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EXISTENCE OF EQUILIBRIUM POINTS FOR ABSTRACT ECONOMIES IN THE TOPOLOGICAL VECTOR SPACE SETTING

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Dedicated to Professor Adrian Petrusel with much admiration

Abstract. In this paper using a general fixed point result in Hausdorff topological vector spaces we establish new equilibrium results for abstract economies.

1. Introduction

In this setting we present a variety of general existence theorems for equilibria of abstract economies. For the model presented and discussed the strategy sets are subsets of Hausdorff topological vector spaces, the constraint and preference correspondences are usually either lower semicontinuous or upper semicontinuous and the set of agents are either countable or uncountable. Our equilibrium results are derived from a fixed point theorem in the literature (see [7]) and our theory improves and generalizes corresponding results in the literature (see [1, 3, 8, 9, 10, 11] and the references therein).

Now we recall a fixed point result [7] in the literature. First we recall the following notions from the literature. For a subset K of a topological space X, we denote by $Cov_X(K)$ the directed set of all coverings of K by open sets of X (usually we write $Cov(K) = Cov_X(K)$). Given two maps $F, G : X \to 2^Y$ (here 2^Y denotes the family of nonempty subsets of Y) and $\alpha \in Cov(Y)$, F and G are said to be α–close if for any $x \in X$ there exists $U_x \in \alpha$, $y \in F(x) \cap U_x$ and $w \in G(x) \cap U_x$.

Let Q be a class of topological spaces. A space Y is an extension space for Q (written $Y \in ES(Q)$) if for any pair (X, K) in Q with $K \subseteq X$ closed, any continuous function $f_0 : K \to Y$ extends to a continuous function $f : X \to Y$. A space Y is an approximate extension space for Q (written $Y \in AES(Q)$) if for any $\alpha \in Cov(Y)$ and any pair (X, K) in Q with $K \subseteq X$ closed, and any continuous function $f_0 : K \to Y$ there exists a continuous function $f : X \to Y$ such that $f|_K$ is α–close to f_0.

Let V be a subset of a Hausdorff topological vector space E. Then we say V is Schauder admissible if for every compact subset K of V and every covering $\alpha \in Cov_V(K)$ there exists a continuous function $\pi_\alpha : K \to V$ such that
(i). π_α and $i : K \to V$ are α–close;
(ii). $\pi_\alpha(K)$ is contained in a subset $C \subseteq V$ with $C \in AES(\text{compact})$.

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An upper semicontinuous map $\phi : X \to CK(Y)$ is said to Kakutani (and we write $\phi \in Kak(X, Y)$); here $CK(Y)$ denotes the family of nonempty, convex, compact subsets of Y.

Theorem 1.1. *Let I be an index set and $\{X_i\}_{i\in I}$ be a family of sets each in a Hausdorff topological vector space E_i. For each $i \in I$ let K_i be a nonempty compact subset of X_i and suppose $F_i : X \equiv \prod_{i\in I} X_i \to K_i$ is upper semicontinuous with nonempty convex compact values (i.e. $F_i \in Kak(X, K_i)$). Also assume $K \equiv \prod_{i\in I} K_i$ is a Schauder admissible subset of the Hausdorff topological vector space $E \equiv \prod_{i\in I} E_i$. Then there exists a $x \in K$ with $x_i \in F_i(x)$ for $i \in I$ (here x_i is the projection of x on X_i).*

Remark 1.2. One could repace K a Schauder admissible subset of E in Theorem 1.1 (and the other results in this paper) with other admissible subsets of E described in [6].

We now state a result from the literature [8] which will be used in Section 2.

Theorem 1.3. *Let X and Y be two topological spaces and A an open (respectively, closed) subset of X. Suppose $F_1 : X \to 2^Y$, $F_2 : A \to 2^Y$ (here 2^Y denotes the family of nonempty subsets of Y) are upper semicontinuous (respectively, lower semicontinuous) such that $F_2(x) \subset F_1(x)$ for all $x \in A$. Then the map $F : X \to 2^Y$ defined by*

$$F(x) = \begin{cases} F_1(x), & x \notin A, \\ F_2(x), & x \in A \end{cases}$$

is upper semicontinuous (respectively, lower semicontinuous).

Finally we note in the next section $C(Y)$ denotes the family of nonempty, convex, closed subsets of Y.

2. Abstract economy results

Let I be the set of agents and we describe the abstract economy as $\Gamma = (X_i, A_i, P_i)_{i\in I}$ where $A_i : X \equiv \prod_{i\in I} X_i \to 2^{E_i}$ is a constraint correspondence, $P_i : X \to 2^{E_i}$ is a preference correspondence and X_i is a choice (or strategy) set which is a subset of a Hausdorff topological vector space E_i. We are interested in finding an equilibrium point for Γ i.e. a point $x \in X$ with $x_i \in A_i(x)$ and $A_i(x) \cap P_i(x) = \emptyset$ for $i \in I$.

Theorem 2.1. *Let $\Gamma = (X_i, A_i, P_i)_{i\in I}$ be an abstract economy with $\{X_i\}_{i\in I}$ a family of nonempty sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$ assume the following conditions:*

$$A_i : X \equiv \prod_{i\in I} X_i \to CK(E_i) \quad \textit{is upper semicontinuous} \tag{2.1}$$

$$U_i = \{x \in X : A_i(x) \cap P_i(x) \neq \emptyset\} \quad \textit{is open in } X \tag{2.2}$$

$$P_i|_{U_i} : U_i \to C(E_i) \quad \textit{is upper semicontinuous} \tag{2.3}$$

$$x_i \notin A_i(x) \cap P_i(x) \ \textit{ if } x \in U_i; \ \textit{here } x_i \textit{ is the projection of } x \textit{ on } E_i \tag{2.4}$$

and

$$(2.5)\qquad \begin{cases} \textit{there exists a nonempty compact subset } K_i \textit{ of } X_i \textit{ with} \\ A_i : X \to CK(K_i) \textit{ and } K \equiv \prod_{i\in I} K_i \textit{ is a Schauder} \\ \textit{admissible subset of } E \equiv \prod_{i\in I} E_i. \end{cases}$$

Then there exists a $x \in X$ with $x_i \in A_i(x)$ and $A_i(x) \cap P_i(x) = \emptyset$ for each $i \in I$.

Proof. Fix $i \in I$ and let $B_i(x) = A_i(x) \cap P_i(x)$ for $x \in U_i$. Note (see [2 pg 470]) the map $B_i : U_i \to 2^{E_i}$ is upper semicontinuous. Let $G_i : X \to 2^{E_i}$ be defined by

$$G_i(x) = \begin{cases} B_i(x)\,(= A_i(x) \cap P_i(x)), & x \in U_i \\ A_i(x), & x \notin U_i \end{cases}$$

and note $B_i(x) \subseteq A_i(x)$ for $x \in U_i$, so Theorem 1.3 guarantees that $G_i : X \to CK(E_i)$ is upper semicontinuous. Also for each $i \in I$ we have $G_i(x) \subseteq A_i(x) \subseteq K_i$ for $x \in X$ so $G_i \in Kak(X, K_i)$. Now Theorem 1.1 guarantees a $x \in K$ with $x_i \in G_i(x)$ for $i \in I$. If $x \in U_i$ for some $i \in I$ then $x_i \in A_i(x) \cap P_i(x)$, which contradicts (2.4). Thus for each $i \in I$ we must have $x \notin U_i$ (i.e. $A_i(x) \cap P_i(x) = \emptyset$) and $x_i \in A_i(x)$. □

We next replace (2.3) with a more general assumption.

Theorem 2.2. *Let $\Gamma = (X_i, A_i, P_i)_{i\in I}$ be an abstract economy with $\{X_i\}_{i\in I}$ a family of nonempty sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$ assume* (2.1), (2.2), (2.4) *and* (2.5) *hold and in addition assume*

$$(2.6)\qquad \begin{cases} \textit{there exists a upper semicontinuous selector} \\ \Phi_i : U_i \to C(E_i) \textit{ of } A_i \cap P_i|_{U_i} : U_i \to 2^{E_i}. \end{cases}$$

Then there exists a $x \in X$ with $x_i \in A_i(x)$ and $A_i(x) \cap P_i(x) = \emptyset$ for each $i \in I$.

Proof. Fix $i \in I$ and let $H_i : X \to 2^{E_i}$ be defined by

$$H_i(x) = \begin{cases} \Phi_i(x), & x \in U_i \\ A_i(x), & x \notin U_i \end{cases}$$

and note $\Phi_i(x) \subseteq A_i(x) \cap P_i(x) \subseteq A_i(x)$ for $x \in U_i$, so Theorem 1.3 guarantees that $H_i : X \to CK(E_i)$ is upper semicontinuous and $H_i(x) \subseteq A_i(x) \subseteq K_i$ for $x \in X$ so $H_i \in Kak(X, K_i)$. Now Theorem 1.1 guarantees a $x \in K$ with $x_i \in H_i(x)$ for $i \in I$. If $x \in U_i$ for some $i \in I$ then $x_i \in \Phi_i(x) \subseteq A_i(x) \cap P_i(x)$, which contradicts (2.4). Thus for each $i \in I$ we must have $x \notin U_i$ (i.e. $A_i(x) \cap P_i(x) = \emptyset$) and $x_i \in A_i(x)$. □

Remark 2.3. (i). Note in Theorem 2.2 we could replace (2.4) with: $x_i \notin \Phi_i(x)$ for $x \in U_i$. (ii). Of course there are other obvious analogues of Theorem 2.1 and Theorem 2.2 for various combinations if A_i is replaced by $co\,A_i$ or $\overline{co}\,A_i$ (respectively, if P_i is replaced by $co\,P_i$ or $\overline{co}\,P_i$).

Corollary 2.4. *Let $\Gamma = (X_i, A_i, P_i)_{i\in I}$ be an abstract economy with $\{X_i\}_{i\in I}$ a family of nonempty sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$ assume* (2.1), (2.2), (2.4) *and* (2.5) *hold and in addition assume*

$$(2.7)\qquad \begin{cases} \textit{there exists a continuous (single valued) selection} \\ f_i : U_i \to E_i \textit{ of } A_i \cap P_i|_{U_i} : U_i \to 2^{E_i}. \end{cases}$$

Then there exists a $x \in X$ *with* $x_i \in A_i(x)$ *and* $A_i(x) \cap P_i(x) = \emptyset$ *for each* $i \in I$.

Proof. Note (2.7) is a special case of (2.6). □

Remark 2.5. If E is a Fréchet space (or a more general space, see [4, 5]) and $A_i \cap P_i|_{U_i} : U_i \to C(E_i)$ is lower semicontinuous then (2.7) holds (see [4, Theorem 1.2] and [5, Theorem 1.2]).

Next we will consider the situation when U_i is open in X is replaced by U_i is closed in X.

Theorem 2.6. *Let* $\Gamma = (X_i, A_i, P_i)_{i\in I}$ *be an abstract economy with* $\{X_i\}_{i\in I}$ *a family of nonempty sets each in a Hausdorff topological vector space* E_i *(here* I *is an index set). For each* $i \in I$ *assume* (2.4) *and* (2.5) *hold and in addition assume*

$$A_i : X \equiv \prod_{i\in I} X_i \to CK(E_i) \quad \text{is lower semicontinuous} \tag{2.8}$$

$$U_i = \{x \in X :\ A_i(x) \cap P_i(x) \neq \emptyset\} \quad \text{is closed in } X \tag{2.9}$$

and

$$\begin{cases} \text{there exists a lower semicontinuous selector} \\ \Phi_i : U_i \to C(E_i) \ \text{ of } A_i \cap P_i|_{U_i} : U_i \to 2^{E_i}. \end{cases} \tag{2.10}$$

For $i \in I$ *suppose*

$$\begin{cases} \text{for any lower semicontinuous map } G_i : X \to CK(E_i) \\ \text{there exists a upper semicontinuous selection} \\ \Psi_i : X \to CK(E_i) \ \text{ of } G_i. \end{cases} \tag{2.11}$$

Then there exists a $x \in X$ *with* $x_i \in A_i(x)$ *and* $A_i(x) \cap P_i(x) = \emptyset$ *for each* $i \in I$.

Remark 2.7. (i), If we assume $A_i \cap P_i|_{U_i} : U_i \to C(E_i)$ is lower semicontinuous, then (2.10) holds (take $\Phi_i = A_i \cap P_i|_{U_i}$).

(ii). If we assume (2.7) holds (i.e. there exists a continuous (single valued) selection $f_i : U_i \to E_i$ of $A_i \cap P_i|_{U_i} : U_i \to 2^{E_i}$) then (2.10) holds (take $\Phi_i = \{f_i\}$).

(iii) If we assume

$$\begin{cases} \text{for any lower semicontinuous map } G_i : X \to CK(E_i) \\ \text{there exists a continuous (single valued) selection} \\ g_i : X \to E_i \ \text{ of } G_i, \end{cases} \tag{2.12}$$

then (2.11) holds (take $\Psi_i = \{g_i\}$).

Proof. Fix $i \in I$ and let

$$F_i(x) = \begin{cases} \Phi_i(x), & x \in U_i \\ A_i(x), & x \notin U_i \end{cases}$$

and note $\Phi_i(x) \subseteq A_i(x) \cap P_i(x) \subseteq A_i(x)$ for $x \in U_i$, so Theorem 1.3 guarantees that $F_i : X \to CK(E_i)$ is lower semicontinuous. Now (2.11) guarantees that there exists a upper semicontinuous selection $\Psi_i : X \to CK(E_i)$ of F_i and note $\Psi_i(x) \subseteq A_i(x) \subseteq K_i$ for $x \in X$ so $\Psi_i \in Kak(X, K_i)$. Now Theorem 1.1 guarantees a $x \in K$ with $x_i \in \Psi_i(x)$ for $i \in I$ so in particular we have $x_i \in F_i(x)$ for $i \in I$ since Ψ_i is a selection of F_i. If $x \in U_i$ for some $i \in I$ then $x_i \in F_i(x) = \Phi_i(x) \subseteq A_i(x) \cap P_i(x)$, which contradicts (2.4). Thus for each $i \in I$ we must have $x \notin U_i$ (i.e. $A_i(x) \cap P_i(x) = \emptyset$) and $x_i \in A_i(x)$. $\square$

Remark 2.8. In the proof of Theorem 2.6 we note that (2.11) (respectively, (2.12)) could be replaced with the assumption: there exists a upper semicontinuous selection $\Psi_i : X \to CK(E_i)$ of F_i (respectively, there exists a continuous (single valued) selection $g_i : X \to E_i$ of F_i). We note that Φ_i in F_i would be $A_i \cap P_i|_{U_i}$ if the assumption in Remark 2.7 (i) holds (i.e. $A_i \cap P_i|_{U_i} : U_i \to C(E_i)$ is lower semicontinuous) and the Φ_i in F_i would be $\{f_i\}$ if the assumption in Remark 2.7 (ii) holds (i.e. there exists a continuous (single valued) selection $f_i : U_i \to E_i$ of $A_i \cap P_i|_{U_i} : U_i \to 2^{E_i}$). Also note with the general assumption above (i.e. there exists a upper semicontinuous selection $\Psi_i : X \to CK(E_i)$ of F_i) there is do not need to assume $A_i : X \to CK(E_i)$ or $\Phi_i : U_i \to C(E_i)$ (a selector of $A_i \cap P_i|_{U_i} : U_i \to 2^{E_i}$) are lower semicontinuous since we do not need to apply Theorem 1.3 in this situation. Also note we do not need to assume U_i is closed in X in this situation.

Motivated by Remark 2.3 and Remark 2.8 we will now consider a more general situation. Let I be the set of agents and we describe the abstract economy as $\Gamma = (X_i, A_i, B_i, P_i)_{i \in I}$ where A_i, $B_i : X \equiv \prod_{i \in I} X_i \to 2^{E_i}$ are constraint correspondences, $P_i : X \to 2^{E_i}$ is a preference correspondence and X_i is a choice (or strategy) set which is a subset of a Hausdorff topological vector space E_i.

Theorem 2.9. *Let $\Gamma = (X_i, A_i, B_i, P_i)_{i \in I}$ be an abstract economy with $\{X_i\}_{i \in I}$ a family of nonempty sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$ assume the following conditions:*

$$U_i = \{x \in X : \overline{co}\,(A_i(x)) \cap P_i(x) \neq \emptyset\} \text{ is open in } X \tag{2.13}$$

$$\begin{cases} \text{there exists a upper semicontinuous selector} \\ \Phi_i : U_i \to C(E_i) \text{ of } \overline{co}\,A_i \cap P_i|_{U_i} : U_i \to 2^{E_i} \end{cases} \tag{2.14}$$

$$cl\,B_i\,(= \overline{B_i}) : X \equiv \prod_{i \in I} X_i \to CK(E_i) \text{ is upper semicontinuous} \tag{2.15}$$

$$\overline{co}\,(A_i(x)) \cap P_i(x) \subseteq \overline{B_i}(x) \text{ for } x \in U_i \tag{2.16}$$

$$x_i \notin \overline{co}\,(A_i(x)) \cap P_i(x) \text{ if } x \in U_i \tag{2.17}$$

and

$$\begin{cases} \text{there exists a nonempty compact subset } K_i \text{ of } X_i \text{ with} \\ \overline{B_i} : X \to CK(K_i) \text{ and } K \equiv \prod_{i \in I} K_i \text{ is a Schauder} \\ \text{admissible subset of } E \equiv \prod_{i \in I} E_i. \end{cases} \tag{2.18}$$

Then there exists a $x \in X$ with $x_i \in \overline{B_i}(x)$ and $\overline{co}\,(A_i(x)) \cap P_i(x) = \emptyset$ for each $i \in I$.

Proof. Fix $i \in I$ and let

$$F_i(x) = \begin{cases} \Phi_i(x), & x \in U_i \\ \overline{B_i}(x), & x \notin U_i \end{cases}$$

and note $\Phi_i(x) \subseteq \overline{co}\,(A_i(x)) \cap P_i(x) \subseteq \overline{B_i}(x))$ for $x \in U_i$, so Theorem 1.3 guarantees that $F_i : X \to CK(E_i)$ is upper semicontinuous. Also for each $i \in I$ we have $F_i(x) \subseteq \overline{B_i}(x) \subseteq K_i$ for $x \in X$ so $F_i \in Kak(X, K_i)$. Now Theorem 1.1 guarantees a $x \in K$ with $x_i \in F_i(x)$ for $i \in I$. If $x \in U_i$ for some $i \in I$ then $x_i \in \Phi_i(x) \subseteq \overline{co}\,(A_i(x)) \cap P_i(x)$, which contradicts (2.17). Thus for each $i \in I$ we must have $x \notin U_i$ (i.e. $\overline{co}\,(A_i(x)) \cap P_i(x) = \emptyset$) and $x_i \in \overline{B_i}(x)$. □

Remark 2.10. Of course there are other obvious analogues of Theorem 2.9 for various combinations if $\overline{co}\,A_i$ is replaced by $co\,A_i$ or A_i (respectively, if P_i is replaced by $co\,P_i$ or $\overline{co}\,P_i$). A similar comment applies to $\overline{B_i}$.

Theorem 2.11. *Let $\Gamma = (X_i, A_i, B_i, P_i)_{i\in I}$ be an abstract economy with $\{X_i\}_{i\in I}$ a family of nonempty sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$ let $U_i = \{x \in X : A_i(x) \cap P_i(x) \neq \emptyset\}$ and*

$$F_i(x) = \begin{cases} \overline{co}\,(A_i(x) \cap P_i(x)), & x \in U_i \\ \overline{B_i}(x), & x \notin U_i. \end{cases}$$

and assume the following conditions:

$$co\,(A_i(x) \cap P_i(x)) \subseteq B_i(x) \quad \text{for} \quad x \in U_i \tag{2.19}$$

$$x_i \notin \overline{co}\,(A_i(x) \cap P_i(x)) \quad \text{if } x \in U_i \tag{2.20}$$

$$\begin{cases} \text{there exists a upper semicontinuous selector} \\ \Psi_i : X \to CK(E_i) \ \text{ of } F_i \end{cases} \tag{2.21}$$

and

$$\begin{cases} \text{there exists a nonempty compact subset } K_i \text{ of } X_i \text{ with} \\ \overline{B_i} : X \to 2^K_i \ \text{ and } K \equiv \prod_{i\in I} K_i \ \text{ is a Schauder} \\ \text{admissible subset of } E \equiv \prod_{i\in I} E_i. \end{cases} \tag{2.22}$$

Then there exists a $x \in X$ with $x_i \in \overline{B_i}(x)$ and $A_i(x) \cap P_i(x) = \emptyset$ for each $i \in I$.

Proof. Fix $i \in I$ and note if $x \in U_i$ then $\overline{co}\,(A_i(x) \cap P_i(x)) \subseteq \overline{B_i}(x) \subseteq K_i$ and so $\Psi_i(x) \subseteq F_i(x) \subseteq K_i$ for $x \in X$ and thus $\Psi_i \in Kak(X, K_i)$. Now Theorem 1.1 guarantees a $x \in K$ with $x_i \in \Psi_i(x) \subseteq F_i(x)$ for $i \in I$. If $x \in U_i$ for some $i \in I$ then $x_i \in \overline{co}\,(A_i(x) \cap P_i(x))$, which contradicts (2.20). Thus for each $i \in I$ we have $x \notin U_i$ (i.e. $A_i(x) \cap P_i(x) = \emptyset$) and $x_i \in \Psi_i(x) \subseteq \overline{B_i}(x)$. □

Remark 2.12. Of course there are other obvious analogues of Theorem 2.11 for various combinations if $\overline{co}\,A_i$ is replaced by $co\,A_i$ or A_i (respectively, if P_i is replaced by $co\,P_i$ or $\overline{co}\,P_i$). A similar comment applies to $\overline{B_i}$.

Remark 2.13. For each $i \in I$ assume

$$U_i \ \text{ is closed in } \ X \tag{2.23}$$

$$A_i \cap P_i|_{U_i} : U_i \to 2^{E_i} \ \text{ is lower semicontinuous} \tag{2.24}$$

and

$$B_i : X \to Cc(E_i) \ \text{ is lower semicontinuous;} \tag{2.25}$$

here $Cc(E_i)$ denotes the family of nonempty convex subsets of E_i. Now for each $i \in I$ let

$$\Lambda_i(x) = \begin{cases} A_i(x) \cap P_i(x), & x \in U_i \\ B_i(x), & x \notin U_i \end{cases}$$

and note $A_i(x) \cap P_i(x) \subseteq co\,(A_i(x) \cap P_i(x)) \subseteq B_i(x)$ for $x \in U_i$ (see (2.19)) and so Theorem 1.3 guarantees that $\Lambda_i : X \to 2^{E_i}$ is lower semicontinuous. Also note $\overline{co}\,(\Lambda_i(x)) \subseteq F_i(x)$ for $x \in X$ (see (2.25) as well).

If in addition for each $i \in I$ we assume

(2.26) $$E_i \ \text{ is a Fréchet space}$$

(or see [1 ,10] for a more general situation) then the above and [1, Theorem 1.1] guarantees that (2.21) holds.

Remark 2.14. For each $i \in I$ assume (2.25) holds and

(2.27) $$A_i \cap P_i : X \to 2^{E_i} \ \text{ is lower semicontinuous.}$$

Note (2.27) guarantees that U_i is open in X for each $i \in I$ and now let Λ_i be as in Remark 2.13 (and note $A_i(x) \cap P_i(x) \subseteq B_i(x)$ for $x \in U_i$). Also note that $\Lambda_i : X \to 2^{E_i}$ is lower semicontinuous since for any closed subset Ω of E_i note

$$\begin{aligned} \{x \in X : \Lambda_i(x) \subseteq \Omega\} &= \{x \in U_i : A_i(x) \cap P_i(x) \subseteq \Omega\} \\ &\cup \ \{x \in X \backslash U_i : B_i(x) \subseteq \Omega\} \\ &= \{x \in X : A_i(x) \cap P_i(x) \subseteq \Omega\} \\ &\cup \ \{x \in X \backslash U_i : B_i(x) \subseteq \Omega\} \\ &= \{x \in X : A_i(x) \cap P_i(x) \subseteq \Omega\} \\ &\cup \ [(X \backslash U_i) \cap \{x \in X : B_i(x) \subseteq \Omega\}] \end{aligned}$$

which is closed in X (note if $x \in X \backslash U_i$ then $A_i(x) \cap P_i(x) = \emptyset$). Also again note $\overline{co}\,(\Lambda_i(x)) \subseteq F_i(x)$ for $x \in X$.

If in addition for each $i \in I$ we assume (2.26) holds then [1, Theorem 1.1] guarantees that (2.21) holds.

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