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SYSTEM OF YOSIDA CAYLEY VARIATIONAL INCLUSIONS INVOLVING XOR-OPERATION

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Abstract. In this article, we introduce a system of Yosida Cayley variational inclusion problem involving XOR-operation in the setting of real ordered Banach space. An existence result is obtained by using resolvent operator technique. We propose a perturbed two-step iterative algorithm for solving system of Yosida Cayley variational inclusions involving XOR-operation. We also discuss convergence and stability analysis of the proposed algorithm. Finally, we provide an example in support of the conditions used in our main result.

1. Introduction

The concept of variational inequalities and its generalized forms provides mathematical models to solve easily a wide range of typical problems related to elasticity, structural analysis, economic equilibria, oceanography, medical and engineering sciences, etc., see for example [1, 5, 12, 13, 19–22, 25, 30, 32, 35–37, 41–43] and references therein.

One of the important generalization of a variational inequality is a variational inclusion introduced by Hassouni and Moudafi [17]. After that system of variational inequalities (inclusions) came into picture and studied by Pang [26], Cohen and Chaplais [10], Bianchi [8] and Ansari and Yao [7], etc.. Pang showed that traffic equilibria, spatial equilibria, nash and general equilibria can be formulated in the form of a variational inequality. For more details, we refer to [2–4, 9, 15, 16, 27, 28, 33, 34, 38–40]and references therein.

Yosida approximation operator has some applications in real world phenomena. For example, heat equation that describes the distribution of heat (or variation in temperatures over time in a fixed region of space), wave equation which is a second order partial differential equation used for the description of waves, linearized equations of coupled sound and heat flow, etc.. For more details, see [11, 31].

Cayley operator is a well defined operator between skew symmetric matrices and special orthogonal matrices. It is an automorphism of a real projective line that permutes the elements $\{1, 0, -1, \infty\}$ in sequence. It has applications in Quaternion homography, Real homography, Complex homography, etc.. For more information, see [18, 29] and references therein.

XOR is a Boolean logic operation that is widely used in cryptography as well as in generating parity bits for error checking and fault tolerance. It stands for "exclusive or", that is to say that resulting bit evaluates to one if only exactly one of the bits is set. Bitwise logical operations in C programming is also an XOR-operation, bits are compared with one another. When two bits are identical, XOR coughs up 0.

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When the two bits are different, XOR spits out 1. We mention some applications of XOR-operation below:

(1) Swapping using XOR.
(2) Encryption with XOR: The XOR Cipher.
(3) Comparing two numbers using XOR.
(4) Conversion of binary values to gray code using XOR.
(5) Checking parity of a number using XOR.

Influenced by the applications of above revealed concepts, in this paper, we introduce and study a system of Yosida Cayley variational inclusions involving XOR-operation in real ordered Banach space. The system under consideration involves Yosida approximation operator, Cayley operator, non-linear single-valued mappings and a set-valued maximal monotone operator. An existence result is proved. Convergence and Stability analyses are discussed, separately. In support of our main result, we provide an example.

2. Preliminaries

Throughout this article, we consider $\widehat{E}$ to be a real ordered Banach space with norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$. Let K be a normal cone with normal constant δ and "$\leq$" be the partial ordering induced by the cone. For any $x, y \in \widehat{E}$, $x \leq y$ if and only if $y - x \in K$. The $lub\{x, y\}$ means the least upper bound of the set $\{x, y\}$ and $glb\{x, y\}$ means the greatest lower bound of the set $\{x, y\}$. Suppose $glb\{x, y\}$ and $lub\{x, y\}$ exist, then some binary operations are defined as follows:

(i) $\vee$ is called AND operation,
(ii) $\wedge$ is called OR operation,
(iii) $\oplus$ is called XOR-operation,
(iv) $\odot$ is called XNOR-operation,
(v) $x \vee y = \sup\{x, y\}$,
(vi) $x \wedge y = \inf\{x, y\}$,
(vii) $x \oplus y = (x - y) \vee (y - x)$,
(viii) $x \odot y = (x - y) \wedge (y - x)$.

Lemma 2.1. [14] *Let $\odot$ be an XNOR operation and $\oplus$ be an XOR operation. Then, the following relations hold:*

(i) $a \odot a = 0$, $a \odot z = z \odot a = -(a \oplus z) = -(z \oplus a)$,
(ii) *if* $a \propto 0$, *then* $-a \oplus 0 \leq a \leq a \oplus 0$,
(iii) $(\lambda a) \oplus (\lambda z) = |\lambda|(a \oplus z)$,
(iv) *if* $a \propto z$, *then* $a \oplus z = 0$ *if and only if* $a = z$,
(v) $(a + b) \odot (y + z) \geq (a \odot y) + (b \odot z)$,
(vi) *if* a, b, y *and* z *are comparative to each other, then*

$$(a \wedge b) \oplus (y \wedge z) \leq \big((a \oplus y) \vee (b \oplus z)\big) \wedge \big((a \oplus z) \vee (b \oplus y)\big),$$

(vii) *if* a, y *and* z *are comparative to each other, then* $(a \oplus z) \leq a \oplus y + y \oplus z$,
(viii) *if* $a \propto z$, *then* $((a \oplus 0) \oplus (z \oplus 0)) \leq (a \oplus z) \oplus 0 = a \oplus z$,
(ix) $(la) \oplus (ma) = |l - m|a = (l \oplus m)a$, *for all* $a, b, y, z \in \widehat{E}$ *and* $l, m, \lambda \in \mathbb{R}$.

Definition 2.2. Let $\widehat{E}$ be a real ordered Banach space. If the following conditions are satisfied:

(i) $\|(x, y, z)\| = \max\{\|x\|, \|y\|, \|z\|\}$, for any $(x, y, z) \in \widehat{E} \times \widehat{E} \times \widehat{E}$,

(ii) $(x_1, y_1, z_1) \leq (x_2, y_2, z_2)$ if and only if $x_1 \leq x_2, y_1 \leq y_2$ and $z_1 \leq z_2$ in $\widehat{E}$,

(iii) $(x_1, y_1, z_1) \propto (x_2, y_2, z_2)$ if and only if $x_1 \propto x_2, y_1 \propto y_2, z_1 \propto z_2$,

(iv) $(x_1, y_1, z_1) \wedge (x_2, y_2, z_2) = (x_1 \wedge x_2, y_1 \wedge y_2, z_1 \wedge z_2)$,
$(x_1, y_1, z_1) \vee (x_2, y_2, z_2) = (x_1 \vee x_2, y_1 \vee y_2, z_1 \vee z_2)$,
$(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1 \oplus x_2, y_1 \oplus y_2, z_1 \oplus z_2)$.

Then $\widehat{E} \times \widehat{E} \times \widehat{E}$ is called an real ordered product Banach space.

Definition 2.3. Let $\widehat{E} \times \widehat{E} \times \widehat{E}$ be an real ordered product Banach space. The sequence $\{(x_n, y_n, z_n)\}$ in $\widehat{E} \times \widehat{E} \times \widehat{E}$, where $\{x_n\}, \{y_n\}$, and $\{z_n\}$ are sequences in $\widehat{E}$, converges to (x^*, y^*, z^*) if and only if $x_n \to x^* \in \widehat{E}, y_n \to y^* \in \widehat{E}$ and $z_n \to z^* \in \widehat{E}$, as $n \to \infty$.

Definition 2.4. For each $i = 1, 2, \cdots, p$, let $h_i : \widehat{E}_i \longrightarrow \widehat{E}_i$ and $T_i : \prod_{j=1}^{p} \widehat{E}_j \to \widehat{E}_i$ be the non-linear single-valued mappings. Then

(i) h_i is said to be λ_{h_i}-ordered rectangular mapping, if there exist a constant $\lambda_{h_i} > 0$ such that

$$\langle h_i(x) \odot h_i(y), -(x \oplus y) \rangle \geq \lambda_{h_i} \|x \oplus y\|^2, \text{ for all } x, y \in \widehat{E}_i,$$

(ii) T_i is said to be κ_i-ordered compression mapping in the i^{th}-argument, if T_i is a comparison mapping, and there exists a constant $0 < \kappa_i < 1$ such that

$$T_i(x_1, x_2, \cdots, x_{i-1}, x_i, x_{i+1}, \cdots, x_p) \oplus T_i(x_1, x_2, \cdots, x_{i-1}, \hat{x}_i, x_{i+1}, \cdots, x_p) \\ \leq \kappa_i(x_i \oplus \hat{x}_i), \text{ for all } x_i, \hat{x}_i \in \widehat{E}_i.$$

Definition 2.5. [23, 24] Let $M : \widehat{E} \to 2^{\widehat{E}}$ be a set-valued mapping. Then

(i) M is said to be a comparison mapping, if for any $v_x \in M(x)$, $x \propto v_x$, and if $x \propto y$, then for any $v_x \in M(x)$ and $v_y \in M(y)$, $v_x \propto v_y$, for all $x, y \in \widehat{E}$,

(ii) a comparison mapping M is said to be γ-ordered rectangular mapping, if for each $x, y \in \widehat{E}$, there exist $\gamma > 0$ and $v_x \in M(x)$ and $v_y \in M(y)$ such that

$$\langle v_x \odot v_y, -(x \oplus y) \rangle \geq \gamma \|x \oplus y\|^2,$$

(iii) a comparison mapping M is said to be λ-weak-ordered different mapping, if for each $x, y \in \widehat{E}$, there exists a constant $\lambda > 0$ and $v_x \in M(x), v_y \in M(y)$ such that

$$\lambda(v_x - v_y) \propto (x - y),$$

(iv) a comparison mapping M is said to be a (γ, λ)-weak-ordered rectangular different set-valued mapping, if M is a γ-ordered rectangular and λ-weak-ordered different comparison mapping and $[I + \lambda M](\mathcal{H}) = \mathcal{H}$, for some $\gamma, \lambda > 0$.

Definition 2.6. Let $M : \widehat{E} \to 2^{\widehat{E}}$ be a (γ, λ)-weak ordered rectangular different set-valued mapping. The resolvent operator $\mathcal{R}_\lambda^M : \widehat{E} \to \widehat{E}$ is defined by

$$\mathcal{R}_\lambda^M(x) = [I + \lambda M]^{-1}(x), \text{ for all } x \in \widehat{E}, \tag{2.1}$$

where $\lambda > 0$ is a constant.

Definition 2.7. The Yosida approximation operator $\mathcal{J}_\lambda^M : \widehat{E} \to \widehat{E}$ is defined by

$$\mathcal{J}_\lambda^M(x) = \frac{1}{\lambda}[I - \mathcal{R}_\lambda^M](x), \text{ for all } x \in \widehat{E}, \tag{2.2}$$

where I is the identity operator and $\lambda > 0$ is a constant.

Definition 2.8. The Cayley operator $\mathcal{C}_\lambda^M : \widehat{E} \to \widehat{E}$ is defined by

$$\mathcal{C}_\lambda^M(x) = [2\mathcal{R}_\lambda^M - I](x), \text{ for all } x \in \widehat{E}, \tag{2.3}$$

where I is the identity operator and $\lambda > 0$ is a constant.

Proposition 2.9. *[6] Let $M : \widehat{E} \to 2^{\widehat{E}}$ be a (γ, λ)-weak ordered rectangular different set-valued mapping with respect to $\mathcal{R}_\lambda^M$. Then, the resolvent operator $\mathcal{R}_\lambda^M$ is well-defined, single valued and Lipschitz-type continuous, that is*

$$\|\mathcal{R}_\lambda^M(x) \oplus \mathcal{R}_\lambda^M(y)\| \leq \theta \|x \oplus y\|, \text{ where } \theta = \frac{1}{(\gamma\lambda - 1)}, \ \gamma\lambda > 1, \text{ for all } x, y \in \widehat{E}. \tag{2.4}$$

Proposition 2.10. *[6] Let $M : \widehat{E} \to 2^{\widehat{E}}$ be a (γ, λ)-weak ordered rectangular different set-valued mapping with respect to $\mathcal{R}_\lambda^M$ and the resolvent operator $\mathcal{R}_\lambda^M$ is θ-Lipschitz-type-continuous. Then, the Cayley operator $\mathcal{C}_\lambda^M$ defined by (2.8) is $(2\theta + 1)$-Lipschitz-type continuous. That is*

$$\|\mathcal{C}_\lambda^M(x) \oplus \mathcal{C}_\lambda^M(y)\| \leq (2\theta + 1)\|x \oplus y\|, \text{ where } \theta = \frac{1}{(\gamma\lambda - 1)}, \ \gamma\lambda > 1, \text{ for all } x, y \in \widehat{E}. \tag{2.5}$$

3. Formulation of problem and existence result

For each $i \in \Lambda = \{1, 2, 3, \cdots, p\}$, let $\widehat{E}_i$ a real ordered Banach space equipped with the norm $\|.\|_i$ and K_i be a normal cone with normal constant δ_i, and let $h_i : \widehat{E}_i \to \widehat{E}_i$ and $T_i : \prod_{j=1}^{p} \widehat{E}_j \to \widehat{E}_i$ be the nonlinear single-valued mappings and $\mathcal{M}_i : \widehat{E}_i \to 2^{\widehat{E}_i}$ be the set-valued mapping. Let $\mathcal{R}_{\lambda_i}^{\mathcal{M}_i} : \widehat{E}_i \to \widehat{E}_i$ be the resolvent operator, $\mathcal{J}_{\lambda_i}^{\mathcal{M}_i} : \widehat{E}_i \to \widehat{E}_i$ be the Yosida approximation operator and $\mathcal{C}_{\lambda_i}^{\mathcal{M}_i} : \widehat{E}_i \to \widehat{E}_i$ be the Cayley operator. We study the following system of Yosida Cayley variational inclusions involving XOR-operation:

For each $\omega_i > 0$, find $(x_1, x_2, \cdots, x_p) \in \prod_{i=1}^{p} \widehat{E}_i$ such that

$$
\text{(3.1)} \quad \begin{cases}
0 \in \Big(\mathcal{J}_{\lambda_1}^{\mathcal{M}_1}(\mathcal{C}_{\lambda_1}^{\mathcal{M}_1}(x_1)) + T_1(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_1 \mathcal{M}_1(h_1(x_1)) \\
0 \in \Big(\mathcal{J}_{\lambda_2}^{\mathcal{M}_2}(\mathcal{C}_{\lambda_2}^{\mathcal{M}_2}(x_2)) + T_2(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_2 \mathcal{M}_2(h_2(x_2)) \\
0 \in \Big(\mathcal{J}_{\lambda_3}^{\mathcal{M}_3}(\mathcal{C}_{\lambda_3}^{\mathcal{M}_3}(x_3)) + T_3(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_3 \mathcal{M}_3(h_3(x_3)) \\
\quad \vdots \\
0 \in \Big(\mathcal{J}_{\lambda_p}^{\mathcal{M}_p}(\mathcal{C}_{\lambda_p}^{\mathcal{M}_p}(x_p)) + T_p(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_p \mathcal{M}_p(h_p(x_p))
\end{cases}
$$

Equivalently, for each $i \in \Lambda$,

$$
\text{(3.2)} \qquad 0 \in \Big(\mathcal{J}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + T_i(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_i \mathcal{M}_i(h_i(x_i)).
$$

Related to the system of Yosida Cayley variational inclusions involving XOR-operation (3.2), we have the following equivalence lemma.

Lemma 3.1. *For each $i \in \Lambda$, let $h_i : \widehat{E}_i \to \widehat{E}_i$ and $T_i : \prod_{j=1}^{p} \widehat{E}_j \to \widehat{E}_i$ be the nonlinear single-valued mappings and $\mathcal{M}_i : \widehat{E}_i \to 2^{\widehat{E}_i}$ be a (γ_i, λ_i)-weak ordered rectangular different set-valued mapping. Let $\mathcal{R}_{\lambda_i}^{\mathcal{M}_i} : \widehat{E}_i \to \widehat{E}_i$ be the resolvent operator, $\mathcal{J}_{\lambda_i}^{\mathcal{M}_i} : \widehat{E}_i \to \widehat{E}_i$ be the Yosida approximation operator and $\mathcal{C}_{\lambda_i}^{\mathcal{M}_i} : \widehat{E}_i \to \widehat{E}_i$ be the Cayley operator. Then the following assertions are equivalent:*

(i) $(x_1, x_2, \cdots, x_p) \in \prod_{i=1}^{p} \widehat{E}_i$ *is a solution of system of Yosida Cayley variational inclusions involving XOR-operations* (3.1).

(ii) $x_i \in \widehat{E}_i$ *is a fixed point of a mapping* $Q_i : \widehat{E}_i \to 2^{\widehat{E}_i}$ *defined by*

$$
Q_i(x_i) = \Big(\mathcal{J}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + T_i(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_i \mathcal{M}_i(h_1(x_i)) + x_i.
$$

(iii) $x_i \in \widehat{E}_i$ *is a solution of the following equation :*

$$
\text{(3.3)} \quad h_i(x_i) = \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) + \lambda_i T_i(x_1, x_2, \cdots, x_p)\big)\big)\big].
$$

Proof. *(i)* $\iff$ *(ii)* : We mention (3.2) below and then adding x_i on both sides of it, we have

$$
\begin{aligned}
&0 \in \Big(\mathcal{J}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + T_i(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_i \mathcal{M}_i(h_i(x_i)) \\
\iff &x_i \in \Big(\mathcal{J}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + T_i(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_i \mathcal{M}_i(h_i(x_i)) + x_i = Q_i(x_i).
\end{aligned}
$$

Hence, x_i is a fixed point of Q_i.

$(ii) \iff (iii)$: Let x_i be a fixed point of Q_i, then

$$x_i \in \ Q_i(x_i) = \Big(\mathcal{J}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + T_i(x_1, x_2, \cdots, x_n)\Big) \oplus \omega_i \mathcal{M}_i(h_i(x_i)) + x_i$$
$$\iff \mathcal{J}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + T_i(x_1, x_2, \cdots, x_n) \in \omega_i \mathcal{M}_i(h_1(x_i))$$
$$\iff \mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + \lambda_i T_i(x_1, x_2, \cdots, x_n) \in \lambda_i \omega_i \mathcal{M}_i(h_i(x_i))$$
$$\iff h_i(x_i) - \omega_i^{-1}\Big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) + \lambda_i T_i(x_1, x_2, \cdots, x_n)\big)\Big) \in [I_i + \lambda_i \mathcal{M}_i](h_i(x_i)),$$
$$\iff h_i(x_i) = \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) + \lambda_i T_i(x_1, x_2, \cdots, x_n)\big)\big)\big].$$

That is, x_i is a solution of (3.3).

$(iii) \iff (i)$: From (3.3) we have

$$h_i(x_i) = \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) + \lambda_i T_i(x_1, x_2, \cdots, x_n)\big)\big)\big]$$
$$\iff h_i(x_i) = [I_i + \lambda_i \mathcal{M}_i]^{-1}\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) + \lambda_i T_i(x_1, x_2, \cdots, x_n)\big)\big)\big]$$
$$\iff \big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) + \lambda_i T_i(x_1, x_2, \cdots, x_n)\big)\big)\big] \in (I_i + \lambda_i \mathcal{M}_i)(h_i(x_i)),$$
$$\iff \big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) + \lambda_i T_i(x_1, x_2, \cdots, x_n)\big)\big)\big] \in h_i(x_i) + \lambda_i \mathcal{M}_i(h_i(x_i)),$$
$$\iff \big(\mathcal{J}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)) + T_i(x_1, x_2, \cdots, x_n)\big) \in \omega_i \mathcal{M}_i(h_i(x_i)),$$
$$\iff 0 \in \big(\mathcal{J}_{\lambda_i}^{\mathcal{M}_i}\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)\big) + T_i(x_1, x_2, \cdots, x_n)\big) \oplus \omega_i \mathcal{M}_i(h_i(x_i)).$$

Therefore, $(x_1, x_2, \cdots, x_p) \in \prod_{i=1}^{p} \widehat{E}_i$ is a solution of system of Yosida Cayley variational inclusions involving XOR-operation (3.1). □

Remark 3.1. It is interesting to note that due to Lemma 3.1, the Yosida approximation operator disappears from Theorem 3.2, Algorithm 4.1 and Theorem 4.1.

Theorem 3.2. *For each $i \in \Lambda$, let K_i be a normal cone with normal constant δ_i, $h_i : \widehat{E}_i \to \widehat{E}_i$ and $T_i : \prod_{j=1}^{p} \widehat{E}_j \to \widehat{E}_i$ be the nonlinear single-valued mappings such that h_i is a comparison and μ_{h_i}-ordered compression mapping, T_i is a comparison, κ_i-ordered compression mapping in the i^{th}-argument and $\kappa_{i,j}$-ordered compression mapping in the j^{th}-argument for each $j \in \Lambda, i \neq j$, respectively. Let $\mathcal{M}_i : \widehat{E}_i \to 2^{\widehat{E}_i}$ be a (γ_i, λ_i)-weak ordered rectangular different set-valued mapping. Let $\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}$ be the resolvent operator and $\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}$ be the Cayley operator such that both the operators are*

Lipschitz-type-continuous with constant θ_i and $(2\theta_i+1)$, respectively. Let $x_i \propto \hat{x}_i$, $\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i)) \propto \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i))$, for all $x_i \propto \hat{x}_i$, for each $i \in \Lambda$ such that the following conditions are satisfied:

$$\begin{cases} \theta_i = \frac{1}{\gamma_i\lambda_i - 1}, \gamma_i\lambda_i > 1 \\ (2\theta_i+1)(\theta_i+1) + \theta_i\lambda_i(1-\kappa_i\delta_i) + \omega_i\sqrt{\left(1-2\lambda_{h_i}+\mu^2_{h_i}\right)} < \omega_i(1-\nu_i), \\ \nu_i = \theta_i\mu_{h_i} + \sum\limits_{\ell\in\Lambda,\ \ell\neq i} \frac{\theta_\ell\lambda_\ell\delta_\ell}{\omega_\ell}\kappa_{\ell,i} < 1, \text{ and } 2\mu_{h_i} < 1+\mu^2_{h_i}. \end{cases} \tag{3.4}$$

Then, the system of Yosida Cayley variational inclusions involving XOR-operation (3.1) admits a unique solution $(x_1^*, x_2^*, \cdots, x_p^*)$.

Proof. It is sufficient to prove that there exist $(x_1^*, x_2^*, \cdots, x_p^*)$ such that fixed point equation (3.3) is satisfied. For each $i \in \Lambda$, we define $\phi_i : \prod\limits_{j=1}^{p} \widehat{E}_j \to \widehat{E}_i$ by

$$\begin{aligned} \phi_i(x_1, x_2, \cdots, x_p) &= x_i + h_i(x_i) - \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i)) - \big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i) \\ &\quad +\lambda_i T(x_1, x_2, \cdots, x_p)\big)\big)\big], \text{for all } (x_1, x_2, \cdots, x_p) \in \prod_{i=1}^{p} \widehat{E}_i. \end{aligned} \tag{3.5}$$

Define $\|.\|_*$ on $\prod\limits_{i=1}^{p} \widehat{E}_i$ by

$$\|(x_1, x_2, \cdots, x_p)\|_* = \sum_{i=1}^{p} \|x_i\|_i, \text{ for all } (x_1, x_2, \cdots, x_p) \in \prod_{i=1}^{p} \widehat{E}_i. \tag{3.6}$$

It is easy to check that $\Big(\prod\limits_{i=1}^{p} \widehat{E}_i, \|.\|_*\Big)$ is a ordered product Banach space. Define a mapping $\psi : \prod\limits_{i=1}^{p} \widehat{E}_i \to \prod\limits_{i=1}^{p} \widehat{E}_i$ such that

$$\psi(x_1, x_2, \cdots, x_p) = (\phi_1(x_1, x_2, \cdots, x_p), \phi_2(x_1, x_2, \cdots, x_p), \cdots, \phi_p(x_1, x_2, \cdots, x_p)), \tag{3.7}$$

for all $(x_1, x_2, \cdots, x_m) \in \prod\limits_{i=1}^{p} \widehat{E}_i$. We will show that ψ is a contraction mapping.

Let $(x_1, x_2, \cdots, x_p), (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p) \in \prod\limits_{i=1}^{p} \widehat{E}_i$. By using (3.5), Lipschitz-type continuity of $\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}$ and $\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}$ and as h_i is μ_{h_i}-ordered compression mapping, for each $i \in \Lambda$, we have

$$\begin{aligned} &\|\phi_i(x_1, x_2, \cdots, x_p) \oplus \phi_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_i \\ &= \Big\|\Big[x_i + h_i(x_i) - \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i)) - \big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i) \end{aligned}$$

$$
\begin{aligned}
&+\lambda_i T(x_1,x_2,\cdots,x_p)))\big]\Big] \oplus \Big[\hat{x}_i + h_i(\hat{x}_i) - \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(\hat{x}_i) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i))\\
&-\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i) + \lambda_i T(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)))\big]\Big]\Big\|_i\\
\le\;& \|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i)\|_i + \Big\|\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i))\\
&-\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i) + \lambda_i T(x_1,x_2,\cdots,x_p)))\big] \oplus \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(\hat{x}_i) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i))\\
&-\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i) + \lambda_i T(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)))\big]\Big\|_i\\
\le\;& \|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i)\|_i + \theta_i\Big\|\big[h_i(x_i) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i))\\
&-\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i) + \lambda_i T(x_1,x_2,\cdots,x_p)))\big] \oplus \big[h_i(\hat{x}_i) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i))\\
&-\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i) + \lambda_i T(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)))\big]\Big\|_i\\
\le\;& \|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i)\|_i + \theta_i\Big\|(h_i(x_i) \oplus h_i(\hat{x}_i)) - \omega_i^{-1}\big[\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i))\\
&-\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i)\big) \oplus \big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i)) - \mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i))\big]\Big\|_i + \omega_i^{-1}\theta_i\lambda_i\|T(x_1,x_2,\cdots,x_p)\\
&\oplus T(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)\|_i\\
\le\;& \|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i)\|_i + \theta_i\|(h_i(x_i) \oplus h_i(\hat{x}_i))\|_i\\
&+\omega_i^{-1}\Big\|_i\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i)) - \mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i)\big) \oplus \big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i)) - \mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i)\big)\Big\|_i\\
&+\omega_i^{-1}\theta_i\lambda_i\|T(x_1,x_2,\cdots,x_p) \oplus T(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)\|_i\\
\le\;& \|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i)\|_i + \theta_i\mu_{h_i}\|x_i \oplus \hat{x}_i\|_i\\
&+\omega_i^{-1}\|\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i)) \oplus \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i))\|_i + \omega_i^{-1}\|\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i) \oplus \mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(\hat{x}_i)\big)\|_i\\
&+\omega_i^{-1}\theta_i\lambda_i\|T(x_1,x_2,\cdots,x_p) \oplus T(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)\|_i\\
\le\;& \|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i)\|_i + \omega_i^{-1}(2\theta_i + 1)(1 + \theta_i)\|x_i \oplus \hat{x}_i\|_i\\
&+\theta_i\mu_{h_i}\|x_i \oplus \hat{x}_i\|_i + \omega_i^{-1}\theta_i\lambda_i\|T(x_1,x_2,\cdots,x_p) \oplus T(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)\|_i.
\end{aligned}
\tag{3.8}
$$

Since h_i is μ_{h_i}-ordered compression and λ_{h_i}-ordered rectangular mapping, we have

$$
\begin{aligned}
\|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i))\|_i^2 \;&\le\; \|x_i \oplus \hat{x}_i\|_i^2 + 2\langle h_i(x_i) \oplus h_i(\hat{x}_i), -(x_i \odot \hat{x}_i)\rangle\\
&\quad+\|h_i(x_i) \oplus h_i(\hat{x}_i)\|_i^2\\
&\le\; \|x_i \oplus \hat{x}_i\|_i^2 - 2\lambda_{h_i}\|x_i \oplus \hat{x}_i\|_i^2 + \mu_{h_i}^2\|x_i \oplus \hat{x}_i\|_i^2\\
&=\; (1 - 2\lambda_{h_i} + \mu_{h_i}^2)\|x_i \oplus \hat{x}_i\|_i^2,
\end{aligned}
$$

that is,

$$
\|(x_i + h_i(x_i)) \oplus (\hat{x}_i + h_i(\hat{x}_i))\|_i \le \sqrt{(1 - 2\lambda_{h_i} + \mu_{h_i}^2)}\;\|x_i \oplus \hat{x}_i\|_i. \tag{3.9}
$$

Since T_i is κ_i-ordered compression mapping in the i^{th}-argument and $\kappa_{i,j}$-ordered compression mapping in the j^{th}-argument for each $j \in \Lambda, i \neq j$, we have

$$
\begin{aligned}
&T_i(x_1,x_2,\cdots,x_p) \oplus T_i(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_p)\\
&\quad\le\; T_i(x_1,x_2,\cdots,x_{i-1},x_i,x_{i+1},\cdots,x_p) \oplus T_i(x_1,x_2,\cdots,x_{i-1},\hat{x}_i,x_{i+1},\cdots,x_p)
\end{aligned}
$$

$$
\begin{aligned}
(3.10)\quad & + \sum_{j\in\Lambda,\ i\neq j} (T_i(x_1, x_2, \cdots, x_{j-1}, x_j, x_{j+1}, \cdots, x_p) \oplus \\
& T_i(x_1, x_2, \cdots, x_{j-1}, \hat{x}_j, x_{j+1}, \cdots, x_p)) \\
\leq\ & \kappa_i(x_i \oplus \hat{x}_i) + \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}(x_j \oplus \hat{x}_j).
\end{aligned}
$$

K_i is a normal cone with normal constant δ_i, we have

$$
\|T_i(x_1, x_2, \cdots, x_p) \oplus T_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_i \leq \kappa_i\delta_i\|x_i \oplus \hat{x}_i\|_i + \delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|x_j \oplus \hat{x}_j\|_i.
$$

(3.11)

Using (3.9) and (3.11), (3.8) becomes

$$
\begin{aligned}
& \|\phi_i(x_1, x_2, \cdots, x_p) \oplus \phi_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_i \\
\leq\ & \Big(\sqrt{(1 - 2\lambda_{h_i} + \mu_{h_i}^2)} + \theta_i\mu_{h_i} + \omega_i^{-1}\big((2\theta_i + 1)(1 + \theta_i) + \theta_i\lambda_i(1 + \kappa_i\delta_i)\big)\Big)(p_i \oplus \hat{p}_i) \\
& + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|x_j \oplus \hat{x}_j\|_j.
\end{aligned}
$$

that is

$$
\begin{aligned}
& \|\phi_i(x_1, x_2, \cdots, x_p) \oplus \phi_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_i \\
(3.12)\quad \leq\ & \vartheta_i(x_i \oplus \hat{x}_i) + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|x_j \oplus \hat{x}_j\|_j,
\end{aligned}
$$

where $\vartheta_i = \Big(\sqrt{(1 - 2\lambda_{h_i} + \mu_{h_i}^2)} + \theta_i\mu_{h_i} + \omega_i^{-1}\big((2\theta_i + 1)(1 + \theta_i) + \theta_i\lambda_i(1 + \kappa_i\delta_i)\big)\Big)$.

From (3.6) and (3.12), we obtain

$$
\begin{aligned}
& \|\psi(x_1, x_2, \cdots, x_p) \oplus \psi(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_* \\
=\ & \sum_{i=1}^{p} \|\phi_i(x_1, x_2, \cdots, x_p) \oplus \phi_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_i \\
\leq\ & \sum_{i=1}^{p} \Big(\vartheta_i\|x_i \oplus \hat{x}_i\|_i + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|x_j \oplus \hat{x}_j\|_j\Big) \\
=\ & \Big(\vartheta_1 + \sum_{\ell=2}^{p} \frac{\theta_\ell\lambda_\ell\delta_\ell}{\omega_\ell}\kappa_{\ell,1}\Big)\|x_1 \oplus \hat{x}_1\|_1 \\
& \Big(\vartheta_2 + \sum_{\ell\Lambda,\ \ell\neq 2} \frac{\theta_\ell\lambda_\ell\delta_\ell}{\omega_\ell}\kappa_{\ell,2}\Big)\|x_2 \oplus \hat{x}_2\|_2 + \cdots \\
& + \Big(\vartheta_p + \sum_{\ell=1}^{p-1} \frac{\theta_\ell\lambda_\ell\delta_\ell}{\omega_\ell}\kappa_{\ell,p}\Big)\|x_p \oplus \hat{x}_p\|_p
\end{aligned}
$$

$$\leq \max\Big\{\vartheta_i + \sum_{\ell\in\Lambda,\ \ell\neq i} \frac{\theta_\ell\lambda_\ell\delta_\ell}{\omega_\ell}\kappa_{\ell,i} : i \in \Lambda\Big\} \sum_{i=1}^{p} \|x_i \oplus \hat{x}_i\|_i,$$

which implies that

$$\|\psi(x_1, x_2, \cdots, x_p) \oplus \psi(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_* \leq \Omega\|(x_1, x_2, \cdots, x_p) \oplus (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)\|_*, \tag{3.13}$$

where $\Omega = \max\Big\{\vartheta_i + \sum_{\ell\in\Lambda,\ \ell\neq i} \frac{\theta_\ell\lambda_\ell\delta_\ell}{\omega_\ell}\kappa_{\ell,i} : i \in \Lambda\Big\}$. By Condition (3.4) we have $0 \leq \Omega < 1$. The inequality (3.13) implies that ψ is a contraction mapping. Therefore, there exists a point $(x_1^*, x_2^*, \cdots, x_p^*) \in \prod_{i=1}^{p} \widehat{E}_i$ such that $\psi(x_1^*, x_2^*, \cdots, x_p^*) = (x_1^*, x_2^*, \cdots, x_p^*)$. From (3.5) and (3.7), it follows that $(x_1^*, x_2^*, \cdots, x_p^*)$ satisfies the equation (3.3), that is, for each $i \in \Lambda$,

$$\begin{aligned} h_i(x_i^*) = \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i^*) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*))\\ - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*) + \lambda_i T(x_1^*, x_2^*, \cdots, x_p^*)\big)\big)\big]. \end{aligned} \tag{3.14}$$

By Lemma 3.1, we conclude that $(x_1^*, x_2^*, \cdots, x_p^*) \in \prod_{i=1}^{p} \widehat{E}_i$ is a unique solution of system of Yosida Cayley variational inclusions involving XOR-operation (3.1). This completes the proof. □

4. Two step iterative algorithm, Convergence and stability analysis

We define a perturbed two-step iterative algorithm based on Lemma 3.1 for finding the approximate solution of system of Yosida Cayley variational inclusions involving XOR-operation (3.1). We study the convergence and stability analysis of the proposed algorithm.

Algorithm 4.1. For each $i \in \Lambda$, let $h_i, T_i, \mathcal{M}_i, \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}$ and $\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}$ be the same as in Theorem (3.1). Let $x_{i,n+1} \propto x_{i,n}$,, we compute the iterative sequences $\{x_{i,n}\}_{n=1}^{\infty}$ and $\{y_{i,n}\}_{n=1}^{\infty}$ where $\{x_{i,n}\}_{n=1}^{\infty} = \{(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\}_{n=1}^{\infty}$, and $\{y_{i,n}\}_{n=1}^{\infty} = \{(y_{1,n}, y_{2,n}, \cdots, y_{p,n})\}_{n=1}^{\infty}$, by the following iterative procedure:

$$\begin{cases} x_{i,n+1} = (1-\varphi_n)x_{i,n} + \varphi_n\Big\{y_{i,n} + h_i(y_{i,n}) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(y_{i,n}) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(y_{i,n}))\\ -\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(y_{i,n}) + \lambda_i T(y_{1,n}, y_{2,n}, \cdots, y_{p,n})\big)\big)\big]\Big\} + \tau_{i,n}\varphi_n\\ y_{i,n} = (1-\varsigma_n)x_{i,n} + \varsigma_n\Big\{x_{i,n} + h_i(x_{i,n}) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_{i,n}) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_{i,n}))\\ -\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_{i,n}) + \lambda_i T(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\big)\big)\big]\Big\} + \eta_{i,n}\varsigma_n. \end{cases} \tag{4.1}$$

Let $\{u_{i,n}\}$ be any sequence in $\widehat{E}_i$ and define $\{z_{i,n}\}$ by
(4.2)

$$\begin{cases} z_{i,n+1} = \Big\| u_{i,n+1} - \Big((1-\varphi_n)u_{i,n} + \varphi_n\Big\{t_{i,n} + h_i(t_{i,n}) \\ -\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(t_{i,n}) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(t_{i,n})) \\ -\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(t_{i,n}) + \lambda_i T(t_{1,n}, t_{2,n}, \cdots, t_{p,n})\big)\big)\big]\Big\} + \tau_{i,n}\varphi_n\Big)\Big\|_i \\ t_{i,n} = (1-\varsigma_n)u_{i,n} + \varsigma_n\Big\{s_{i,n} + h_i(s_{i,n}) - \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(s_{i,n}) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(s_{i,n})) \\ -\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(s_{i,n}) + \lambda_i T(s_{1,n}, s_{2,n}, \cdots, s_{p,n})\big)\big)\big] + \eta_{i,n}\varsigma_n, \end{cases}$$

where $0 \le \varphi_n, \varsigma_n \le 1$, $\sum\limits_{n=0}^{\infty} \varphi_n = \infty$, for all $n \ge 0$. For each $i \in \Lambda$, here $\{\tau_{i,n}\}$ and $\{\eta_{i,n}\}$ are two sequences in $\widehat{E}_i$ introduced to take into account the possible inexact computation provided that $\tau_{i,n} \oplus 0 = \tau_{i,n}$ and $\eta_{i,n} \oplus 0 = \eta_{i,n}$, for all $n \ge 0$.

Theorem 4.1. *For each $i \in \Lambda$, let $\widehat{E}_i, h_i, T_i, \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}, \mathcal{C}^{\mathcal{M}_i}_{\lambda_i}$ and $\omega_i > 0$ be the same as in Theorem 3.2 and all the conditions of Theorem 3.2 remain the same. Suppose that $\Omega < \min\{1, \frac{1}{\delta}\}$, for each $i \in \Lambda$, where $\delta = \max\{\delta_1, \delta_2, \cdots, \delta_p\}$ and Ω is same as in* (3.13). *If $\lim\limits_{n\to\infty} \|\tau_{1,n} \vee (-\tau_{1,n}), \tau_{2,n} \vee (-\tau_{2,n}), \cdots, \tau_{p,n} \vee (-\tau_{p,n})\|_* = \lim\limits_{n\to\infty} \|\eta_{1,n} \vee (-\eta_{1,n}), \eta_{2,n} \vee (-\eta_{2,n}), \cdots, \eta_{p,n} \vee (-\eta_{p,n})\|_* = 0$, then*

- (I) *the iterative sequence $\{(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\}_{n=1}^{\infty}$ generated by Algorithm* 4.1 *converges strongly to the unique solution $\{(x_1^*, x_2^*, \cdots, x_p^*)\}$ of system of Yosida Cayley variational inclusions involving XOR-operation (3.1).*
- (II) $\lim\limits_{n\to\infty}(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) = (x_1^*, x_2^*, \cdots, x_p^*)$ *if and only if* $\lim\limits_{n\to\infty}\Big(\sum\limits_{i=1}^{p} z_{i,n}\Big) = 0$, *where $z_{i,n}$ is same as in* (4.2). *Then, the sequence $\{(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\}_{n=1}^{\infty}$ generated by* (4.1) *is $\mathcal{R}^{M_i}_{\lambda_i}$-stable, for each $i \in \Lambda$.*

Proof. (I). Suppose $(x_1^*, x_2^*, \cdots, x_p^*)$ is a unique solution of system of Yosida Cayley variational inclusions involving XOR-operation (3.1). Applying Lemma 3.1, for each $i \in \Lambda$, we have

$$h_i(x_i^*) = \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(x_i^*) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i^*)) - \big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i^*) + \lambda_i T(x_1^*, x_2^*, \cdots, x_p^*)\big)\big)\big]. \tag{4.3}$$

Using Algorithm 4.1, Lemma 2.1, Proposition 2.9, Proposition 2.10 and (4.3), we evaluate

$$\begin{aligned} 0 &\le \|x_{i,n+1} \oplus x_i^*\|_i \\ &= \Big\|\Big[(1-\varphi_n)x_{i,n} + \varphi_n\Big\{y_{i,n} + h_i(y_{i,n}) - \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(y_{i,n}) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(y_{i,n})) \\ &\quad -\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(y_{i,n}) + \lambda_i T(y_{1,n}, y_{2,n}, \cdots, y_{p,n})\big)\big)\big]\Big\} + \tau_{i,n}\varphi_n\Big] \\ &\quad \oplus\Big[(1-\varphi_n)x_i^* + \varphi_n\Big\{x_{i,n}^* + h_i(x_i^*) - \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(x_i^*) - \omega_i^{-1}\big(\mathcal{R}^{\mathcal{M}_i}_{\lambda_i}(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i^*)) \\ &\quad -\big(\mathcal{C}^{\mathcal{M}_i}_{\lambda_i}(x_i^*) + \lambda_i T(x_1^*, x_2^*, \cdots, x_p^*)\big)\big)\big]\Big\}\Big]\Big\|_i \\ &\le (1-\varphi_n)\|x_{i,n} \oplus x_i^*\|_i + \varphi_n\|\tau_{i,n} \oplus 0\|_i + \varphi_n\Big\|\Big(y_{i,n} + h_i(y_{i,n}) - \mathcal{R}^{\mathcal{M}_i}_{\lambda_i}\big[h_i(y_{i,n}) \end{aligned}$$

$$-\omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(y_{i,n})) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(y_{i,n}) + \lambda_i T(y_{1,n}, y_{2,n}, \cdots, y_{p,n})\big)\big)\big]\Big)\oplus$$
$$\Big(x_{i,n}^* + h_i(x_i^*) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i^*) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*))$$
$$-\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*) + \lambda_i T(x_1^*, x_2^*, \cdots, x_p^*)\big)\big)\big]\Big)\Big\|_i. \tag{4.4}$$

Using the same arguments as for (3.12), we evaluate

$$\|x_{i,n+1} \oplus x_i^*\|_i \leq (1-\varphi_n)\|x_{i,n} \oplus x_i^*\|_i + \varphi_n\|\tau_{i,n} \oplus 0\|_i + \varphi_n\Big(\vartheta_i(y_{i,n} \oplus x_i^*)$$
$$+\omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|y_{j,n} \oplus x_j^*\|_j\Big), \tag{4.5}$$

where $\vartheta_i = \Big[\sqrt{(1-2\lambda_{h_i} + \mu_{h_i}^2)} + \theta_i\mu_{h_i} + \omega_i^{-1}\big((2\theta_i+1)(1+\theta_i) + \theta_i\lambda_i(1+\kappa_i\delta_i)\big)\Big]$.

Using the same arguments as for (3.13), (4.5) becomes

$$\|(x_{1,n+1}, x_{2,n+1}, \cdots, x_{p,n+1}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* = \sum_{i=1}^{p} \|x_{i,n+1} \oplus x_i^*\|_i$$
$$\leq \sum_{i=1}^{p}\Big((1-\varphi_n)\|x_{i,n} \oplus x_i^*\|_i + \varphi_n\|\tau_{i,n} \oplus 0\|_i$$
$$+\varphi_n\Big(\vartheta_i(y_{i,n} \oplus x_i^*) + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j}\kappa_{i,j}\|y_{j,n} \oplus x_j^*\|_j\Big)\Big),$$
$$\leq (1-\varphi_n)\|(x_{1,n}, x_{2,n}, \cdots, x_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_*$$
$$+\varphi_n\|(\tau_{1,n}, \tau_{2,n}, \cdots, \tau_{p,n}) \oplus (0, 0, \cdots, 0)\|_*$$
$$+\varphi_n\Omega\|(y_{1,n}, y_{2,n}, \cdots, y_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_*. \tag{4.6}$$

Using the same conditions as used for (4.4)-(4.6), we calculate

$$0 \leq \|y_{i,n} \oplus x_i^*\|_i$$
$$= \Big\|\Big[(1-\varsigma_n)x_{i,n} + \varsigma_n\Big\{x_{i,n} + h_i(x_{i,n}) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_{i,n}) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_{i,n}))$$
$$-\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_{i,n}) + \lambda_i T(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\big)\big)\big]\Big\} + \eta_{i,n}\varsigma_n\Big]$$
$$\oplus\Big[(1-\varsigma_n)x_i^* + \varsigma_n\Big\{x_{i,n}^* + h_i(x_i^*) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i^*) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*))$$
$$-\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*) + \lambda_i T(x_1^*, x_2^*, \cdots, x_p^*)\big)\big)\big]\Big\}\Big]\Big\|_i$$
$$\leq (1-\varsigma_n)\|x_{i,n} \oplus x_i^*\|_i + \varsigma_n\|\eta_{i,n} \oplus 0\|_i + \varsigma_n\Big\|\Big(x_{i,n} + h_i(x_{i,n}) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_{i,n})$$
$$-\omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_{i,n})) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_{i,n}) + \lambda_i T(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\big)\big)\big]\Big) \oplus$$
$$\Big(x_i^* + h_i(x_i^*) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i^*) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*))$$

$$-\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*)+\lambda_i T(x_1^*,x_2^*,\cdots,x_p^*))\big)\big]\Big)\Big\|_i. \tag{4.7}$$

From (4.7), we obtain

$$\begin{aligned}\|y_{i,n}\oplus x_i^*\|_i \quad\le\quad & (1-\varsigma_n)\|x_{i,n}\oplus x_i^*\|_i+\varsigma_n\|\eta_{i,n}\oplus 0\|_i+\varsigma_n\Big(\vartheta_i(x_{i,n}\oplus x_i^*)\\ & +\omega_i^{-1}\theta_i\lambda_i\delta_i\sum_{j\in\Lambda,\ i\neq j}\kappa_{i,j}\|x_{j,n}\oplus x_j^*\|_j\Big),\end{aligned} \tag{4.8}$$

where $\vartheta_i=\Big(\sqrt{(1-2\lambda_{h_i}+\mu_{h_i}^2})+\theta_i\mu_{h_i}+\omega_i^{-1}\big((2\theta_i+1)(1+\theta_i)+\theta_i\lambda_i(1+\kappa_i\delta_i)\big)\Big)$.

Using the same arguments as for (4.6), (4.8) becomes

$$\begin{aligned}&\|(y_{1,n},y_{2,n},\cdots,y_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\le\ \sum_{i=1}^{p}\Big((1-\varsigma_n)\|x_{i,n}\oplus x_i^*\|_i+\varsigma_n\|\eta_{i,n}\oplus 0\|_i\\ &\quad +\varphi_n\Big(\vartheta_i(x_{i,n}\oplus x_i^*)+\omega_i^{-1}\theta_i\lambda_i\delta_i\sum_{j\in\Lambda,\ i\neq j}\kappa_{i,j}\|x_{j,n}\oplus x_j^*\|_j\Big)\Big)\\ &\le\ (1-\varsigma_n)\|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\quad +\varsigma_n\|(\eta_{1,n},\eta_{2,n},\cdots,\eta_{p,n})\oplus(0,0,\cdots,0)\|_*\\ &\quad +\varsigma_n\Omega\|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*,\\ &\le\ \big(1-\varsigma_n(1-\Omega)\big)\|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\quad +\varsigma_n\|(\eta_{1,n},\eta_{2,n},\cdots,\eta_{p,n})\oplus(0,0,\cdots,0)\|_*\\ &\le\ \|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*+\varsigma_n\|(\eta_{1,n},\eta_{2,n},\\ &\quad \cdots,\eta_{p,n})\oplus(0,0,\cdots,0)\|_*,\ \text{since }\big(1-\varsigma_n(1-\Omega)\big)\le 1.\end{aligned} \tag{4.9}$$

Combining (4.6) and (4.9), we obtain

$$\begin{aligned}&\|(x_{1,n+1},x_{2,n+1},\cdots,x_{p,n+1})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\le\ (1-\varphi_n)\|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\quad +\varphi_n\|(\tau_{1,n},\tau_{2,n},\cdots,\tau_{p,n})\oplus(0,0,\cdots,0)\|_*\\ &\quad +\varphi_n\Omega\big(\|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\quad +\varsigma_n\|(\eta_{1,n},\eta_{2,n},\cdots,\eta_{p,n})\oplus(0,0,\cdots,0)\|_*\big)\\ &\le\ \big(1-\varphi_n(1-\Omega)\big)\|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\quad +\Omega\varphi_n\varsigma_n\|\eta_{1,n}\vee(-\eta_{1,n}),\eta_{2,n}\vee(-\eta_{2,n}),\cdots,\eta_{p,n}\vee(-\eta_{p,n})\|_*\\ &\quad +\varphi_n\|\tau_{1,n}\vee(-\tau_{1,n}),\tau_{2,n}\vee(-\tau_{2,n}),\cdots,\tau_{p,n}\vee(-\tau_{p,n})\|_*\\ &\le\ \big(1-\varphi_n(1-\Omega)\big)\|(x_{1,n},x_{2,n},\cdots,x_{p,n})\oplus(x_1^*,x_2^*,\cdots,x_p^*)\|_*\\ &\quad +\varphi_n(1-\Omega)\xi_n,\end{aligned} \tag{4.10}$$

where
$\xi_n=\frac{\Omega\varsigma_n\|\eta_{1,n}\vee(-\eta_{1,n}),\eta_{2,n}\vee(-\eta_{2,n}),\cdots,\eta_{p,n}\vee(-\eta_{p,n})\|_*+\|\tau_{1,n}\vee(-\tau_{1,n}),\tau_{2,n}\vee(-\tau_{2,n}),\cdots,\tau_{p,n}\vee(-\tau_{p,n})\|_*}{(1-\Omega)}$.

On setting $\kappa_n = \|(x_{1,n}, x_{2,n}, \cdots, x_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_*$ and $\mu_n = \varphi_n(1-\Omega)$, inequality (4.10) can be re-written as

$$\kappa_{n+1} \leq (1-\mu_n)\kappa_n + \mu_n \xi_n. \tag{4.11}$$

From Lemma 3.1 and using $\lim_{n\to\infty} \|\eta_{1,n} \vee (-\eta_{1,n}), \eta_{2,n} \vee (-\eta_{2,n}), \cdots, \eta_{p,n} \vee (-\eta_{p,n})\|_* = \lim_{n\to\infty} \|\tau_{1,n} \vee (-\tau_{1,n}), \tau_{2,n} \vee (-\tau_{2,n}), \cdots, \tau_{p,n} \vee (-\tau_{p,n})\|_* = 0$, we deduce that $\kappa_n \to 0$, as $n \to \infty$, and so $\{(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\}$ converges strongly to a unique solution $(x_1^*, x_2^*, \cdots, x_p^*)$ of system of Yosida Cayley variational inclusions involving XOR-operation (3.1).

(II). Let $H_i(x_i^*) = x_i^* + h_i(x_i^*) - \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i^*) - \omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*)) - \big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i^*) + \lambda_i T(x_1^*, x_2^*, \cdots, x_p^*)\big)\big)\big]$. Using Algorithm 4.1 and Lemma 2.1, we obtain

$$\begin{aligned}
0 &\leq \|u_{i,n+1} \oplus x_i^*\|_i \\
&\leq \|u_{i,n+1} \oplus \big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big)\|_i \\
&\quad + \|\big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big) \oplus x_i^*\|_i \\
&\leq \|u_{i,n+1} \oplus \big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big)\|_i \\
&\quad + \|\big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big) \oplus \big((1-\varphi_n)x_i^* + \varphi_n H_i(x_i^*)\big)\|_i \\
&\leq \|u_{i,n+1} - \big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big)\|_i \\
&\quad + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i + \varphi_n \|H_i(t_{i,n}) \oplus H_i(x_i^*))\|_i + \varphi_n \|\tau_{i,n} \oplus 0\|_i \\
&\leq z_{i,n+1} + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i + \varphi_n \|H_i(t_{i,n}) \oplus H_i(x_i^*))\|_i + \varphi_n \|\tau_{i,n} \oplus 0\|_i \\
&\leq z_{i,n+1} + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i \\
&\quad + \varsigma_n\Big(\vartheta_i(t_{i,n} \oplus x_i^*) + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|t_{j,n} \oplus x_j^*\|_j\Big) \\
&\quad + \varphi_n \|\tau_{i,n} \oplus 0\|_i,
\end{aligned} \tag{4.12}$$

where $\vartheta_i = \Big(\sqrt{(1-2\lambda_{h_i} + \mu_{h_i}^2)} + \theta_i\mu_{h_i} + \omega_i^{-1}\big((2\theta_i+1)(1+\theta_i) + \theta_i\lambda_i(1+\kappa_i\delta_i)\big)\Big)$.

Using the same argument as for (4.6), (4.12) becomes

$$\begin{aligned}
&\|(u_{1,n+1}, u_{2,n+1}, \cdots, u_{p,n+1}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&\quad \leq \sum_{i=1}^{p}\Big(z_{i,n+1} + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i + \varphi_n\|\tau_{i,n} \oplus 0\|_i \\
&\qquad + \varphi_n\Big(\vartheta_i(t_{i,n} \oplus x_i^*) + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|t_{j,n} \oplus x_j^*\|_j\Big)\Big), \\
&\quad \leq \sum_{i=1}^{p}\Big(z_{i,n+1}\Big) + (1-\varphi_n)\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&\qquad + \varphi_n\|(\tau_{1,n}, \tau_{2,n}, \cdots, \tau_{p,n}) \oplus (0, 0, \cdots, 0)\|_* \\
&\qquad + \varphi_n\Omega\|(t_{1,n}, t_{2,n}, \cdots, t_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_*.
\end{aligned} \tag{4.13}$$

Using the same argument as for (4.9), it follows that

$$\begin{aligned}
&\|(t_{1,n}, t_{2,n}, \cdots, t_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
\leq\ & \sum_{i=1}^{p} \Big((1-\varsigma_n)\|u_{i,n} \oplus x_i^*\|_i + \varsigma_n\|\eta_{i,n} \oplus 0\|_i \\
&+\varphi_n\Big(\vartheta_i(u_{i,n} \oplus x_i^*) + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|u_{j,n} \oplus x_j^*\|_j\Big)\Big), \\
\leq\ & (1-\varsigma_n)\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&+\varsigma_n\|(\eta_{1,n}, \eta_{2,n}, \cdots, \eta_{p,n}) \oplus (0, 0, \cdots, 0)\|_* \\
&+\varsigma_n\Omega\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_*, \\
\leq\ & \big(1-\varsigma_n(1-\Omega)\big)\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&+\varsigma_n\|(\eta_{1,n}, \eta_{2,n}, \cdots, \eta_{p,n}) \oplus (0, 0, \cdots, 0)\|_* \\
\leq\ & \|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* + \varsigma_n\|(\eta_{1,n}, \eta_{2,n}, \\
&\cdots, \eta_{p,n}) \oplus (0, 0, \cdots, 0)\|_*, \text{ since } \big(1-\varsigma_n(1-\Omega)\big) \leq 1.
\end{aligned} \tag{4.14}$$

Combining (4.13) and (4.14), we obtain

$$\begin{aligned}
&\|(u_{1,n+1}, u_{2,n+1}, \cdots, u_{p,n+1}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
\leq\ & \sum_{i=1}^{p}\Big(z_{i,n+1}\Big) + (1-\varphi_n)\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&+\varphi_n\|(\tau_{1,n}, \tau_{2,n}, \cdots, \tau_{p,n}) \oplus (0, 0, \cdots, 0)\|_* \\
&+\varphi_n\Omega\big(\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&+\varsigma_n\|(\eta_{1,n}, \eta_{2,n}, \cdots, \eta_{p,n}) \oplus (0, 0, \cdots, 0)\|_*\big) \\
\leq\ & \sum_{i=1}^{p}\Big(z_{i,n+1}\Big) + \big(1-\varphi_n(1-\Omega)\big)\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&+\Omega\varphi_n\varsigma_n\|\eta_{1,n} \vee (-\eta_{1,n}), \eta_{2,n} \vee (-\eta_{2,n}), \cdots, \eta_{p,n} \vee (-\eta_{p,n})\|_* \\
&+\varphi_n\|\tau_{1,n} \vee (-\tau_{1,n}), \tau_{2,n} \vee (-\tau_{2,n}), \cdots, \tau_{p,n} \vee (-\tau_{p,n})\|_* \\
\leq\ & \sum_{i=1}^{p}\Big(z_{i,n+1}\Big) + \big(1-\varphi_n(1-\Omega)\big)\|(x_{1,n}, x_{2,n}, \cdots, x_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\
&+\varphi_n(1-\Omega)\xi_n,
\end{aligned} \tag{4.15}$$

where ξ_n is same as defined in Part (I).

Assume that $\lim\limits_{n\to\infty} \sum_{i=1}^{p}\Big(z_{i,n+1}\Big) = 0$, hence $\lim\limits_{n\to\infty}(u_{1,n+1}, u_{2,n+1}, \cdots, u_{p,n+1}) = (x_1^*, x_2^*, \cdots, x_p^*)$, where $\lim\limits_{n\to\infty} \|\eta_{1,n} \vee (-\eta_{1,n}), \eta_{2,n} \vee (-\eta_{2,n}), \cdots, \eta_{p,n} \vee (-\eta_{p,n})\|_* = \lim\limits_{n\to\infty} \|\tau_{1,n} \vee (-\tau_{1,n}), \tau_{2,n} \vee (-\tau_{2,n}), \cdots, \tau_{p,n} \vee (-\tau_{p,n})\|_* = 0$.

Conversely, suppose that $\lim\limits_{n\to\infty}(u_{1,n+1}, u_{2,n+1}, \cdots, u_{p,n+1}) = (x_1^*, x_2^*, \cdots, x_p^*)$. From (4.3) and applying $\lim\limits_{n\to\infty} \|\eta_{1,n} \vee (-\eta_{1,n}), \eta_{2,n} \vee (-\eta_{2,n}), \cdots, \eta_{p,n} \vee (-\eta_{p,n})\|_* =$

$\lim_{n\to\infty} \|\tau_{1,n} \vee (-\tau_{1,n}), \tau_{2,n} \vee (-\tau_{2,n}), \cdots, \tau_{p,n} \vee (-\tau_{p,n})\|_* = 0$, we have

$$\begin{aligned} z_{i,n} &= \|u_{i,n+1} - \big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big)\|_i \\ &= \|u_{i,n+1} \oplus \big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big)\|_i \\ &\leq \|u_{i,n+1} \oplus x_i^*\|_i \\ &\quad + \|\big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big) \oplus x_i^*\|_i \\ &\leq \|u_{i,n+1} \oplus x_i^*\|_i \\ &\quad + \|\big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big) \oplus \big((1-\varphi_n)x_i^* + \varphi_n H_i(x_i^*)\big)\|_i \\ &\leq \|u_{i,n+1} - x_i^*\|_i \\ &\quad + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i + \varphi_n \|H_i(t_{i,n}) \oplus H_i(x_i^*))\|_i + \varphi_n \|\tau_{i,n} \oplus 0\|_i \\ &\leq \|u_{i,n+1} - x_i^*\|_i + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i \\ &\quad + \varphi_n \|H_i(t_{i,n}) \oplus H_i(x_i^*))\|_i + \varphi_n \|\tau_{i,n} \oplus 0\|_i \\ &\leq \|u_{i,n+1} - x_i^*\|_i + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i \\ &\quad + \varsigma_n \Big(\vartheta_i(t_{i,n} \oplus x_i^*) + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|t_{j,n} \oplus x_j^*\|_j\Big) \\ &\quad + \varphi_n \|\tau_{i,n} \oplus 0\|_i. \end{aligned} \tag{4.16}$$

From (4.16), it follows that

$$\begin{aligned} \sum_{i=1}^{p}\Big(z_{i,n+1}\Big) &= \sum_{i=1}^{p} \|u_{i,n+1} - \big((1-\varphi_n)u_{i,n} + \varphi_n H_i(t_{i,n}) + \varphi_n \tau_{i,n}\big)\|_i \\ &\leq \sum_{i=1}^{p}\Big(\|u_{i,n+1} - x_i^*\|_i + (1-\varphi_n)\|u_{i,n} \oplus x_i^*\|_i + \varsigma_n\Big(\vartheta_i(t_{i,n} \oplus x_i^*) \\ &\quad + \omega_i^{-1}\theta_i\lambda_i\delta_i \sum_{j\in\Lambda,\ i\neq j} \kappa_{i,j}\|t_{j,n} \oplus x_j^*\|_j\Big) + \varphi_n\|\tau_{i,n} \oplus 0\|_i\Big) \\ &\leq \|(u_{1,n+1}, x_{2,n+1}, \cdots, x_{p,n+1}) - (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\ &\quad + \big(1 - \varphi_n(1-\Omega)\big)\|(u_{1,n}, u_{2,n}, \cdots, u_{p,n}) \oplus (x_1^*, x_2^*, \cdots, x_p^*)\|_* \\ &\quad + \varphi_n(1-\Omega)\xi_n. \end{aligned} \tag{4.17}$$

It follows from (4.17) that

$$\lim_{n\to\infty} \sum_{i=1}^{p}\Big(z_{i,n+1}\Big) = 0.$$

Hence, the iterative sequence $\{(x_{1,n}, x_{2,n}, \cdots, x_{p,n})\}_{n=1}^{\infty}$ generated by Algorithm (4.1) is stable with respect to $\mathcal{R}_{\lambda_i}^{M_i}$-stable, for each $i \in \Lambda$. □

In support of conditions mentioned in Theorem 3.2, we provide the following example.

Example 4.2. For each $i \in \Lambda = \{1, 2, 3, \cdots, p\}$, and let $\widehat{E}_i = \mathbb{R}, i \in \Lambda = \{1, 2, 3, \cdots, p\}$ with the usual inner product and norm and $K_i = \{x_i \in \mathcal{H}_i : 0 \leq$

$x_i \leq 1\}$ be a normal cone with normal constant $\delta_i = \frac{1}{i}$. Let $h_i : \widehat{E}_i \to \widehat{E}_i$ and $T_i : \prod_{j=1}^{p} \widehat{E}_j \to \widehat{E}_i$ be the single-valued mappings such that for all $x_i \in \widehat{E}_i$

$$h_i(x_i) = \frac{x_i}{13i} + \frac{1}{35i-7}$$

and

$$T_i(x_1, x_2, \cdots, x_{j-1}, x_j, x_{j+1}, \cdots, x_p) = \frac{x_j}{30ij}, \text{ for each } j \in \Lambda.$$

We evaluate,

$$\begin{aligned} h_i(x_i) \oplus h_i(\hat{x}_i) &= \Big(\frac{x_i}{13i} + \frac{1}{35i-7}\Big) \oplus \Big(\frac{\hat{x}_i}{13i} + \frac{1}{35i-7}\Big) \\ &\leq \Big(\frac{x_i}{13i} \oplus \frac{\hat{x}_i}{13i}\Big) + \Big(\frac{1}{35i-7} \oplus \frac{1}{35i-7}\Big) \\ &= \frac{1}{13i}(x_i \oplus \hat{x}_i) \\ &\leq \frac{1}{10i}(x_i \oplus \hat{x}_i), \end{aligned}$$

that is,

$$h_i(x_i) \oplus h_i(\hat{x}_i) \leq \frac{1}{10i}(x_i \oplus \hat{x}_i).$$

and

$$\begin{aligned} \langle h_i(x_i) \odot h_i(\hat{x}_i), -(x_i \oplus \hat{x}_i)\rangle_i &= \Big\langle \Big(\frac{x_i}{13i} + \frac{1}{35i-7}\Big) \oplus \Big(\frac{\hat{x}_i}{13i} + \frac{1}{35i-7}\Big), x_i \oplus \hat{x}_i \Big\rangle_i \\ &= \Big\langle \frac{1}{13i}(x_i \oplus \hat{x}_i), x_i \oplus \hat{x}_i \Big\rangle_i \\ &= \frac{1}{13i}\|x_i \oplus \hat{x}_i\|_i^2 \\ &\geq \frac{1}{10i}\|x_i \oplus \hat{x}_i\|_i^2, \end{aligned}$$

that is

$$\langle h_i(x_i) \odot h_i(\hat{x}_i), -(x_i \oplus \hat{x}_i)\rangle_i \geq \frac{1}{10i}\|x_i \oplus \hat{x}_i\|_i^2.$$

Thus, h_i is $\frac{1}{10i}$-ordered compression and $\frac{1}{10i}$-ordered rectangular mapping.
Also,

$$\begin{aligned} & T_i(x_1, x_2, \cdots, x_{j-1}, x_j, x_{j+1}, \cdots, x_p) \oplus T_i(x_1, x_2, \cdots, x_{i-1}, \hat{x}_i, x_{i+1}, \cdots, x_p) \\ = \ & \frac{x_i}{30i^2} \oplus \frac{\hat{x}_i}{30i^2} = \frac{1}{30i^2}(x_i \oplus \hat{x}_i) \\ \leq \ & \frac{1}{30i}(x_i \oplus \hat{x}_i). \end{aligned}$$

That is, T_i is $\frac{1}{30i}$-ordered compression mapping in the i^{th} argument.

So, we have

$T_i(x_1, x_2, \cdots, x_p) \oplus T_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_p)$

$$\begin{aligned}
&\leq T_i(x_1, x_2, \cdots, x_{i-1}, x_i, x_{i+1}, \cdots, x_p) \oplus T_i(x_1, x_2, \cdots, x_{i-1}, \hat{x}_i, x_{i+1}, \cdots, x_p) \\
&\quad + \sum_{j \in \Lambda,\ i \neq j} (T_i(x_1, x_2, \cdots, x_{j-1}, x_j, x_{j+1}, \cdots, x_p) \oplus \\
&\quad T_i(x_1, x_2, \cdots, x_{j-1}, \hat{x}_j, x_{j+1}, \cdots, x_p)) \\
&= \frac{1}{30i^2}(x_i \oplus \hat{x}_i) + \sum_{j \in \Lambda,\ i \neq j} \frac{1}{30ij}(x_j \oplus \hat{x}_j) \\
&\leq \frac{1}{30i}(x_i \oplus \hat{x}_i) + \sum_{j \in \Lambda,\ i \neq j} \frac{1}{30ij}(x_j \oplus \hat{x}_j).
\end{aligned}$$

Let $M_i : \widehat{E}_i \to 2^{\widehat{E}_i}$ be a set-valued mapping such that

$$M_i(x_i) = \{5i^2 x_i\}, \text{ for all } x_i \in \widehat{E}_i$$

and $v_{x_i} = 5i^2 x_i \in M_i(x_i)$ and $v_{y_i} = 5i^2 y_i \in M_i(y_i)$.
Then,

$$\begin{aligned}
\langle v_{x_i} \odot v_{y_i}, -(x_i \oplus y_i) \rangle_i &= \langle (5i^2 x_i) \odot (5i^2 y_i), -(x_i \oplus y_i) \rangle_i \\
&= \langle (5i^2 x_i) \oplus (5i^2 y_i), (x_i \oplus y_i) \rangle_i \\
&= 5i^2 \|x_i \oplus y_i\|_i^2 \\
&\geq 2i^2 \|x_i \oplus y_i\|_i^2,
\end{aligned}$$

that is

$$\langle v_{x_i} \odot v_{y_i}, -(x_i \oplus y_i) \rangle_i \geq 2i^2 \|x_i \oplus y_i\|_i^2.$$

Clearly, M is a $2i^2$-ordered rectangular compression mapping and also it is easy to verify that M is $\frac{1}{i}$-weak-ordered different comparison mapping. For $\lambda_i = \frac{1}{i}$, we have $[I_i \oplus \lambda_i M_i](\widehat{E}_i) = \widehat{E}_i$. Hence, M_i is an $(2i^2, \frac{1}{i})$-weak ordered rectangular different set-valued mapping.

The resolvent operator is given by

$$\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(x_i) = \frac{1}{5i-1} x_i, \text{ for all } x_i \in \widehat{E}_i. \tag{4.18}$$

It is easy to examine that the resolvent operator defined (4.18) is a comparison and single-valued mapping.
Also,

$$\begin{aligned}
\left\| \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(x_i) \oplus \mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(y_i) \right\|_i &= \left\| \left(\frac{x_i}{5i-1} \right) \oplus \left(\frac{y_i}{5i-1} \right) \right\|_i \\
&= \frac{1}{5i-1} \|x_i \oplus y_i\|_i \\
&\leq \frac{1}{2i-1} \|x_i \oplus y_i\|_i, \text{ for all } x_i, y_i \in \widehat{E}_i.
\end{aligned}$$

So, the resolvent operator $\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}$ is $\frac{1}{2i-1}$-Lipschitz-type continuous.

The Cayley operator is given by

$$\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i) = [2\mathcal{R}_{\lambda_i}^{\mathcal{M}_i} - I_i](x_i) = \frac{3-5i}{5i-1} x_i, \text{ for all } x_i \in \widehat{E}_i. \tag{4.19}$$

It is easy to check that the Cayley operator defined by (4.19) is comparison and single-valued mapping. Also,

$$\begin{aligned}\Big\|\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)\oplus\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(y_i)\Big\|_i &= \Big\|\Big(\frac{3-5i}{5i-1}x_i\Big)\oplus\Big(\frac{3-5i}{5i-1}y_i\Big)\Big\|_i\\ &= \frac{3-5i}{5i-1}\|x_i\oplus y_i\|_i\\ &\leq \frac{2i+1}{2i-1}\|x_i\oplus y_i\|_i,\end{aligned}$$

that is,

$$\Big\|\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)\oplus\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(y_i)\Big\|_i \leq \frac{2i+1}{2i-1}\|x_i\oplus y_i\|_i, \text{ for all } x_i,y_i\in\widehat{E}_i.$$

That is, the Cayley operator $\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}$ is $\frac{2i+1}{2i-1}$-Lipschitz-type continuous. In particular $\omega_i = 2i\in\Lambda$ and we define $\phi_i:\prod_{j=1}^{p}\widehat{E}_j\to\widehat{E}_i$ by

$$\begin{aligned}&\phi_i(x_1,x_2,\cdots,x_p)\\ =\;& x_i+h_i(x_i)-\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}\big[h_i(x_i)-\omega_i^{-1}\big(\mathcal{R}_{\lambda_i}^{\mathcal{M}_i}(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i))-\big(\mathcal{C}_{\lambda_i}^{\mathcal{M}_i}(x_i)\\ &+\lambda_i T(x_1,x_2,\cdots,x_p)\big)\big)\big]\\ =\;& x_i+\frac{x_i}{13i}+\frac{1}{35i-7}-\Big(\frac{x_i}{13i(5i-1)}+\frac{1}{7(5i-1)}-\frac{3-5i}{2i(5i-1)^3}x_i\\ &+\frac{3-5i}{2i(5i-1)^2}x_i-\frac{1}{60i^4(5i-1)}x_i\Big)\\ =\;& \Big(1+\frac{1}{13i}-\frac{1}{13i(5i-1)}+\frac{(5i-2)(3-5i)}{2i(5i-1)^3}-\frac{1}{60i^4(5i-1)}\Big)x_i.\end{aligned}$$

For $\mu_{h_i},\lambda_{h_i}=\frac{1}{10i},\kappa_i=\frac{1}{30i},\kappa_{ij}=\frac{1}{30ij},\lambda_i,\delta_i=\frac{1}{i},\omega_i=2i$ and $\theta_i=\frac{1}{2i-1}$, the condition (3.4) is also satisfied. So, all the conditions of Theorem 3.2 are satisfied. Therefore, $(0,0,\cdots,0)$ is a fixed point of the mapping $\psi(.,.,\cdots,.) = (\phi_1(.),\phi_2(.),\cdots,\phi_p(.))$ defined by (3.7). By Lemma 3.1, $(0,0,\cdots,0)$ is a solution of system of Yosida Cayley variational inclusions involving XOR-operation (3.1).

5. Conclusion

This work is dedicated to introduce and study a system of Yosida Cayley variational inclusions involving XOR-operation. Using resolvent operator technique, we obtain the solution of our system in real ordered Banach space. In order to define two step perturbed algorithm, we prove an equivalence Lemma. Convergence as well as stability analysis are also discussed. An example is constructed in support of our main result.

We remark that our results may be generalized further in higher order Banach spaces as well as in other higher dimensional spaces. Engineers using the functional analysis may find concrete applications of our results.

References

[1] R. P. Agarwal, Y. J. Cho and N. J. Huang, *Sensitivity analysis for strongly nonlinear quasi-variational inclusions*, Appl. Math. Lett. **13** (2000), 19–24.

[2] R. P. Agarwal, N. J. Huang and M. Y. Tan, *Sensitivity Analysis for a new system of generalized Nonlinear mixed quasi-variational inclusions*, Appl. Math Lett. **17** (2004) 345-3-52.

[3] I. Ahmad, A. Abdullah and S. S. Irfan, *Nonlinear System of mixed ordered variational inclusions involving XOR-operation*, Aust. J. Math Anal. **18** (2021), pp. 15.

[4] I. Ahmad, C. T. Pang, R. Ahmad and M. Ishtyak, *System of Yosida inclusions involving XOR operator* J. Nonlinear Convex Anal. **18** (2017), 831–845.

[5] T. O. Alakoya, L. O. Jolaoso and O. T. Mewomo, *A general iterative method for finding common fixed point of finite family of demicontractive mappings with accretive variational inequality probems in Banach spaces*, Nonlinear Stud. **27** (2020) 213–236.

[6] I. Ali, R. Ahmad and C. F. Wen, *Cayley inclusion probem involving XOR-operation*, Mathematics **7** (2019): 302.

[7] Q. H. Ansari, J. C. Yao, *A fixed point theorem and its applications to a system of variational inequalities*, Bull. Austral. Math Soc., **59** (1999), 433-442.

[8] M. Bianchi, *Pseudo P-monotone operators and variational inequalities. Report 6, Istituto di Econometria e Mathematica per le Decisioni Economiche*, Universita Cattolica del Sacro Cuore, Milan, Italy (1993).

[9] Y. J. Cho, Y. P. Fang, N. J. Huang, H. J. Hwang, *Algorithms for systems of nonlinear variational inequalities*, J. Korean Math Soc. **41** (2004), 489-499.

[10] G. Cohen, F. Chaplais, *Nested monotony for variational inequalities over a product of spaces and convergence of iterative algorithms*, J. Optim. Theory Appl. **59** (1988), 360–390.

[11] A. De, *Hille-Yosida Theorem and Some Applications*, PhD Thesis, Department of Mathematics and its Applications, Central European University, Budapest, Hungary.

[12] X. P. Ding, *Perturbed proximal point algorithms for generalized quasi variational inclusions*, J. Math Anal. Appl. **210** (1997), 88–101.

[13] X. P. Ding and C. L. Luo, *Perturbed proximal point algorithms for generalized quasi-variational-like inclusions*, J. Comput. Appl. Math. **113** (2000), 153–165.

[14] Y. H. Du, *Fixed points of increasing operators in ordered Banach spaces and applications*, Appl. Anal. **38** (1990), 1–20.

[15] M. Eslamian and A. Kamandi, *A novel algorithm for approximating common solution of a system of monotone inclusion problems and common fixed point problem*, J. Ind. Manag. Optim. (2021).

[16] A. G. Gebrie and B. Bekele, *Viscosity self-adaptive method for generalized split system of variational inclusion problem*, Bull. Iran Math Soc. **47** (2021), 897–917.

[17] A. Hassouni and A. Moudafi, *A Perturbed algorithm for variational inclusions*, J. Math Anal. Appl. **185** (1994), 706–712.

[18] G Helmberg, *Introduction to spectral Theory in Hilbert Space: The Cayley Transform*, Applied Mathematics and Mechanics. 1969;6 #6, North Holland, 1969.

[19] C. Izuchukwu, A. Mebawondu and O. T. Mewomo, *A new method for solving split variational inequality problems without co-coerciveness*, J. Fixed Point Theory Appl. **22** (2020): Article ID 98:23 Pages.

[20] J. U. Jeong, *Generalized set-valued variational inclusions and resolvent equations in Banach spaces*, Comput. Math Appl. **47** (2004), 1241–1247.

[21] L. O. Jolaoso, A. Taiwo, T. O. Alakoya and O. T. Mewomo, *A unified algorithm for solving variational inequality and fixed point problems with application to the split equality problem*, Comput. Appl. Math. **39(1)** (2020).

[22] P. Kumam, I. K. Argyros, N. A. Alreshidi, W. Kumam and W. Jirakitpuwapat, *A Self-Adaptive Extra-Gradient Methods for a Family of Pseudomonotone Equilibrium Programming with Application in Different Classes of Variational Inequality Problems*, Symmetry **12** (2020): #523.

[23] H. G. Li, *Approximation solution for generalized nonlinear ordered variatinal inequality and ordered equations in ordered Banach space*, Nonlinear Anal. Forum, **13** (2008), 205–214.

[24] H. G. Li, *Approximation solution for a new class general nonlinear ordered variatinal inequalities and ordered equations in ordered Banach space*, Nonlinear Anal. Forum **14** (2009), 89–97.
[25] D. T. Luc and N. X. Tan, *Existence conditions in variational inclusions with constraints*, Optimization **53** (2004), 505–515.
[26] J. S. Pang, *Asymmetric variational inequality problems over product sets: applications and iterative methods*, Math Programming **31** (1985), 206–219.
[27] J. W. Peng, *System of generalized set-valued quasi-variational-like inequalities*, J. Comput. Appl. Math, **13)** (2001), 111-124.
[28] Y. Qiu and X. Liu, *Iterative Algorithms for a System of Variational Inclusions involving Set-valued Quasi-contractive mappings in Banach spaces*, Numer. Func. Anal. Opt. **42** (2021), 865–882.
[29] Z. A. Rather, R. Ahmad and C. F. Wen, *Variational-like inequality problem involving generalized Cayley Operator*, Axioms **1** (2021), p.133.
[30] H. Rehman, P. Kumam, I. K. Argyros, M. Shutaywi and Z. Shah, *Optimization based methods for solving the equilibrium problems with applications in variational inequality problems and solution of Nash equilibrium models*, Mathematics **5** (2020), 1–28.
[31] H. A. Ricardo, *Modern Introduction to Linear Algebra*, Taylor and Francis Group. Boca Raton : CRC Press, 2010.
[32] R. T. Rockafellar, *Monotone operators and the proximal point algorithm*, SIAM J. Control Optim. , **14** (1976), 877–898.
[33] D. R. Sahu and A. K. Singh, *Inertial iterative algorithms for common solution of variational inequality and system of variational inequalities*, J. Appl. Math Comput. **65** (2021), 351-378.
[34] S. Shafi and L. N. Mishra, *Iterating a system of variational-like inclusions problems in Banach spaces*, Journal of Nonlinear Modelling and Analysis **3** (2021), 283-300.
[35] B. Tan, X, Qin and J. C. Yao, *Two modified inertial projection algorithms for bilevel pseudomonotone variational inequalities with applications to optimal control problems*, Numerical Algorithms **88** (2021), 1757–1786.
[36] N. X. Tan, *On the existence of solutions of quasivariational inclusion problem*, J. Optim. Theory Appl. **123** (2004), 619–638.
[37] D. V. Thong, D. V. Hieu, *Mann-type algorithms for variational inequality problems and fixed point problems*, Optimization **69** (2020), 2305–2326.
[38] R. U. Verma, *On a new system of nonlinear variational inequalities and associated iterative algorithms*, Math Sci. Res Hot-Line **3** (2001), 65–68.
[39] R. U. Verma, *Projection Methods, algorithms and a new system of nonlinear variational inequalities*, Comput. Math Appl. **41** (2001), 1025–1031.
[40] R. U. Verma, *Iterative algorithms and a new system of nonlinear quasivariational inequalities*, Adv. Nonlinear Var. Inequal. **4** (2001), 117–124.
[41] R. U. Verma, *A-monotonicity and applications to nonlinear variational inclusion problems*, J. Appl. Math Stoc. Anal. **17** (2004), 193–195.
[42] R. U. Verma, *Sensitivity analysis for generalized strongly monotone variational inclusions based on the (A,η)-resolvent operator technique*, Appl. Math Lett. **16** (2006), 1409–1413.
[43] Y. Yao, M. Postolache and J. C. Yao, *Strong convergence of an extragradient algorithm for variational inequality and fixed point problems*, Sci. Bull. Series A **82(1)** (2021).

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