

$$V_0 = \frac{\delta p_1\,(\alpha - \beta) - \delta p_2\,(1 + \alpha) + p_1\,(1 + \beta)}{(1 - \delta)\,(1 + \beta)}. \tag{2.1}$$

Moreover, the demand function of retailer in the two-stage supply chain can be obtained:

$$Q_1 = NP_r\,(\mathrm{v} - V_0 \geq 0) = N \int_{V_0}^{V_1} f\,(v)\,dv = N\left(1 - \frac{V_0}{V_1}\right),$$

$$Q_2 = NP_r\,(p_2 \leq \mathrm{v} < V_0) = N \int_{p_2}^{V_0} f\,(v)\,dv = \frac{N\,(V_0 - p_2)}{V_1}.$$

Table 1. Symbols and meanings

Symbol	Description
p_1	Retail price of original sales stage
p_2	Retail price of discount stage
w_1	Wholesale price of original sales stage
w_2	Wholesale price of discount stage
α	High price regret
β	Out of stock regret
δ	Consumers strategic degree
V	Valuation for the original product
V_1	Maximum valuation for the original product
N	The total number of consumers in the market

3. Centralized and decentralized decision-making model

Using the demand function of original price products and discount products, the profit of manufacturers and retailers are obtained as follows:

$$\pi_M(w_1, w_2) = (w_1 - c) N \left(1 - \frac{V_0}{V_1}\right) + (w_2 - c) N \left(\frac{V_0}{V_1} - \frac{p_2}{V_1}\right),$$

$$\pi_R(p_1, p_2) = (p_1 - w_1) N \left(1 - \frac{V_0}{V_1}\right) + (p_2 - w_2) N \left(\frac{V_0}{V_1} - \frac{p_2}{V_1}\right).$$

3.1. Centralized decision-making model.

In the centralized decision-making situation, manufacturers and retailers are a whole system, and then maximize the overall total profit of the supply chain. The total profit of two-stage supply chain under centralized decision-making is stated as follows:

$$\pi^C(p_1, p_2) = (p_1 - c) N \left(1 - \frac{V_0}{V_1}\right) + (p_2 - c) N \left(\frac{V_0}{V_1} - \frac{p_2}{V_1}\right). \tag{3.1}$$

Derivate p_2 in (3.1), and then obtain that

$$P_2 = \frac{V_0 + c}{2}. \tag{3.2}$$

Let $A = \delta(\alpha - \beta) + 1 + \beta$. Combining (2.1) and (3.2), we can get the equilibrium price p_2, i.e.

$$P_2^{C*} = \frac{(2V_1 + 4c) A - (V_1 + 3c) \delta (1 + \alpha)}{2[3A - 2\delta(1 + \alpha)]}. \tag{3.3}$$

Using derivation and substitution, we can obtain the equilibrium solutions of other variable in the supply chain system, i.e.

$$V_0^{C*} = \frac{(2V_1 + c) A - (V_1 + c) \delta (1 + \alpha)}{3A - 2\delta(1 + \alpha)},$$

$$P_1^{C*} = \frac{(4V_1 + 2c) A^2 - 4A\delta(1 + \alpha) V_1 + (V_1 - c) \delta^2 (1 + \alpha)^2}{2A[3A - 2\delta(1 + \alpha)]},$$

$$\pi_1^{C*} = N(V_1 - c)^2 \frac{4A^3 - 8A^2\delta\,(1+\alpha) + 5A\delta^2(1+\alpha)^2 - \delta^3(1+\alpha)^3}{2A[3A - 2\delta\,(1+\alpha)]^2 V_1},$$

$$\pi_2^{C*} = N(V_1 - c)^2 \frac{4A^2 - 4A\delta\,(1+\alpha) + \delta^2(1+\alpha)^2}{4[3A - 2\delta\,(1+\alpha)]^2 V_1},$$

$$\pi^{C*} = N(V_1 - c)^2 \frac{4A^2 - 4A\delta\,(1+\alpha) + \delta^2(1+\alpha)^2}{4A[3A - 2\delta\,(1+\alpha)]^2 V_1}.$$

3.2. **Decentralized decision-making model.** In the decentralized decision-making situation, manufacturers and retailers in the supply chain tend to perform their own duties, focus on themselves, and seek benefits as the ultimate goal of their decisions. Traditionally, manufacturers dominate retailers in the supply chain. That is, the relationship between manufacturers and retailers is a Stackelberg game in which manufacturers are the leader and retailers are the follower.

The total profit of manufacturers under decentralized decision-making is stated as follows:

$$\pi_M\,(w_1, w_2) = (w_1 - c)\,N\left(1 - \frac{V_0}{V_1}\right) + (w_2 - c)\,N\left(\frac{V_0}{V_1} - \frac{p_2}{V_1}\right). \tag{3.4}$$

The total profit of retailers under decentralized decision-making is written as follows:

$$\pi_R\,(p_1, p_2) = (p_1 - w_1)\,N\left(1 - \frac{V_0}{V_1}\right) + (p_2 - w_2)\,N\left(\frac{V_0}{V_1} - \frac{p_2}{V_1}\right). \tag{3.5}$$

Using the inverse induction method, it follows from (3.4) and (3.5) that the equilibrium solutions of each variable in the supply chain system can be obtained as follows:

$$w_1^{D*} = \frac{(121V_1 + 103c)\,A^2 - (62V_1 + 2c)\,A\delta\,(1+\alpha) + 8\,(V_1 - c)\,\delta^2(1+\alpha)^2}{224A^2 - 64A\delta\,(1+\alpha)},$$

$$w_2^{D*} = \frac{(23V_1 + 33c)\,A - (6V_1 + 10c)\,\delta\,(1+\alpha)}{56A - 16\delta\,(1+\alpha)},$$

$$P_1^{D*} = \frac{(92V_1 + 20c)\,A^2 - (47V_1 - 15c)\,A\delta\,(1+\alpha) + 6\,(V_1 - c)\,\delta^2(1+\alpha)^2}{112A^2 - 32A\delta\,(1+\alpha)},$$

$$P_2^{D*} = \frac{(69V_1 + 43c)\,A - (18V_1 + 14c)\,\delta\,(1+\alpha)}{112A - 32\delta\,(1+\alpha)},$$

$$V_0^{D*} = \frac{(23V_1 + 5c)\,A - (6V_1 + 2c)\,\delta\,(1+\alpha)}{28A - 8\delta\,(1+\alpha)}.$$

Then, the profit under centralized decision-making can be easily obtained:

$$\pi_{M1}^{D*} = \frac{N(V_1 - c)^2\left[121A^2 - 62A\delta\,(1+\alpha) + 8\delta^2(1+\alpha)^2\right][5A - 2\delta\,(1+\alpha)]}{128A[7A - 2\delta\,(1+\alpha)]^2 V_1},$$

$$\pi_{R1}^{D*} = \frac{N(V_1 - c)^2\left[63A^2 - 32A\delta\,(1+\alpha) + 4\delta^2(1+\alpha)^2\right][5A - 2\delta\,(1+\alpha)]}{128A[7A - 2\delta\,(1+\alpha)]^2 V_1},$$

$$\pi_{M2}^{D*} = \frac{N(V_1 - c)^2[23A - 6\delta\,(1+\alpha)]^2}{128[7A - 2\delta\,(1+\alpha)]^2 V_1},$$

$$\pi_{R2}^{D*} = \frac{N(V_1 - c)^2[23A - 6\delta\,(1+\alpha)]^2}{256[7A - 2\delta\,(1+\alpha)]^2 V_1},$$

$$\pi_{M}^{D*} = \frac{N(V_1 - c)^2\left[81A^2 - 36A\delta\,(1+\alpha) + 4\delta^2(1+\alpha)^2\right]}{64A\,[7A - 2\delta\,(1+\alpha)]\,V_1},$$

$$\pi_{R}^{D*} = \frac{N(V_1 - c)^2\left[\begin{array}{l} 1159A^3 - 848A^2\delta\,(1+\alpha) \\ +204A\delta^2(1+\alpha)^2 - 16\delta^3(1+\alpha)^3 \end{array}\right]}{256A[7A - 2\delta\,(1+\alpha)]^2 V_1}.$$

3.3. **Model comparison and analysis.**

Proposition 3.1. *Under the centralized and decentralized decision-making situations, we find that 1) If* $2132A - 6837\delta\,(1+\alpha) > 0$, $\pi_1^{C*} > \pi_1^{D*}$*; 2) If* $-1739A + 6784\delta\,(1+\alpha) < 0$, $\pi_2^{C*} < \pi_2^{D*}$*; 3) If* $2263A - 5346\delta\,(1+\alpha) > 0$, $\pi^{C*} > \pi_M^{D*} + \pi_R^{D*}$.

Proof. It follows from

$$\pi_1^{C*} - \left(\pi_{M1}^{D*} + \pi_{R1}^{D*}\right) = \frac{N(V_1 - c)^2\left[\begin{array}{l} 2132A^5 - 6837A^4\delta\,(1+\alpha) \\ +7536A^3\delta^2(1+\alpha)^2 - 3800A^2\delta^3(1+\alpha)^3 \\ +995A\delta^4(1+\alpha)^4 - 160\delta^5(1+\alpha)^5 \end{array}\right]}{64A[3A - 2\delta\,(1+\alpha)]^2[7A - 2\delta\,(1+\alpha)]^2 V_1}$$

that when $2132A - 6837\delta\,(1+\alpha) > 0$, we can obtain that $\pi_1^{C*} > \pi_1^{D*}$.

Note that,

$$\pi_2^{C*} - \left(\pi_{M2}^{D*} + \pi_{R2}^{D*}\right) = \frac{N(V_1 - c)^2\left[\begin{array}{l} -1739A^4 + 6784A^3\delta\,(1+\alpha) \\ -5928A^2\delta^2(1+\alpha)^2 + 1792A\delta^3(1+\alpha)^3 \\ -176\delta^4(1+\alpha)^4 \end{array}\right]}{256[3A - 2\delta\,(1+\alpha)]^2[7A - 2\delta\,(1+\alpha)]^2 V_1}.$$

Then, it is easy to observe that $\pi_2^{C*} < \pi_2^{D*}$ if $-1739A + 6784\delta\,(1+\alpha) < 0$.

It follows from

$$\pi^{C*} - \left(\pi_{M}^{D*} + \pi_{R}^{D*}\right) = \frac{N(V_1 - c)^2\left[\begin{array}{l} 2263A^4 - 5346A^3\delta\,(1+\alpha) \\ +4508A^2\delta^2(1+\alpha)^2 - 1464A\delta^3(1+\alpha)^3 \\ +160\delta^4(1+\alpha)^4 \end{array}\right]}{256A\,[3A - 2\delta\,(1+\alpha)]\,[7A - 2\delta\,(1+\alpha)]^2 V}$$

that when $2263A - 5346\delta\,(1+\alpha) > 0$, $\pi^{C*} > \pi_M^{D*} + \pi_R^{D*}$. The proof is completed. $\square$

Proposition 3.1 shows that, the profit of the system in the original sales stage depends on the relationship among the high price regret, out of stock regret and consumers' strategic degree under the centralized and decentralized decision-making situations. As to the discount stage, the results are the same.

Proposition 3.2. *Under the centralized and decentralized decision-making situations, the price difference and the value indifference equilibrium point difference are respectively affected by the consumers strategic degree and consumers expected regret in the two stages of the supply chain. The impact trend are shown as follows:*

1) *If* $263A - 752\delta(1+\alpha) > 0$, *then we have* $\frac{\partial\left(p_1^{D*}-p_1^{C*}\right)}{\partial\delta} > 0$, $\frac{\partial\left(V_0^{D*}-V_0^{C*}\right)}{\partial\delta} < 0$ *and* $\frac{\partial\left(p_2^{D*}-p_2^{C*}\right)}{\partial\delta} < 0$.
2) *If* $263A - 1015\delta(1+\alpha) > 0$, *then* $\frac{\partial\left(p_1^{D*}-p_1^{C*}\right)}{\partial\alpha} > 0$, $\frac{\partial\left(V_0^{D*}-V_0^{C*}\right)}{\partial\alpha} < 0$ *and* $\frac{\partial\left(p_2^{D*}-p_2^{C*}\right)}{\partial\alpha} < 0$.
3) *If* $-263A + 752\delta(1+\alpha) < 0$, *then* $\frac{\partial\left(p_1^{D*}-p_1^{C*}\right)}{\partial\beta} < 0$, $\frac{\partial\left(V_0^{D*}-V_0^{C*}\right)}{\partial\beta} > 0$ *and* $\frac{\partial\left(p_2^{D*}-p_2^{C*}\right)}{\partial\beta} > 0$.

Proof. It follows from

$$\frac{\partial p_1^{D*}}{\partial\delta} - \frac{\partial p_1^{C*}}{\partial\delta} = \frac{(V_1-c)\,\delta(1+\alpha)(1+\beta)\begin{bmatrix} 263A^4 - 752A^3\delta(1+\alpha) \\ +560A^2\delta^2(1+\alpha)^2 \\ -160A\delta^3(1+\alpha)^3 + 16\delta^4(1+\alpha)^4 \end{bmatrix}}{16A^2[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2}$$

that when $263A - 752\delta(1+\alpha) > 0$, we can find that $\frac{\partial\left(p_1^{D*}-p_1^{C*}\right)}{\partial\delta} > 0$.

It can be seen from

$$\frac{\partial p_1^{D*}}{\partial\alpha} - \frac{\partial p_1^{C*}}{\partial\alpha} = \frac{(V_1-c)\,\delta\begin{bmatrix} 263A^5 - 1015A^4\delta(1+\alpha) \\ +1312A^3\delta^2(1+\alpha)^2 - 720A^2\delta^3(1+\alpha)^3 \\ +176A\delta^4(1+\alpha)^4 - 16\delta^5(1+\alpha)^5 \end{bmatrix}}{16A^2[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2}$$

that if $263A - 1015\delta(1+\alpha) > 0$, then $\frac{\partial\left(p_1^{D*}-p_1^{C*}\right)}{\partial\alpha} > 0$.

Since

$$\frac{\partial p_1^{D*}}{\partial\beta} - \frac{\partial p_1^{C*}}{\partial\beta} = \frac{(V_1-c)(1+\alpha)(1-\delta)\begin{bmatrix} -263A^4 + 752A^3\delta(1+\alpha) \\ -560A^2\delta^2(1+\alpha)^2 + 160A\delta^3(1+\alpha)^3 \\ -16\delta^4(1+\alpha)^4 \end{bmatrix}}{16A^2[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2},$$

we can observe that $\frac{\partial\left(p_1^{D*}-p_1^{C*}\right)}{\partial\beta} < 0$ if $-263A + 752\delta(1+\alpha) < 0$.

It is easy to follow that

$$\frac{\partial V_0^{D*}}{\partial\delta} - \frac{\partial V_0^{C*}}{\partial\delta} = \frac{-8A(V_1-c)(1+\alpha)(1+\beta)[5A-2\delta(1+\alpha)]}{[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2} < 0,$$

$$\frac{\partial V_0^{D*}}{\partial\alpha} - \frac{\partial V_0^{C*}}{\partial\alpha} = \frac{-8A(V_1-c)\,\delta[A-\delta(1+\alpha)][5A-2\delta(1+\alpha)]}{[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2} < 0,$$

$$\frac{\partial V_0^{D*}}{\partial\beta} - \frac{\partial V_0^{C*}}{\partial\beta} = \frac{8A(V_1-c)\,\delta(1-\delta)(1+\alpha)[5A-2\delta(1+\alpha)]}{[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2} > 0,$$

$$\frac{\partial p_2^{D*}}{\partial \delta}-\frac{\partial p_2^{C*}}{\partial \delta}=\frac{(V_1-c)(1+\alpha)(1+\beta)\begin{bmatrix}-71A^2+20A\delta(1+\alpha)\\-4\delta^2(1+\alpha)^2\end{bmatrix}}{4[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2}<0,$$

$$\frac{\partial p_2^{D*}}{\partial \alpha}-\frac{\partial p_2^{C*}}{\partial \alpha}=\frac{(V_1-c)\delta[A-\delta(1+\alpha)]\begin{bmatrix}-71A^2+20A\delta(1+\alpha)\\-4\delta^2(1+\alpha)^2\end{bmatrix}}{4[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2}<0,$$

$$\frac{\partial p_2^{D*}}{\partial \beta}-\frac{\partial p_2^{C*}}{\partial \beta}=\frac{(V_1-c)\delta(1-\delta)(1+\alpha)\begin{bmatrix}71A^2-20A\delta(1+\alpha)\\-4\delta^2(1+\alpha)^2\end{bmatrix}}{4[3A-2\delta(1+\alpha)]^2[7A-2\delta(1+\alpha)]^2}>0.$$

The proof is completed. □

4. Revenue-sharing contract

In this section, a new two-stage revenue-sharing contract $\left\{w_1^{RS}, w_2^{RS}, \lambda_1, \lambda_2\right\}$ is presented to coordinate the supply chain. The retailers receive λ_1 proportion of revenue and share $1-\lambda_1$ proportion of sales revenue to the manufacturers in the original sales stage. Moreover, the retailers receive λ_2 proportion of revenue and share $1-\lambda_2$ proportion of sales revenue to the manufacturers in the original sales stage. Here, $\lambda_1\lambda_2 \in [01]$. Now, the total profit of manufacturers and retailers under two-stage revenue-sharing contract are proposed as follows:

$$\begin{aligned}\pi_M^{RS}&=(w_1-c)N\left(1-\frac{V_0}{V_1}\right)+(1-\lambda_1)p_1N\left(1-\frac{V_0}{V_1}\right)\\&\quad+(w_2-c)N\left(\frac{V_0}{V_1}-\frac{p_2}{V_1}\right)+(1-\lambda_2)p_2N\left(\frac{V_0}{V_1}-\frac{p_2}{V_1}\right),\end{aligned}\tag{4.1}$$

$$\pi_R^{RS}=(\lambda_1p_1-w_1)N\left(1-\frac{V_0}{V_1}\right)+(\lambda_2p_2-w_2)N\left(\frac{V_0}{V_1}-\frac{p_2}{V_1}\right).\tag{4.2}$$

Proposition 4.1. *Under the decentralized decision-making situations, the two-stage revenue-sharing contract $\left\{w_1^{RS}, w_2^{RS}, \lambda_1, \lambda_2\right\}$ designed by the manufacturers coordinates the supply chain in which the contract parameters should satisfy:* $w_1^{RS}=\frac{\left[2A^2-\delta^2(1+\alpha)^2\right]V_1\lambda_1+[2A-\delta(1+\alpha)]^2c\lambda_1-2A[2A-\delta(1+\alpha)](V_1-c)\lambda_2}{2A[3A-2\delta(1+\alpha)]}$, $w_2^{RS}=\lambda_2 c$ *and* $\lambda_2<\frac{[\delta(\beta-1)+1+\beta]\lambda_1}{\delta(\alpha-\beta)+1+\beta}$.

Proof. For the collaboration mechanism of two-stage revenue-sharing contract, the optimal retail price under the centralized decision-making is equal to the sales price in which the profit of retailers is maximized at the discount stage. That is, $p_2^{RS}=p_2^{C*}$.

Derivate p_2 in the second term of (8), and obtain:

$$p_2^{RS}=\frac{\lambda_2V_0+w_2}{2\lambda_2}.\tag{4.3}$$

When (4.3) is equal to (3.3), the incentives are achieved. Then, it follows that

$$w_2=\frac{[2A-\delta(1+\alpha)]V_1\lambda_2+[4A-3\delta(1+\alpha)]c\lambda_2-[3A-2\delta(1+\alpha)]V_0\lambda_2}{3A-2\delta(1+\alpha)}.$$

Bring (4.3) into (2.1), and obtain:

$$V_0 = \frac{\begin{array}{l} 2A\left[3A - 2\delta\left(1+\alpha\right)\right]p_1 - \left[2A - \delta\left(1+\alpha\right)\right]\delta\left(1+\alpha\right)V_1 \\ -\left[4A - 3\delta\left(1+\alpha\right)\right]\delta\left(1+\alpha\right)c \end{array}}{2\left(1-\delta\right)\left(1+\beta\right)\left[3A - 2\delta\left(1+\alpha\right)\right]}. \tag{4.4}$$

Taking (4.3) and (4.4) into (4.2), the total profit function expression of retailers for p_1 can be obtained:

$$\begin{array}{l} \pi_R^{RS}\left(p_1\right) = \dfrac{\left(\lambda_1 p_1 - w_1\right)N \left[\begin{array}{l} \left[6A^2 - 8A\delta\left(1+\alpha\right) + 3\delta^2\left(1+\alpha\right)^2\right]V_1 \\ -2A\left[3A - 2\delta\left(1+\alpha\right)\right]p_1 \\ +\left[4A - 3\delta\left(1+\alpha\right)\right]\delta\left(1+\alpha\right)c \end{array}\right]}{2(1-\delta)(1+\beta)[3A-2\delta(1+\alpha)]V_1} \\ +\frac{\lambda_2 N}{4V_1}\left[\dfrac{\begin{array}{l} 2A\left[3A - 2\delta\left(1+\alpha\right)\right]p_1 - A\left[3A - 2\delta\left(1+\alpha\right)\right]V_1 \\ -A\left[4A - 3\delta\left(1+\alpha\right)\right]c \end{array}}{(1-\delta)(1+\beta)[3A-2\delta(1+\alpha)]}\right]^2 \end{array}.$$

In order to ensure that the retailers profit function has an optimal solution in the original sales stage, $\partial^2\pi_R^{RS}\left(p_1\right)/\partial p_1^2 < 0$ should be meet. By calculation, we find that

$$\lambda_2 < \frac{A - \delta\left(1+\alpha\right)}{A}\lambda_1.$$

Furthermore, we can conclude that

$$p_1^{RS} = \frac{\begin{array}{l} 2A\left[A - \delta\left(1+\alpha\right)\right]\left[A - \delta\left(1+\alpha\right)\right]w_1 \\ +\left[A - \delta\left(1+\alpha\right)\right]\left[6A^2 - 8A\delta\left(1+\alpha\right) + \delta^2\left(1+\alpha\right)^2\right]V_1\lambda_1 \\ +\left[A - \delta\left(1+\alpha\right)\right]\left[4A - 3\delta\left(1+\alpha\right)\right]\delta\left(1+\alpha\right)c\lambda_1 \\ -2A^2\left[2A - \delta\left(1+\alpha\right)\right]V_1\lambda_2 - 2A^2\left[4A - 3\delta\left(1+\alpha\right)\right]c\lambda_2 \end{array}}{4A\left[3A - 2\delta\left(1+\alpha\right)\right]\left[\left(A - \delta\left(1+\alpha\right)\right)\lambda_1 - A\lambda_2\right]}. \tag{4.5}$$

Let $p_1^{RS} = p_1^{C*}$. It is easy to obtain that

$$w_1^{RS} = \frac{\begin{array}{l} \left[2A^2 - \delta^2\left(1+\alpha\right)^2\right]V_1\lambda_1 + \left[2A - \delta\left(1+\alpha\right)\right]^2 c\lambda_1 \\ -2A\left[2A - \delta\left(1+\alpha\right)\right]\left(V_1 - c\right)\lambda_2 \end{array}}{2A\left[3A - 2\delta\left(1+\alpha\right)\right]} \tag{4.6}$$

and then

$$w_2^{RS} = \lambda_2 c. \tag{4.7}$$

□

Proposition 4.2. *Under two-stage revenue-sharing contract, if the revenue sharing ratio λ_1 in original sales stage and revenue sharing ratio λ_2 in discount sales stage satisfy $\xi\left(\lambda_2\right) \le \lambda_1 \le \eta\left(\lambda_2\right)$ and $\frac{A}{A-\delta(1+\alpha)}\lambda_2 < \lambda_1 < 1$, then the perfect coordination of the supply chain is achieved. Here, $\xi\left(\lambda_2\right) \le \lambda_1 \le \eta\left(\lambda_2\right)$ and they are defined as follows:*

$$\begin{array}{l} \xi\left(\lambda_2\right) = \frac{[3A-2\delta(1+\alpha)]^2\left[16\delta^3(1+\alpha)^3 - 204\delta^2(1+\alpha)^2 + 848\delta(1+\alpha)\right]}{256[A-\delta(1+\alpha)]^3[7A-2\delta(1+\alpha)]^2} \\ -\frac{[2A-\delta(1+\alpha)][6A-5\delta(1+\alpha)]}{4A[A-\delta(1+\alpha)]^3}\lambda_2 \end{array},$$

$$\eta(\lambda_2)=\frac{[3A-2\delta(1+\alpha)]\left[\begin{array}{c}16\,[7A-2\delta(1+\alpha)]\,[2A-\delta(1+\alpha)]^2\\-\,[3A-2\delta(1+\alpha)]\,[9A-2\delta(1+\alpha)]^2\end{array}\right]}{64[A-\delta(1+\alpha)]^3[7A-2\delta(1+\alpha)]}-\frac{[2A-\delta(1+\alpha)][6A-5\delta(1+\alpha)]}{4A[A-\delta(1+\alpha)]^3}\lambda_2.$$

Proof. Substitute (4.3) and (4.5)-(4.7) into (4.1) and (4.2), and obtain that:

$$\pi_M^{RS}=N(V_1-c)^2\frac{\begin{array}{c}-4[A-\delta(1+\alpha)]^3\lambda_1+[3A-2\delta(1+\alpha)]\,[2A-\delta(1+\alpha)]^2\\-A\,[2A-\delta(1+\alpha)]\,[6A-5\delta(1+\alpha)]\,\lambda_2\end{array}}{4A[3A-2\delta(1+\alpha)]^2V_1},$$

and

$$\pi_R^{RS}=N(V_1-c)^2\frac{\begin{array}{c}4[A-\delta(1+\alpha)]^3\lambda_1\\+A\,[2A-\delta(1+\alpha)]\,[6A-5\delta(1+\alpha)]\,\lambda_2\end{array}}{4A[3A-2\delta(1+\alpha)]^2V_1}.$$

In order to enable manufacturers and retailers to actively participate in the revenue sharing contract, it is necessary to establish a reasonable feasible region of $\{\lambda_1,\lambda_2\}$. Within the range of this feasible region, it should be able to ensure that the profit obtained by both parties after participating in the revenue-sharing contract is no less than that before participating. In other words, to solve the feasible region for Pareto improvement of both manufacturers and retailers, $\pi_M^{RS}(\lambda_1,\lambda_2)-\pi_M^{D*}\geq 0$ and $\pi_R^{RS}(\lambda_1,\lambda_2)-\pi_R^{D*}\geq 0$ should be satisfied. Therefore, it follows immediately to conclude the conclusion of Proposition 4.2. □

5. Numerical analysis

Using Matlab.R2017 simulation tool, some intuitive results are verified through numerical examples in this section. Let $V_1=100$, $c=50$ and $N=100$.

5.1. **Sensitivity analysis of consumer heterogeneity.** The consumers strategic degree plays an important role in the supply chain. For further visual verification, the impact of consumers strategic degree on profit difference, price difference and strategic behavior threshold difference between the original and discount sales stages. It can be seen in Figure 1 in which $\alpha=\frac{2}{3}$ and $\beta=\frac{3}{4}$ are assumed.

It can be seen from on the left of Figure 1 that, when the consumers' strategic degree increase, the profit in the original sales stage under the decentralized decision-making model is first less and then greater than that under the centralized decision-making model. Moreover, the profit in the discount sales stage is first higher and then less than that under the centralized decision-making model. In the whole, the profit of the two stages in decentralized decision-making is always less than that under the centralized decision-making.

As can be seen on the right of Figure 1 that, with the increase of consumers' strategic degree, the price difference between centralized and decentralized decision-making in the original sales stage gradually increases first and then decreases. Under the two decision-making situations, price difference and strategic behavior threshold difference gradually decrease in the discount sales stage. This is due to the fact that, the price reduction of the decentralized decision-making is less than that of the centralized decision-making in the original sales stage. Moreover, the increase

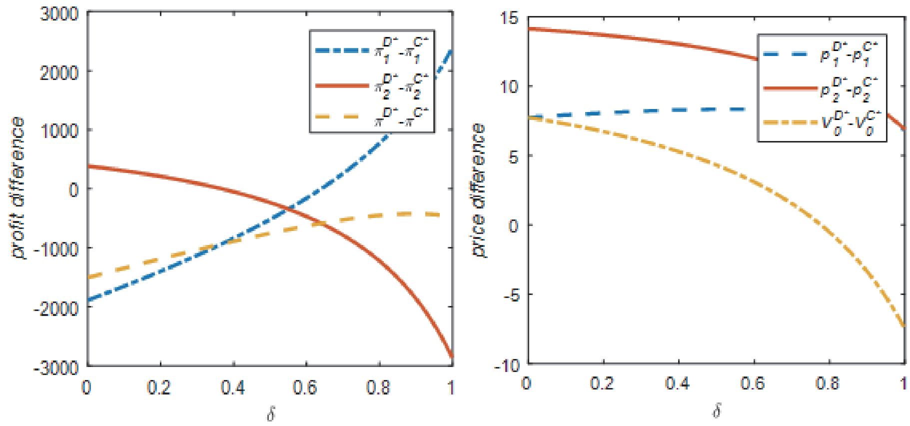

FIGURE 1. The impact of consumers strategic degree on profit difference, price difference and strategic behavior threshold difference

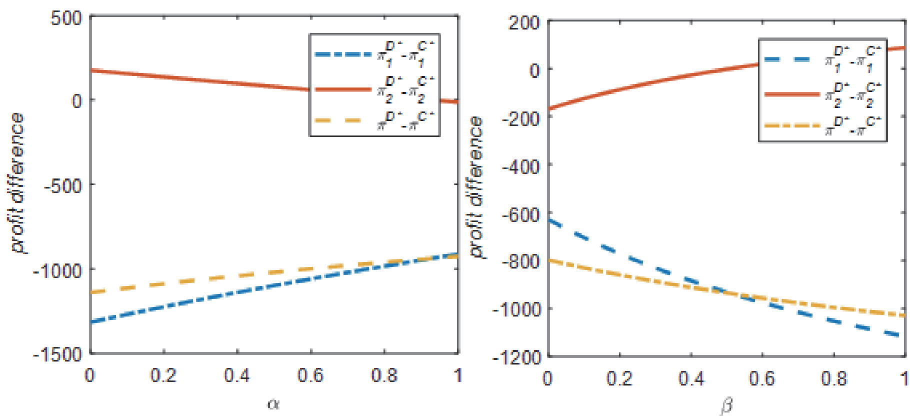

FIGURE 2. The impact of consumers expected regret on profit difference

of sales price and strategic behavior threshold of consumers under the decentralized decision-making is less than that of the centralized decision-making in the discount sales stage.

5.2. **Sensitivity analysis of expected regret.** The consumers' expected regret will affect their own decisions and the interests of enterprises. In this section, the impact of consumers expected regret on profit difference , price difference and strategic behavior threshold difference between the original and discount sales stages are studied in Figures 2 and 3 respectively.

On the left of Figure 2, $\beta = \frac{3}{4}$ and $\delta = \frac{1}{2}$ are set. With the increase of consumers' high price regret, the profit in the original sales stage under the decentralized decision-making is less than that under the centralized decision-making. The profit in the discount sales stage under the decentralized decision-making is first more and then less than that under the centralized decision-making. On the right of Figure 2, let $\alpha = \frac{2}{3}$ and $\delta = \frac{1}{2}$. As the out of stock regret increase, the profit

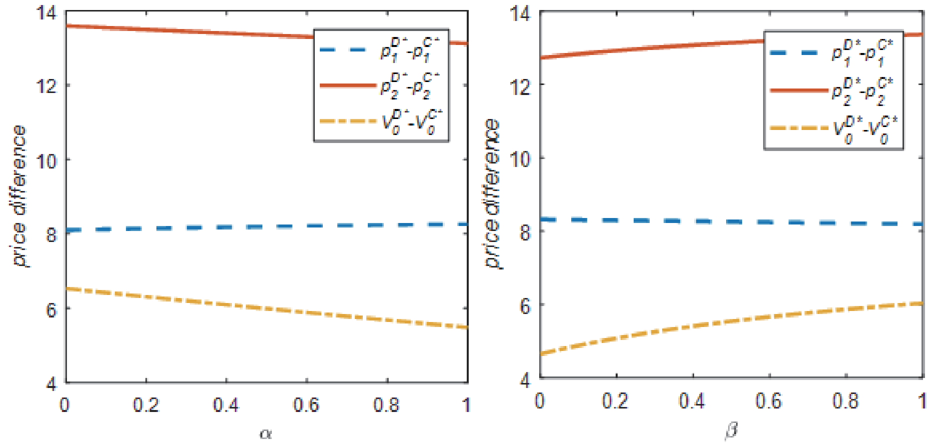

FIGURE 3. The impact of consumers expected regret on price difference and strategic behavior threshold difference

in the original sales stage under the decentralized decision-making is less than that under the centralized decision-making. The profit in the discount sales stage under the decentralized decision-making is first more and then less than that under the centralized decision-making. In above, the total profit of the two stages under the decentralized decision-making are always less than that under the centralized decision-making whether high price regret or out of stock regret situations.

On the left of Figure 3, assume that $\delta = \frac{1}{3}$ and $\beta = \frac{3}{4}$. With the increase of high price regret, the price difference of the centralized and decentralized decision-making gradually increase in the original sales stage. Moreover, the price difference and strategic behavior threshold difference in the discount sales stage decrease gradually. This is mainly based on the following reasons. Firstly, in general, retailers may reduce the sales price in the original sales stage under both centralized and decentralized decision-making situations. Secondly, the decline of the sales price in the original sales stage under decentralized decision-making is relatively low. Thirdly, in the discount sales stage, the increase of the sales price and strategic behavior threshold under the decentralized decision-making is less than that in centralized decision-making. On the right of Figure 3, let $\alpha = \frac{2}{3}$ and $\delta = \frac{1}{3}$. As the out of stock regret increase, the price difference gradually decrease in the original sales stage while price difference and strategic behavior threshold difference gradually increase in the discount sales stage under both the centralized and decentralized decision-making. In the centralized decision-making situation, retailers increase the sales price in the original sales stage. The increased range of sales price in the decentralized decision-making is less than that in the centralized decision-making under the original sales stage, while the reduction range of sales price in the decentralized decision-making is also less than that in the centralized decision-making under the discount sales stage.

5.3. **Sensitivity analysis of revenue sharing ratios.** In Figure 4, the profit differences of manufacturers and retailers before and after coordination are denoted by $\Delta\pi_M = \pi_M^{RS} - \pi_M^D$ and $\Delta\pi_R = \pi_R^{RS} - \pi_R^D$ respectively. Now, assume that

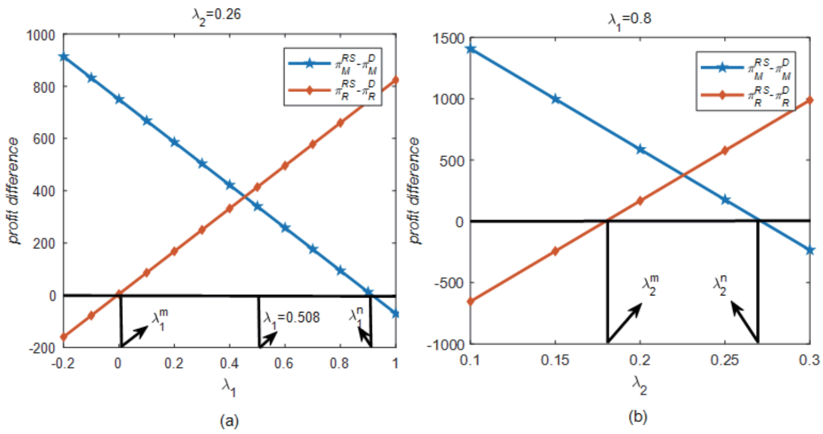

FIGURE 4. The impact of revenue sharing ratios on profit difference

$\alpha = \frac{2}{3}$, $\beta = \frac{3}{4}$ and $\delta = \frac{1}{2}$. It is easy to obtain that the profits of manufacturers before and after coordination are $\pi_M^D = 4.1751\mathrm{e}+03$ and $\pi_M^{RS}(\lambda_1, \lambda_2) \approx 7060 - 820\lambda_1 - 8212\lambda_2$ respectively. The profits of retailers before and after coordination are $\pi_R^D = 2.1311\mathrm{e}+03$ and $\pi_R^{RS}(\lambda_1, \lambda_2) \approx 820\lambda_1 + 8212\lambda_2$ respectively.

As long as $\Delta\pi_M, \Delta\pi_R \geq 0$, the revenue-sharing contract can be introduced to coordinate the supply chain. It follows immediately that

$$\frac{2.1311\mathrm{e}+03}{820} - \frac{8212\lambda_2}{820} \leq \lambda_1 \leq \frac{7060 - 4.1751\mathrm{e}+03}{820} - \frac{8212\lambda_2}{820}.$$

Furthermore, the revenue sharing ratios should satisfy: $1.9524\lambda_2 < \lambda_1 < 1$. The impact of revenue sharing ratios on profit difference before and after coordination are shown in Figure 4. On the one hand, suppose that $\lambda_2 = 0.26$ in Figure 4(a) which shows how both $\Delta\pi_M$ and $\Delta\pi_R$ vary with λ_1. On the other hand, assume $\lambda_1 = 0.8$ in Figure 4(b) which shows how both $\Delta\pi_M$ and $\Delta\pi_R$ vary with λ_2.

As can be seen from Figure 4(a) that, when λ_2 remains unchanged, the profit change of manufacturers and retailers are linear functions of λ_1 before and after coordination. Note that, λ_1 has a positive effect on the increasing and decreasing changes of retailers' profits, and a negative effect on the increasing and decreasing changes of manufacturers' profits. Similarly in Figure 4(b), when λ_1 remains unchanged, λ_2 has a positive effect on the increasing and decreasing changes of retailers' profits, and negative effect on the increasing and decreasing changes of manufacturers' profits. In particular, when $\lambda_2 = 0.26$ and $\lambda_1 \in [\lambda_1^m, \lambda_1^n]$, there are always $\Delta\pi_M, \Delta\pi_R \geq 0$. In addition, $\lambda_1 > 1.9524\lambda_2$ should be satisfied. In conclusion, $\lambda_1 \in (1.9524\lambda_2, \lambda_1^n]$ can ensure that the supply chain would realize Pareto improvement. By calculating, we can obtain that $\lambda_1^m = 0.005$, $\lambda_1^n = 0.914$ and $1.9524\lambda_2 = 0.508$. Similarly, it can be seen from Figure 4 (b) that, when $\lambda_1 = 0.8$ and $\lambda_2 \in [\lambda_2^m, \lambda_2^n]$, there are always $\Delta\pi_M, \Delta\pi_R \geq 0$. Now, the supply chain can realize Pareto improvement. By calculating, we can obtain that $\lambda_2^m = 0.18$ and $\lambda_2^n = 0.271$. Therefore, both the revenue sharing ratio of the original sales stage and the revenue sharing ratio of the discount stage have a certain amount of flexibility.

6. Conclusions

In this paper, by considering consumers expected regret, a two-stage dynamic game model composed of manufacturers, retailers and strategic consumers is established. The impact of consumers expected regret and consumer heterogeneity on the two-stage supply chain are analyzed. The main conclusions are proposed as follows.

As the consumers' strategic degree increase, consumers often tend to wait until the discount stage to buy products. On the one hand, to obtain more customer choices and then generate final purchase behavior, manufacturers and retailers may only take price reduction measures to avoid excessive loss of profits in the original sales stage. On the other hand, since the demand for products in discount stage will increase significantly, manufacturers and retailers can seize the opportunity to appropriately increase prices and reduce discounts.

As the consumers high price regret increases, consumers are unwilling to buy products with uncertain value at high prices. This leads to consumers prefer to purchase products at the discount stage. For high regret consumers, thus manufacturers can take a quality commitment strategy, and retailers adopt a price commitment strategy.

Many companies attract consumers through promotional activities. Products out of stock occurs frequently, and then out of stock regret make consumers want to obtain products in time. This psychological demand is greater than the increase of cost. For out of stock regret consumers, thus manufacturers and retailers should strengthen the relationship between each other, and pay more attention to the inventory level of products. In case of stock shortage, retailers can make up for consumers in a certain way, so as to reduce consumers' dissatisfaction.

The two-stage revenue-sharing contract proposed in this paper can adjust the strategic behavior of consumers. When consumers strategic degree, high price regret and out of stock regret meet certain conditions, the perfect coordination of the supply chain can be achieved through the revenue-sharing contract. Then, manufacturers and retailers can achieve a win-win situation. In the actual market, however, there are many manufacturers and retailers, and the types of consumers are different. Therefore, exploring two-stage dynamic pricing and coordination strategies of different consumer types is the future research.

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