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NOVEL SOLITARY WAVE AND LUMP WAVE SOLUTION IN FLUID DYNAMICS

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Abstract. In this script, the central inspiration is the appliance of the unified scheme to build the soliton solution, which encompasses some controlling constraints of the standard Drinfel'd-Sokolov–Wilson (DSW) equivalence. Also unified scheme used for finding the exact solution of the standard Drinfel'd-Sokolov–Wilson (DSW) equivalence. Here the graphical solution of the equivalence represents various physical phenomena like as Lump wave, Dark Kink, Bright Kink, Singular Kink, Kink with interaction and Periodic soliton behavior. This scheme is able and highly operative mathematical tools for taking out the solution of NLPDEs in physical mathematics and all other fields of engineering.

1. Introduction

In the study of accurate results of NLPDEs shows a vital role in the inquisition of nonlinear physical spectacles like as fluid dynamics, visual strands, electrical conduction shapes, plasma physical science, artificial intelligence, engineering, physics, earth sciences, and bioinformatics and so on. In short, it is a fundamental ingredient of all modern sciences. In future, it may be focused on fluid mechanics, optimal control and biochemical problems. At present, various effective methods for gaining exact solution of NLPDEs had been introduced, like as solitary waves of the DSW equation were examined the algebraic approach [2, 8], the Darboux conversion of the DSW equation was assembled with a Lax hand [6], some management laws of the DSW scheme were gained via the multiplier technique [16] and Noether's technique [27], the double reductions of the scheme were calculated with the formulation of the relationship between balances and preservations formulas [15], the nonlocal balance and its bargains for the DSW equation were calculated by the Painlev 卒 e approaches [18],the Homogeneous balance scheme [20, 24] and the scheme of B 辰 cklund transformation [13], Hyperbolic tangent function expansion scheme [1, 22],scheme of Bifurcation [19, 21, 23], Jacobi elliptic function scheme [4, 5, 12] ,The Hirotas bilinear scheme [7] Et Cetra.
In this script, we consider the standard DSW equivalence, [14]

$$\begin{aligned} &\ell_t + pvv_x = 0, \\ &v_t + qv_{xxx} + r\ell v_x + s\ell_x v = 0, \end{aligned} \tag{1.1}$$

In the above equation p, q, r, s are some nonzero constraints. At present, The Drinfel'd-Sokolov–Wilson (DSW) equivalence and the joined Drinfel'd-Sokolov–Wilson (DSW) equivalence, a special case of DSW equation had been investigated through

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many writers [3,9–11,17,25,26]. In this investigation, we use complicated deviation of ξ defined by $\xi = kx - \omega t$, Now transfer the equation (1.1) in ODE which write

$$-\omega\ell' + pvkv' = 0, \tag{1.2}$$

$$-\omega v' + qk^3 v''' + rk\ell v' + skv\ell' = 0, \tag{1.3}$$

In the above equations the das represents the differentiation by the help of ξ Integrating (1.2) we get,

$$\ell = \frac{pkv^2}{2\omega} + h,$$

Putting the value of ℓ and ℓ' in equation (1.3) we obtain,

$$2\omega qk^3 v''' + (2\omega rhk - 2\omega^2)v' + (rpk^2 + 2psk^2)v^2 v' = 0, \tag{1.4}$$

We divide this article into five sections. The overview section is our first section. We have talked about the depiction of the strategy in the 2nd section. The 3rd section we have applied the technique in Drinfel'd-Sokolov–Wilson (DSW) equivalence or appliance. The 4th section has delivered the graphical representation and discussion section. Finally, we have given our conclusion in the 5th section.

2. Enlargement of the unified method

In this portion, we will narrate the unified method for determining different types of roaming wave results of nonlinear evolution equations. Suppose the NLPDEs in two free variables x and t, is given by

$$R(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, \dots) = 0, \tag{2.1}$$

With wave conversion:

$$u(x,t) = u(\xi), \xi = kx - \omega t, \tag{2.2}$$

Transformed Equivalence (2.1) to ordinary differential equation (ODE):

$$S(u, u', u'', u''', \dots) = 0, \tag{2.3}$$

Let the trail solution of ODE Equivalence (2.3) is the following system:

$$U(\xi) = M_0 + \sum_{i=1}^{n} [M_i F(\xi)^i + N_i F(\xi)^{-i}], \tag{2.4}$$

Where $M_i (i = 0, 1, 2, 3, \dots, n)$ and $N_i (i = 0, 1, 2, 3, \dots, n)$ are coefficients to be investigated afterward such that M_n and N_n cannot be zero at a time. Suppose an ODE namely Riccati differential equivalence:

$$F' = (F(\xi))^2 + \tau, \tag{2.5}$$

It is satisfied by $u(\xi)$. The result of the seeing Riccati differential equivalence is given below:

Case-1: Hyperbolic function solutions (When $\tau < 0$):

$$F(\xi) = \begin{cases} \dfrac{\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh\left(2\sqrt{-\tau}\,(\xi+C)\right)}{H\sinh\left(2\sqrt{-\tau}\,(\xi+C)\right)+B}, \\ \dfrac{-\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh\left(2\sqrt{-\tau}\,(\xi+C)\right)}{H\sinh\left(2\sqrt{-\tau}\,(\xi+C)\right)+B}, \\ \sqrt{-\tau} + \dfrac{-2H\sqrt{-\tau}}{H+\cosh\left(2\sqrt{-\tau}\,(\xi+C)\right)-\sinh\left(2\sqrt{-\tau}\,(\xi+C)\right)}, \\ -\sqrt{-\tau} + \dfrac{2H\sqrt{-\tau}}{H+\cosh\left(2\sqrt{-\tau}\,(\xi+C)\right)+\sinh\left(2\sqrt{-\tau}\,(\xi+C)\right)}, \end{cases} \tag{2.6}$$

Where H and B are two real random coefficients, and C arbitrary coefficients.
Case-2: Trigonometric function results (When $\tau > 0$):

$$F(\xi) = \begin{cases} \dfrac{\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos\left(2\sqrt{\tau}\,(\xi+C)\right)}{H\sin\left(2\sqrt{\tau}\,(\xi+C)\right)+B}, \\ \dfrac{-\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos\left(2\sqrt{\tau}\,(\xi+C)\right)}{H\sin\left(2\sqrt{\tau}\,(\xi+C)\right)+B}, \\ i\sqrt{\tau} + \dfrac{-2iH\sqrt{\tau}}{H+\cos\left(2\sqrt{\tau}\,(\xi+C)\right)-i\sin\left(2\sqrt{\tau}\,(\xi+C)\right)}, \\ -i\sqrt{\tau} + \dfrac{2iH\sqrt{\tau}}{H+\cos\left(2\sqrt{\tau}\,(\xi+C)\right)+i\sin\left(2\sqrt{\tau}\,(\xi+C)\right)}, \end{cases} \tag{2.7}$$

Where H and B are two real random coefficients, and C arbitrary coefficients.
Case-3: Rational function results. (When $\tau = 0$)

$$F(\xi) = -\frac{1}{\xi + C}, \tag{2.8}$$

Where $H \neq 0, B$ and C are real random coefficients. We determine the positive integer n in Equivalence (2.3) by taking into account the homogeneous balance between the highest number of derivatives and the nonlinear terms in Equivalence (2.3). Moreover, the degree of U as $D(U(\xi)) = n$ which gives the order of others expression as follows:

$$D\left(\frac{d^qU}{d\xi^q}\right) = n + q, D\left(U^p\left(\frac{d^qu}{d\xi^q}\right)^s\right) = pn + s(n+q) \tag{2.9}$$

Inserting Equivalence (2.4) into Equivalence (2.3) and making use of Equivalence (2.5) and then extracting all terms of like powers of $F(\xi)$ together, then set each coefficient of them to zero yield an over-determined system of algebraic equations and then solving this system of algebraic equations M_i $(i = 0, 1, 2, 3, \ldots, n)$, $N_i = (i = 0, 1, 2, 3, \ldots, n)$ k and ω. We obtain several sets of solutions. Finally, substituting M_i $(i = 0, 1, 2, \ldots, n), N_i$ $(i = 0, 1, 2, \ldots, n)$, k and ω into Eq. (2.4) and using the trail results of Equivalence (2.5), clear results of Equivalence (2.2) can be obtained immediately depending on the value b.

3. Appliance of the unified scheme

In the present paragraph we apply the unified scheme for equivalence (1.4) and since here the irregular term is v^2 and the maximum number of derivative is v''' .So the balance number is $n = 1$. So the result of equivalence (1.4) proceeds the following system

$$v(\xi) = M_0 + M_1.F(\xi) + N_1.\frac{1}{F(\xi)} \ , \tag{3.1}$$

Differentiating (3.1) with respect to ξ and placing the values of v, v' and v'' in equation (1.4) and comparing the coefficient of $F(\xi)^i$ equivalent to zero (where $i = 0, \pm1, \pm2 \ldots$). Explaining those systems of equivalences, we obtain the results set for the equation (1.4) is:

Set 1:

$$k = k, \quad \omega = \frac{(24\tau k^2q^2 + 3hqr)k}{3q}, \quad M_0 = 0,$$

$$M_1 = \pm 2\sqrt{-\frac{24\tau k^2q^2 + 3hqr}{pr + 2ps}}k, \quad N_1 = \mp 2\sqrt{-\frac{24\tau k^2q^2 + 3hqr}{pr + 2ps}}k\tau,$$

Set 2:

$$k = k, \quad \omega = \frac{(-12\tau k^2q^2 + 3hqr)k}{3q}, \quad M_0 = 0,$$

$$M_1 = \pm 2\sqrt{-\frac{-12\tau k^2q^2 + 3hqr}{pr + 2ps}}k, \quad N_1 = \pm 2\sqrt{-\frac{-12\tau k^2q^2 + 3hqr}{pr + 2ps}}k\tau,$$

Set 3:

$$k = k, \quad \omega = \frac{1}{3}\frac{p(r + 2s)(6\tau k^2q^2 + 3hqr)k}{(pr + 2ps)q}, \quad M_0 = 0,$$

$$M_1 = \pm 2\sqrt{-\frac{6\tau k^2q^2 + 3hqr}{pr + 2ps}}k, \quad N_1 = 0,$$

Set 4:

$$k = k, \quad \omega = \frac{(6\tau k^2q^2 + 3hqr)k}{3q}, M_0 = 0,$$

$$M_1 = 0, \quad N_1 = \pm 2\sqrt{-\frac{6\tau k^2q^2 + 3hqr}{pr + 2ps}}k\tau,$$

Case I: Hyperbolic function solutions (When $\tau < 0$):

Family 1:

$$v_{1,2}(x,t) = \pm\frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k(\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi + C)))}{H\sinh(2\sqrt{-\tau}(\xi + C)) + B}$$
$$- \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sinh(2\sqrt{-\tau}(\xi + C)) + B)}{\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi + C))},$$

$$v_{3,4}(x,t) = \pm\frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k(-\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C)))}{H\sinh(2\sqrt{-\tau}(\xi+C))+B}$$
$$-\frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sinh(2\sqrt{-\tau}(\xi+C))+B)}{-\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C))},$$

$$v_{5,6}(x,t) = \pm 2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(\sqrt{-\tau} - \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C)) - \sinh(2\sqrt{-\tau}(\xi+C))})$$
$$-\frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{\sqrt{-\tau} - \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))-\sinh(2\sqrt{-\tau}(\xi+C))}},$$

$$v_{7,8}(x,t) = \pm 2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(-\sqrt{-\tau} + \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C)) + \sinh(2\sqrt{-\tau}(\xi+C))})$$
$$-\frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{-\sqrt{-\tau} + \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))+\sinh(2\sqrt{-\tau}(\xi+C))}},$$

$$v_{9,10}(x,t) = \pm\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k(\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C)))}{H\sinh(2\sqrt{-\tau}(\xi+C))+B}$$
$$+\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sinh(2\sqrt{-\tau}(\xi+C))+B)}{\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C))},$$

$$v_{11,12}(x,t) = \pm\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k(-\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C)))}{H\sinh(2\sqrt{-\tau}(\xi+C))+B}$$
$$+\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sinh(2\sqrt{-\tau}(\xi+C))+B)}{-\sqrt{-(H^2+B^2)\tau} - H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C))},$$

$$v_{13,14}(x,t) = \pm 2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(\sqrt{-\tau} - \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))-\sinh(2\sqrt{-\tau}(\xi+C))})$$
$$+\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{\sqrt{-\tau}-\frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))-\sinh(2\sqrt{-\tau}(\xi+C))}},$$

$$v_{15,16}(x,t) = \pm 2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(-\sqrt{-\tau} + \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))+\sinh(2\sqrt{-\tau}(\xi+C))})$$
$$+\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{-\sqrt{-\tau}+\frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))+\sinh(2\sqrt{-\tau}(\xi+C))}},$$

$$v_{17,18}(x,t) = \pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k(\sqrt{-(H^2+B^2)\tau}-H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C)))}{H\sinh(2\sqrt{-\tau}(\xi+C))+B},$$

$$v_{19,20}(x,t) = \pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k(-\sqrt{-(H^2+B^2)\tau}-H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C)))}{H\sinh(2\sqrt{-\tau}(\xi+C))+B},$$

$$v_{21,22}(x,t) = \pm 2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(\sqrt{-\tau} - \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))-\sinh(2\sqrt{-\tau}(\xi+C))}),$$

$$v_{23,24}(x,t) = \pm 2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(-\sqrt{-\tau} + \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))+\sinh(2\sqrt{-\tau}(\xi+C))}),$$

$$v_{25,26}(x,t) = \pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sinh(2\sqrt{-\tau}(\xi+C))+B)}{\sqrt{-(H^2+B^2)\tau}-H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C))},$$

$$v_{27,28}(x,t) = \pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sinh(2\sqrt{-\tau}(\xi+C))+B)}{-\sqrt{-(H^2+B^2)\tau}-H\sqrt{-\tau}\cosh(2\sqrt{-\tau}(\xi+C))},$$

$$v_{29,30}(x,t) = \pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{\sqrt{-\tau}-\frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))-\sinh(2\sqrt{-\tau}(\xi+C))}},$$

$$v_{31,32}(x,t) = \pm \frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{-\sqrt{-\tau} + \frac{2H\sqrt{-\tau}}{H+\cosh(2\sqrt{-\tau}(\xi+C))+\sinh(2\sqrt{-\tau}(\xi+C))}},$$

Case II: Trigonometric function solutions (When $\tau > 0$):
Family 2:

$$v_{33,34}(x,t) = \pm \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k(\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C)))}{H\sin(2\sqrt{\tau}(\xi+C))+B}$$
$$- \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sin(2\sqrt{\tau}(\xi+C))+B)}{\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C))},$$

$$v_{35,36}(x,t) = \pm \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k(-\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C)))}{H\sin(2\sqrt{\tau}(\xi+C))+B}$$
$$- \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sin(2\sqrt{\tau}(\xi+C))+B)}{-\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C))},$$

$$v_{37,38}(x,t) = \pm 2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(I\sqrt{\tau} - \frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C)) - I\sin(2\sqrt{\tau}(\xi+C))})$$
$$- \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{I\sqrt{\tau} - \frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))-I\sin(2\sqrt{\tau}(\xi+C))}},$$

$$v_{39,40}(x,t) = \pm 2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(-I\sqrt{\tau} + \frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C)) + I\sin(2\sqrt{\tau}(\xi+C))})$$
$$- \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{-I\sqrt{\tau} + \frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))+I\sin(2\sqrt{\tau}(\xi+C))}},$$

$$v_{41,42}(x,t) = \pm \frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k(\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C)))}{H\sin(2\sqrt{\tau}(\xi+C))+B}$$
$$+ \frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sin(2\sqrt{\tau}(\xi+C))+B)}{\sqrt{(H^2-B^2)\tau} - H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C))},$$

$$v_{43,44}(x,t)=\pm\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k(-\sqrt{(H^2-B^2)\tau}-H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C)))}{H\sin(2\sqrt{\tau}(\xi+C))+B}$$
$$+\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sin(2\sqrt{\tau}(\xi+C))+B)}{-\sqrt{(H^2-B^2)\tau}-H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C))},$$

$$v_{45,46}(x,t)=\pm 2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(I\sqrt{\tau}-\frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))-I\sin(2\sqrt{\tau}(\xi+C))})$$
$$+\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{I\sqrt{\tau}-\frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))-I\sin(2\sqrt{\tau}(\xi+C))}},$$

$$v_{47,48}(x,t)=\pm 2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(-I\sqrt{\tau}+\frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))+I\sin(2\sqrt{\tau}(\xi+C))})$$
$$+\frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{-I\sqrt{\tau}+\frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))+I\sin(2\sqrt{\tau}(\xi+C))}},$$

$$v_{49,50}(x,t)=\pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k(\sqrt{(H^2-B^2)\tau}-H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C)))}{H\sin(2\sqrt{\tau}(\xi+C))+B},$$

$$v_{51,52}(x,t)=\pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k(\sqrt{-(H^2-B^2)\tau}-H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C)))}{H\sin(2\sqrt{\tau}(\xi+C))+B},$$

$$v_{53,54}(x,t)=\pm 2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(I\sqrt{\tau}-\frac{2HI\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))-I\sin(2\sqrt{\tau}(\xi+C))}),$$

$$v_{55,56}(x,t)=\pm 2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}$$
$$k(-I\sqrt{\tau}+\frac{2HI\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))+I\sin(2\sqrt{\tau}(\xi+C))}),$$

$$v_{57,58}(x,t)=\pm\frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sin(2\sqrt{\tau}(\xi+C))+B)}{\sqrt{(H^2-B^2)\tau}-H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C))},$$

$$v_{59,60}(x,t) = \pm \frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau(H\sin(2\sqrt{\tau}(\xi+C))+B)}{-\sqrt{(H^2-B^2)\tau}-H\sqrt{\tau}\cos(2\sqrt{\tau}(\xi+C))},$$

$$v_{61,62}(x,t) = \pm \frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{I\sqrt{\tau}-\frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))-I\sin(2\sqrt{\tau}(\xi+C))}},$$

$$v_{63,64}(x,t) = \pm \frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k\tau}{-I\sqrt{\tau}+\frac{2IH\sqrt{\tau}}{H+\cos(2\sqrt{\tau}(\xi+C))+I\sin(2\sqrt{\tau}(\xi+C))}},$$

Case III: Rational function solutions. (When $\tau = 0$):
Family 3:

$$v_{65,66}(x,t) = \mp \frac{2\sqrt{-\frac{24\tau k^2q^2+3hqr}{pr+2ps}}k}{\xi+C},$$

$$v_{67,68}(x,t) = \mp \frac{2\sqrt{-\frac{-12\tau k^2q^2+3hqr}{pr+2ps}}k}{\xi+C},$$

$$v_{69,70}(x,t) = \mp \frac{2\sqrt{-\frac{6\tau k^2q^2+3hqr}{pr+2ps}}k}{\xi+C},$$

4. Outcomes and consideration

In the present part, we will deliberate the physical clarification and graphical demonstration of the gained particular and solitary wave result of Drinfel'd-Sokolov–Wilson (DSW) equivalence.

4.1. **Physical Clarification.** In the present unit, using the unified scheme the DSW equivalence affords particular roaming wave results. The results

$$v_{1,2}(x,t), v_{3,4}(x,t), v_{5,6}(x,t), v_{7,8}(x,t),$$
$$v_{9,10}(x,t), v_{11,12}(x,t), v_{13,14}(x,t), v_{15,16}(x,t)$$

$$v_{17,18}(x,t), v_{19,20}(x,t), v_{21,22}(x,t), v_{23,24}(x,t),$$
$$v_{25,26}(x,t), v_{27,28}(x,t), v_{29,30}(x,t), v_{31,32}(x,t)$$

all are trigonometric hyperbolic function results.

$$v_{33,34}(x,t), v_{35,36}(x,t), v_{37,38}(x,t), v_{39,40}(x,t),$$
$$v_{41,42}(x,t), v_{43,44}(x,t), v_{45,46}(x,t), v_{47,48}(x,t)$$

$$v_{49,50}(x,t), v_{51,52}(x,t), v_{53,54}(x,t), v_{55,56}(x,t),$$
$$v_{57,58}(x,t), v_{59,60}(x,t), v_{61,62}(x,t), v_{63,64}(x,t)$$

all are trigonometric function results. And rational function result is $v_{65,66}(x,t)$, $v_{67,68}(x,t)$, $v_{69,70}(x,t)$. The explanation of $v_{17}(x,t)$ is a composite system and the structure appearances in imaginary system which characterizes in Fig. 1. Which looks like the Dark Kink kind of particular roaming wave result with

$$b = -2, p = 1, k = 1, q = 1, h = 5, r = 1, s = 1, A = 1, B = 1, C = 1$$

within the movements $-10 \leq x, t \leq 10$. And the result of $v_{19}(x,t)$ is a composite system and the structure shows in imaginary form which symbolizes in Fig. 2. It appearances the Singular-Kink form particular roaming wave result with

$$b = -2, p = 1, k = 1, q = 1, h = 5, r = 1, s = 1, A = 1, B = 1, C = 1$$

within the movements $-10 \leq x, t \leq 10$. Then the result of $v_{27}(x,t)$ is a multipart form and the structure shows in imaginary form which characterizes in Fig. 3. which looks like the Bright Kink style particular roaming wave explanation with

$$b = -2, p = 1, k = 1, q = 1, h = 5, r = 1, s = 1, A = 1, B = 1, C = 1$$

within the shifts $-10 \leq x, t \leq 10$. So the clarification of $v_{29}(x,t)$ is a multipart system and the structure shows in imaginary form which symbolizes in Fig. 4. It forms the Kink with interaction form particular roaming wave result with

$$b = -2, p = 1, k = 1, q = 1, h = 5, r = 1, s = 1, A = 1, B = 1, C = 1$$

within the movements $-10 \leq x, t \leq 10$. And the result $v_{56}(x,t)$ is a composite form and the structure displays in imaginary form which signifies in Fig. 5. It appearances the Periodic-style form exact roaming wave result with

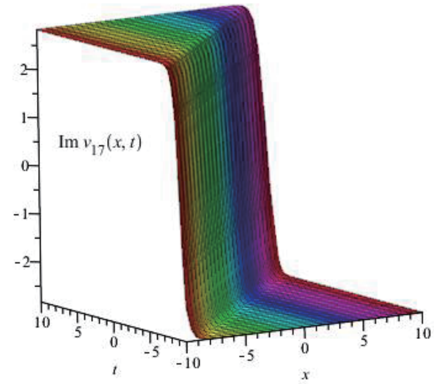
$$b = 2, p = 1, k = 1, q = 1, h = 5, r = 1, s = 1, A = 1, B = 1, C = 1$$

within the shifts $-10 \leq x, t \leq 10$. And the result of $v_{67}(x,t)$ is a multipart system and the structure confirmations in real form which indicates in Fig. 6. It forms the Lump wave-kind exact roaming wave result with $b = 0, p = 2\sqrt{-3}, k = 1, q = 1, h = 0.5, r = \sqrt{-9}, s = 1, A = 1, B = 1, C = 1$ within the movements $-10 \leq x, t \leq 10$. Finally the solution of $v_{69}(x,t)$ is a composite system and the structure shows in imaginary form which symbolizes in Fig. 7. It presences the Lump wave form exact roaming wave result with

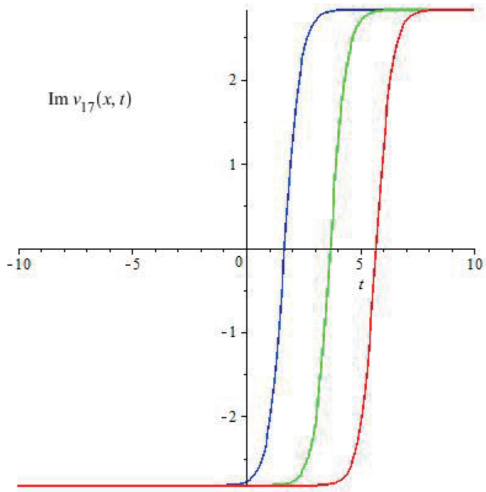
$$b = 0, p = \sqrt{-3}, k = 1, q = 1, h = 0.5, r = \sqrt{-9}, s = 1, A = 1, B = 1, C = 1$$

within the shifts $-10 \leq x, t \leq 10$.

4.2. **Graphical Explanation.** In this segment, we will discuss the achieved solutions by graphically with 3D, 2D and density plots. For the diverse condition on the parameters, the solutions are expressed as complex and real valued function form.



[3D]



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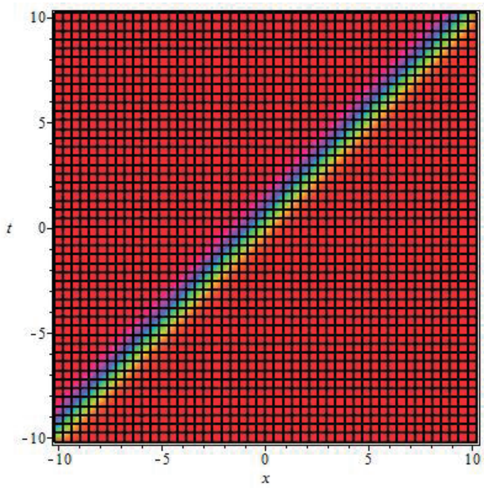
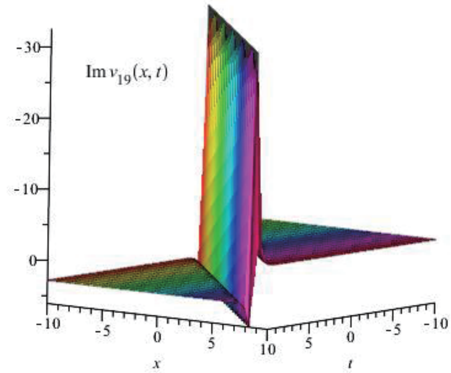


[Density]

FIGURE 1. Structure of $v_{17}(x,t)$for imaginary form.The figure 3D form, 2D form and Density plot are depicted for t=1.



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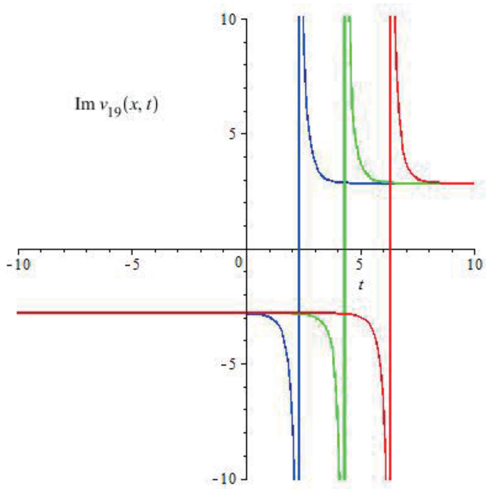
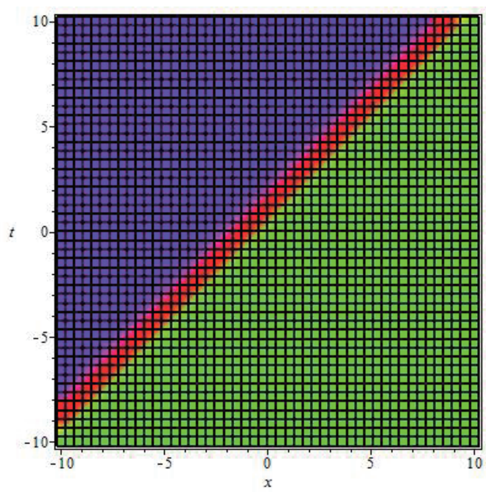
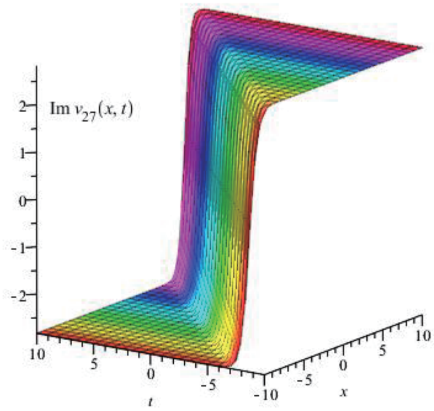


[2D]



[Density]

FIGURE 2. Structure of $v_{19}(x, t)$for imaginary form.The figure 3D form, 2D form and Density plot are depicted for t=1.



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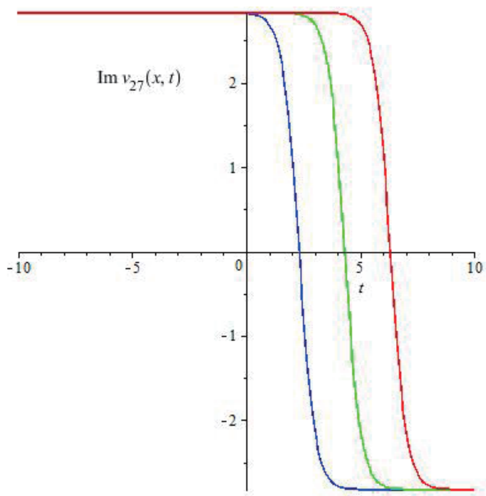

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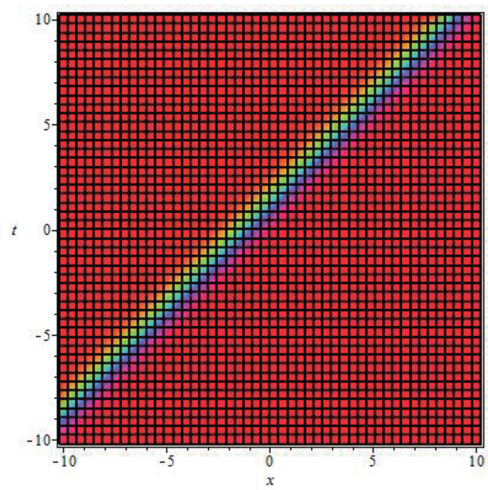
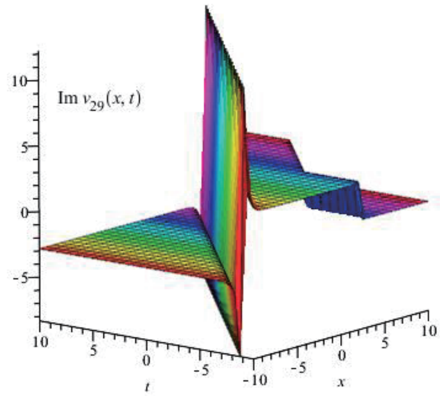


[Density]

FIGURE 3. Structure of $v_{27}(x,t)$ for imaginary form.The figure 3D form, 2D form and Density plot are depicted for t=1.



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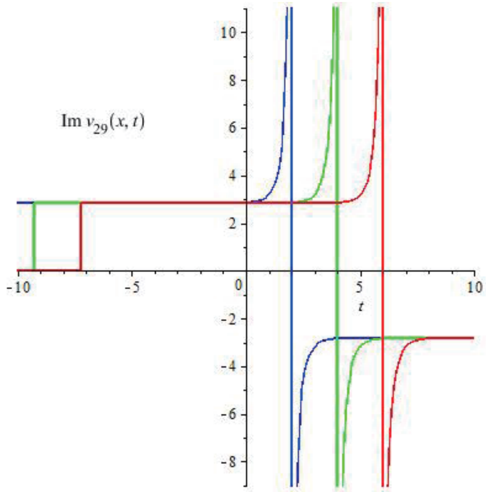

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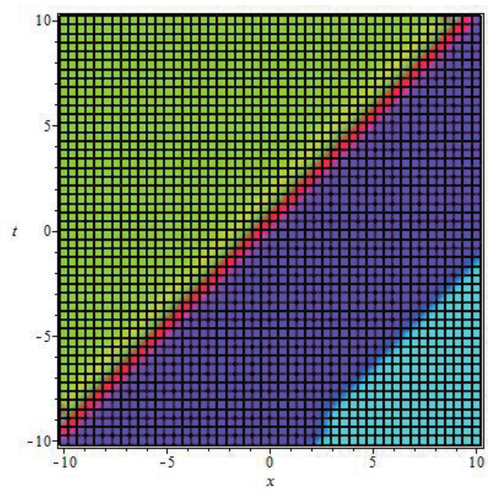
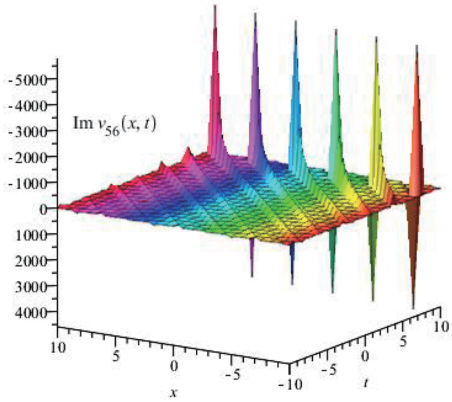


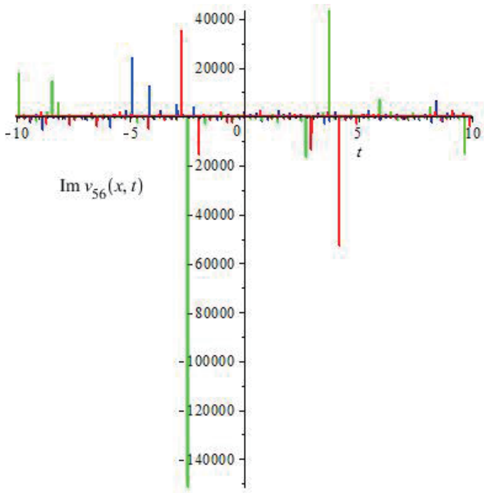
[Density]

. .

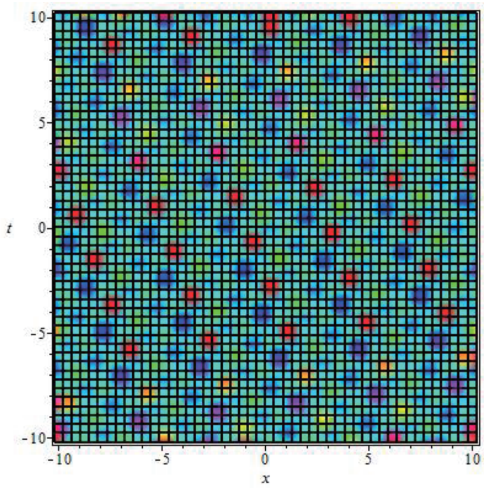
FIGURE 4. Structure of $v_{29}(x,t)$ for imaginary form.The figure 3D form, 2D form and Density plot are depicted for t=1.



[3D]

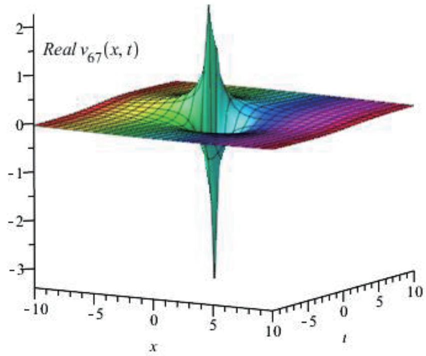


[2D]

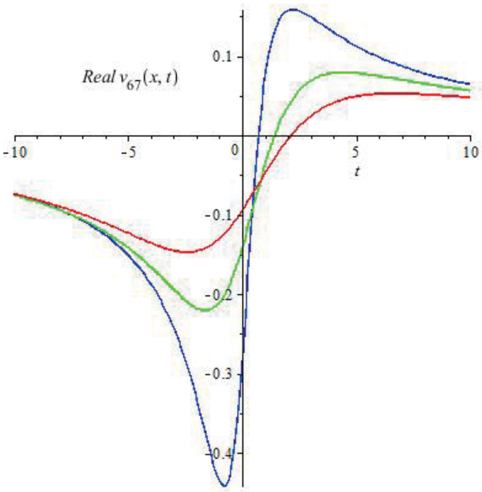


[Density]

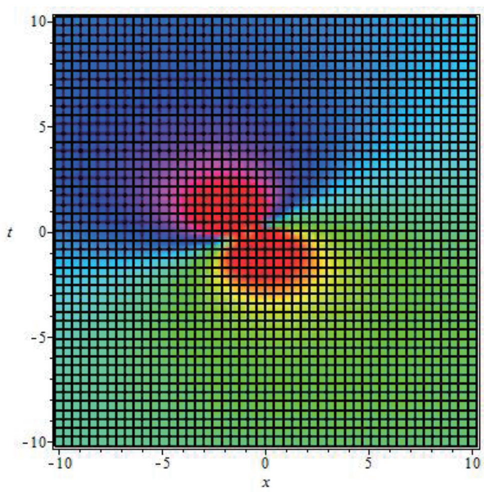
FIGURE 5. Structure of $v_{56}(x,t)$ for imaginary form.The figure 3D form, 2D form and Density plot are depicted for t=1.



[3D]

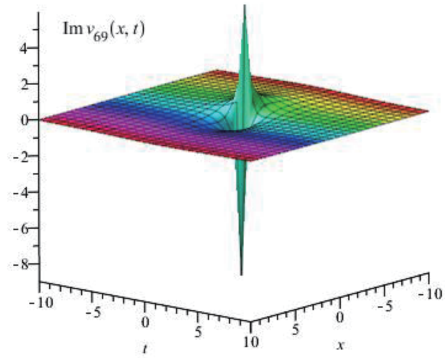


[2D]

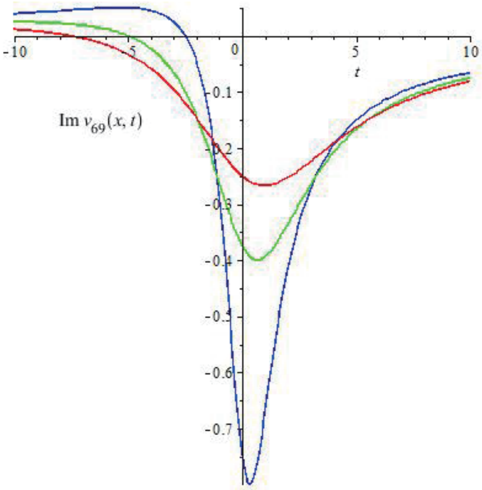


[Density]

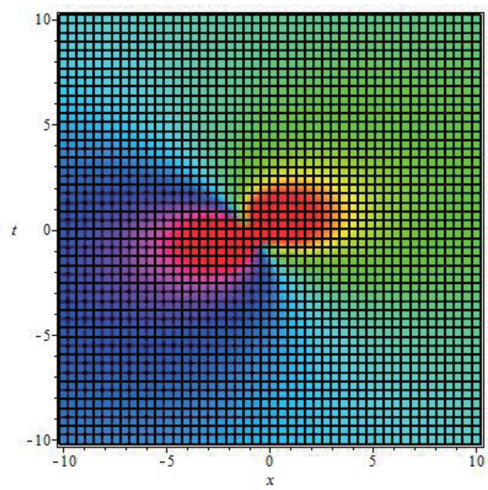
FIGURE 6. Structure of $v_{67}(x,t)$ for real form.The figure 3D form, 2D form and Density plot are depicted for t=1.



[3D]



[2D]



[Density]

FIGURE 7. Structure of $v_{69}(x, t)$for imaginary form.The figure 3D form, 2D form and Density plot are depicted for t=1.

5. Conclusion

In this article, we applied the unified method successfully and this method gives some novel particular roaming wave results of various mathematical and physical graphs for specific unrestricted constraints. For some special constraints we have some special types of displays from this equation. Finally, we think that this scheme is a controlling and straightforward mathematical tool for gaining exact roaming wave results and the present scheme can be applied to more kinds of nonlinear problems or equations.

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