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FLEXIBLE JOB SHOP SCHEDULING PROBLEM WITH SETUP TIMES AND ROUTING STRUCTURE: A REAL-LIFE CASE

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Abstract. In this paper, we introduce a novel approach for the flexible job shop scheduling problem that includes a routing structure into the flexible job shop scheduling problem. The paper proposes a mathematical model for this problem, which is based on a vehicle routing problem's routing structure to schedule a manufacturing system with a facility flexibility and setup times. The proposed approach is applied to a real-life scheduling problem in the food industry. This problem is solved for six facilities and sixty-six types of products, by using a standard software, and the computational results are analyzed to validate the effectiveness of the proposed mathematical model.

1. Introduction

The decision-making process by optimizing multiple objectives and assigning production resources to jobs for specific times is called scheduling [20]. Scheduling problems try to establish the most efficient usage of machinery, facilities, and resources for manufacturing while meeting the orders on time. By using efficient scheduling techniques, customers' demands can be responded to on time, lead times can be shortened, inventory turnover rates can be reduced, and capacities can be used efficiently.

The scheduling problem is a necessary decision-making process in the manufacturing and service industries. Especially determining and analyzing the necessary constraints plays a significant role in solving scheduling problems. Concepts like raw material stock, efficient production time, and manufacturing line occupancy are the most critical parameters that define necessary constraints.

In manufacturing, using machines under or over capacity, delays in due time of the orders, while using one machine constantly keeping the others empty, having excessive stocks, over time, and low labor usage create massive problems in workplaces. Scheduling is the most efficient tool to overcome these problems.

The production planning function aims to optimize the firm's product mix and the long-term allocation of resources, taking into account resource needs, demand forecasts, and stock levels. However, scheduling can be defined as using appropriate machinery, labor, and materials, to achieve the most recent production goals. It puts forward a clear path taking into account the conditions. The most significant difference between production planning and scheduling is the timeframe in which they are used and the detail of the method.

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There is a set of machines or facilities and a set of jobs in a job scheduling problem. According to the predefined routine, each job is processed in a machine or plant, and each machine or plant can only do one type of job. In modernized production facilities, a machine can process more than one job. These machines with multiple programming fall into the category of flexible job scheduling problems [5].

Scheduling problems have been studied since 1960 with different aspects. Shapiro proposed various mathematical programming models and methods for production planning and scheduling in his study [22]. Pan [19] provided the examination and comparison of mixed-integer linear programming formulas for workflow problems. Kim and Egbelu developed a mixed-integer linear programming model for the job scheduling problem, in which each job has predetermined alternative process plans. They also developed two algorithms, each consisting of three models. These three models consist of selecting process plan combinations, calculating the lower limit in production time, and scheduling [13]. Brandimarte proposed an approach based on taking a multi-objective programming problem with process plan flexibility and breaking it down into machine loading and scheduling sub-problems. Brandimarte's work only dealt with machine loading issues, namely choosing an action plan for each job and assigning processes to machines [4]. Low and Wu [15] have addressed the problem of flexible scheduling to minimize overall latency, taking into account setup times. This problem is formulated as a mixed-integer programming model, and then the quadratic terms in the constraints are linearized. Thomalla proposed a discrete-time integer programming model for the flexible job shop scheduling problem in order to minimize the sum of the weighted quadratic delays of the jobs. He also developed an algorithm based on Lagrangian stretch [23]. Choi and Choi set up a mixed-integer programming model for the flexible job shop scheduling problem, as well as a search algorithm with a dispatch rule applied to obtain an upper bound on the construction time of a subproblem [6]. Gomes developed a flexible job shop scheduling problem with limited intermediate buffers and a multi-objective discrete-time model that divides the schedule into several intervals of equal duration. This model was later generalized to account for recirculation, where jobs could visit certain groups of machines multiple times [10]. Gao formulated non-fixed machine suitability constraints in which each machine is subject to an arbitrary number of preventive maintenance tasks. He proposed a hybrid genetic algorithm to solve the problem [8]. Fattahi developed a mixed-integer programming model and six different search structures that they used to solve flexible job shop scheduling problems [7]. Liu, Sun, Yan, and Kang [14] presented the adaptive annealing genetic algorithm in 2011, demonstrating that it is more efficient than conventional methods for workplace planning, scheduling, one-piece production, and small batch production. Moradi, Ghomi, and Zandieh [16] developed four algorithms in 2011 to solve the flexible job shop scheduling problem with preventive maintenance activities under multi-objective optimization approaches.

Ho and Haugland approached the vehicle routing problem with the tabu search heuristic with time windows and split deliveries [11]. They explained the vehicle routing problem as distributing goods, people, or information between warehouses and customers. Pop et al. published their study about generalized vehicle routing problems and extensions [21]. The generalized vehicle routing problem is an extension of the vehicle routing problem. This study aims to provide two new models based on integer programming. Alpaslan Takan and Kasimbeyli proposed a new hybrid subgradient method for solving the capacitated vehicle routing problem, which is nonconvex and nondifferentiable due to integer variable constraints [1]. This method is based on the Gasimov's modified subgradient algorithm, which was developed for solving general constrained optimization problems without convexity and differentiability conditions [9]. Combinatorial optimization problems have a nonconvex structure due to the decision variables they contain. This situation has led to the importance of exact solution methods developed for problems with such a structure [2, 3, 12, 17].

In this paper, we study the flexible job shop scheduling problem. We consider a real-life problem of scheduling specialized facilities in the food industry. In order to solve this problem, we construct a mathematical model by using the vehicle routing problem's routing structure and scheduling problem's timeframe. In the scheduling part of the problem, the mathematical model given in [18] is taken as a reference. Ozguven et al. developed a mixed-integer linear programming model for flexible job shop scheduling problems without setup times between operations. In their study, every job has an ordered set of operations, and by using this order, they create a route for operations. However, in our study, each job is defined uniquely, without ordered sub-operations. In addition, the setup times between jobs are taken into account. We use the vehicle routing problem's constraints to obtain routes of jobs for each facility. To indicate the start and endpoint of the routes, we define dummy starting and ending jobs, which are assigned to each facility.

The following is how the paper is structured. Section 2 describes the flexible job shop scheduling problem and explains the mixed-integer programming model for this problem. Section 3 presents computational results, and finally, Section 4 draws some conclusions from the study.

2. Problem definition

The makespan (C_{max}) is the time it takes for the last job to be completed (when it leaves the system) in scheduling theory. A lower C_{max} indicates higher efficiency. Therefore, minimization of the "makespan" is the objective of this study.

This paper proposes a mathematical model for a flexible job shop scheduling problem, which is based on a vehicle routing problem's routing structure applied to schedule a manufacturing system with facility flexibility and setup times. The obtained solution assigns each process to one of the alternative facilities, and consequently arranges the operation in the proper sequence to minimize the makespan. We consider a real-life problem that occurs in a food production plant. Different varieties of food products are produced in this plant. Each type of product is subjected to different processes that have its own processing times according to the characteristics of the facilities as well as the product types. There are six facilities in the plant for the production of these products. Most of the product types can be manufactured in all facilities. However, some products cannot be processed in some facilities due to existing technological constraints.

In the current situation, scheduling plans are made manually based on the knowledge and experience of production planning managers. The given deadline, personnel status, line occupancy, raw material, material stock status, and arrival dates are considered while creating the schedules. Since more than one type of product is processed in the same facility, a setup time occurs when switching from one product to another. The setup time is due to the differences in the pan, mold, and recipe used in producing the products. Due to these differences, the setup times differ from one variety to another.

In this study, we aim to create the most appropriate monthly production schedule by minimizing the makespan and considering the setup times and other constraints. By assigning the products to the facilities in the proper order and at the right time, the plant will save on labor and ease its burden in terms of cost because overwork of the facilities will be prevented, thereby easing the workload of the employees. It also minimizes order completion times and ensures efficient use of time. Due to the definitions of scheduling problems in the literature, each product will be called a job in the problem description.

2.1. **Sets and parameters.** The sets that we use in this paper are defined as follows:

Jobs will be denoted by $i, j, q \in \{1, 2..., N\}$,

$i_0 = \{Dummy\ Starting\ Job\} \cup i$,

$i_1 = i \cup \{Dummy\ Ending\ Job\}$,

$i_2 = \{Dummy\ Starting\ Job\} \cup i \cup \{Dummy\ Ending\ Job\}$,

$h = \{1, 2..., K\}$ *Facilities.*

Parameters

- s_{ijh} denotes the setup time on facility h between jobs j and i;
- d_{ih} denotes the process time of job i on facility h;
- u_{ih} takes value 1 if job i can be processed on facility h, otherwise 0;
- H_i denotes the set of facilities that job i can be processed;
- $H_i = \{h : u_{ih} = 1\}$;
- α be a sufficiently small number to indicate starting time of facilities, $\alpha > 0$;
- M be a sufficiently large number.

2.2. **Decision variables.**

$$x_{ih} = \begin{cases} 1, & \text{if job } i \text{ is assigned to facilty } h, \\ 0, & \text{otherwise.} \end{cases}$$

$$z_{ijh} = \begin{cases} 1, & \text{if job } j \text{ is assigned to facilty } h \text{ after job } i, \\ 0, & \text{otherwise.} \end{cases}$$

y_{ih} = the starting time of job i on facility h.

t_{ih} = the ending time of job i on facility h.

C_{max} = the completion time of the final job.

f_{ih} = arbitrary positive real number.

2.3. **Mathematical model.** The mathematical model of the flexible job shop scheduling problem considered in this paper is as follows:

$$\min C_{max} \tag{2.1}$$

s.t.

$$\sum_{j} z_{ijh} = 1, \qquad \forall h, \quad i = 0, \tag{2.2}$$

$$\sum_{i} z_{ijh} = 1, \qquad \forall h, \quad j = N+1, \tag{2.3}$$

$$\sum_{i \in i_0, i \neq j} z_{ijh} - \sum_{q \in i_1, j \neq q} z_{jqh} = 0, \quad \forall h, \quad \forall j, \tag{2.4}$$

$$\sum_{j \in i_1} z_{ijh} = x_{ih}, \qquad \forall i, \quad \forall h, \quad i \neq j, \tag{2.5}$$

$$\sum_{i \in i_0} z_{ijh} = x_{jh}, \qquad \forall j, \quad \forall h, \quad i \neq j, \tag{2.6}$$

$$y_{ih} = \alpha, \quad \forall h, \quad i = 0, \tag{2.7}$$

$$\sum_{h \in H_i} x_{ih} = 1, \qquad \forall i, \tag{2.8}$$

$$x_{ih} \leq u_{ih}, \qquad \forall i, \quad \forall h, \tag{2.9}$$

$$x_{ih} = 1, \qquad \forall h, \quad i = 0, \tag{2.10}$$

$$x_{ih} = 1, \qquad \forall h, \quad i = N+1, \tag{2.11}$$

$$t_{ih} = y_{ih} + d_{ih}x_{ih}, \qquad \forall i \in i_2, \quad \forall h, \tag{2.12}$$

$$y_{jh} \geq y_{ih} - M(1 - z_{ijh}) + s_{ijh}z_{ijh} + d_{ih}x_{ih}, \qquad \forall i, \quad \forall j, \quad \forall h, \tag{2.13}$$

$$f_{ih} - f_{jh} + Nz_{ijh} \leq N - 1, \qquad \forall i \in i_1, \quad \forall j \in i_1, \quad \forall h, \tag{2.14}$$

$$t_{ih} \leq C_{max}, \qquad \forall h, i = N + 1, \tag{2.15}$$

$$x_{ih} \in \{0, 1\}, z_{ijh} \in \{0, 1\}, y_{ih} \geq 0, f_{ih} \geq 0, t_{ih} \geq 0 \quad \forall i, \quad \forall j, \quad \forall p. \tag{2.16}$$

The objective function (2.1) minimizes the maximum completion time. The constraint sets (2.2) and (2.3) assures that the dummy starting and ending jobs are the first and the last jobs, respectively, on all facilities. Constraint set (2.4) guarantees the flow between jobs. Once a job is finished on a facility, the next one is started. Constraints (2.5) ensure that once a job is assigned to a facility if it is not the dummy ending job, there must be a premise job. Similarly, constraints (2.6) ensure that once a job is assigned to a facility and if it is not the dummy starting job, there must be the following job. The starting time is accomplished with the constraint set (2.7). Constraint set (2.8) and (2.9) make sure that every job must be assigned to a suitable facility. The constraint set (2.10) and (2.11) assures that the dummy starting and ending jobs are assigned to all facilities. The completion time of a job is calculated with equation (2.12). Constraint set (2.13) makes sure that starting time of a job is greater than the starting time of the preceding job on that facility. Since we construct the structure of this problem as a vehicle routing problem, subtour elimination is guaranteed with constraints (2.14). Constraint set (2.15) obtains the completion time of the last job. Finally, the constraints (2.16) define all of the decision variables.

3. Computational results

The mathematical model described in Section 2 is used to solve a real-life problem that occurs in a food production plant with six facilities. Seventy-two types of products are produced in these facilities. However, it was decided that the distributors did not request six types of products over time and were excluded from the production plan. Therefore, the developed model is solved for sixty six types of products. The production times of each product in the facilities vary. In addition, some products cannot be processed in some facilities. The former production schedule for the plant is given in Table 1. As it can be seen from Table 1, the former makespan of the plant is 15169.2 minutes, and sixteen jobs were assigned to Facility-5. The former total production time of all facilities is 69206.2.

We solve the same problem by keeping the exact parameters and implementing the GAMS 33.2 software and run the experiments on a Windows PC with a 2.8 GHz IntelCore i7 processor and 8 GB RAM. The obtained results are given in

Table 1. Former Production Schedule of Plant

Facility-1	Facility-2	Facility-3	Facility-4	Facility-5	Facility-6
14	15	40	68	60	28
8	22	33	66	39	5
2	7	37	67	51	20
1	12	53	72	46	16
23	11	38	64	57	19
10	18	44	69	36	26
31	13	32	65	49	27
3	24	35	71	56	30
17	25	41		61	9
		50		59	21
		34		48	
		43		52	
		42		54	
		45		55	
				47	
				58	
Process Times for Facilities					
8471.94	8196.15	13581.1	14620	15169.2	9167.8

Table 2. Improved Production Schedule of Plant

Facility-1	Facility-2	Facility-3	Facility-4	Facility-5	Facility-6
50	12	32	71	51	2
10	11	39	66	50	26
3	15	42	69	56	21
9	17	35	64	58	23
31	7	44	67	61	28
1	13	34	68	57	30
27	18	45	72	49	20
14	16	37	65	52	
8	22	41		54	
	24	33		55	
	19	43		47	
	25	60		46	
		38		48	
		53		36	
		40		59	
Process Times for Facilities					
8515.63	11264.6	14143.5	14160	14076	6607.61

Table 2. The improved makespan of the plant is 14160 minutes. The improved total production time of all facilities is decreased to 68767.287.

The comparison of the completion time of each facility is demonstrated in Figure 1. While the completion time of Facility-1, Facility-2, and Facility-3 increases, the completion time of Facility-4, Facility-5, and Facility-6 decreases.

The improved production routes for products are given in Figure 2.

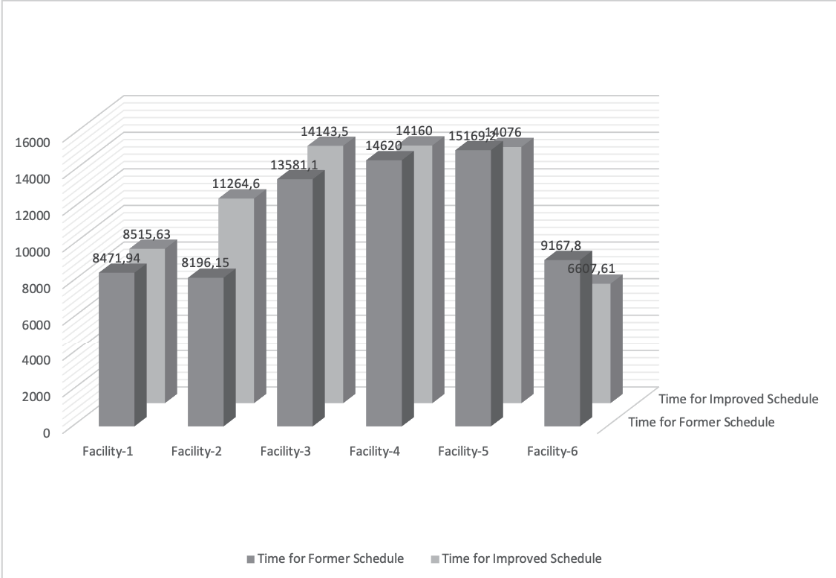

FIGURE 1. Production routes for each facility.

As a result, we can conclude that, the mathematical model developed in this paper, improves the production schedule, where a difference of 1009.2 minutes, or approximately 16.82 hours. According to the former production schedule, the factory was producing an average of 18.97 products per minute, while according to the improved production schedule, it will produce 20.32 products per minute. It is possible to produce more than 1944 boxes of products in one day, by using the proposed schedule. At the same time, when the production times of all facilities are added up, there a production time of 69206.2 minutes was obtained according to the former scheduling. This time was reduced to 68767,287 minutes with the improved new schedule. There is a reduction in the total production time of 438,913 minutes.

4. CONCLUSION

The paper presents a mathematical model for the flexible job shop scheduling problem based on a vehicle routing problem's routing structure to schedule a manufacturing system with facility flexibility and setup times. This mathematical model is used to solve a real-life problem in a food production plant with six facilities and sixty-six products. A new, improved production schedule is created, and as a result according to this production schedule, workers' working time per product is saved. At the same time, the 438,913 minutes reduction was achieved by the plant, according to the improved schedule. This new schedule will reduce the use of the natural gas, electricity, and water required in the production.

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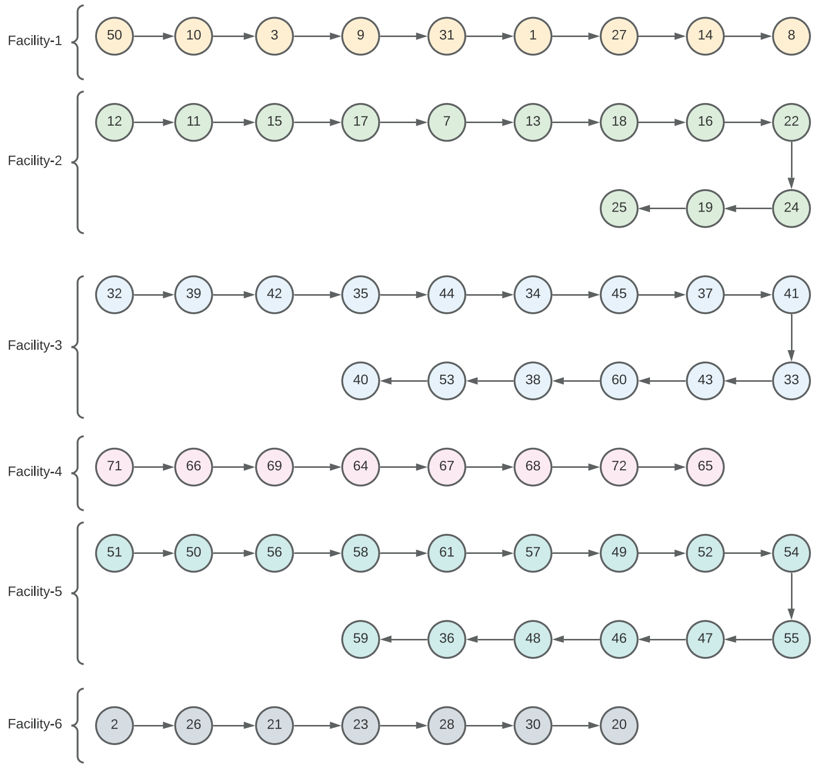

Figure 2. Production routes for each facility.

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